

## **An Introduction to the Mechanical Properties of Ceramics**

This book is a comprehensive introduction to the mechanical properties of ceramics, and is designed primarily as a textbook for undergraduate and graduate students in materials science and engineering.

Over the past 25 years ceramics have become key materials in the development of many new technologies as scientists have been able to design these materials with new structures and properties. An understanding of the factors that influence their mechanical behavior and reliability is essential. Some of these new applications are structural, and for these it is important to understand the factors that control their mechanical behavior. Non-structural applications are also being developed, but in each case it is necessary to design mechanically reliable materials. This is a particular challenge for materials that are inherently brittle. This book will introduce the reader to current concepts in the field. It contains problems and exercises to help readers develop their skills.

Although designed principally as a textbook for advanced undergraduates and graduate students, this book will also be of value as a supplementary text for more general courses and to industrial scientists and engineers involved in the development of ceramic-based products, materials selection and mechanical design.

# An Introduction to the Mechanical Properties of Ceramics

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The book is dedicated to the memory of  
Christina Cushing Green

# An Introduction to the Mechanical Properties of Ceramics

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The Pennsylvania State University



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## Preface

The aim of this book is to provide a text for a senior undergraduate course on the mechanical behavior of ceramics. There are, however, some advanced sections that would allow the book to be used at the graduate level (marked ††). The format of the book owes much to the text, *Mechanical Properties of Matter*, by A. H. Cottrell, which helped me through graduate school. In teaching a course in this area, it has always been frustrating that there are so few texts aimed primarily at ceramics. There is often the concern of discerning whether ideas applied to other materials could also be used to understand ceramic materials. I have also been fortunate in being involved in the field of structural ceramics at a time it has undergone remarkable developments and I have tried to incorporate my interpretation of these recent advances into the text.

I would be amiss in not acknowledging the support I have received in undertaking this project. I owe much to Pat Nicholson, Dave Embury and Dick Hoagland, who patiently introduced me to this field of research and to Tom Wheat, who taught me about the processing of ceramics. I am particularly grateful to Fred Lange, who took a chance on me and became my mentor. His enthusiastic, intuitive advice and sense of fun encouraged me to pursue many new ideas. I also appreciate the interaction with my other colleagues at Rockwell International Science Center. The undergraduates at Penn State in Ceramic Science and Engineering should be acknowledged for suffering through the various versions of this book. I should particularly thank Fred Fitch for patiently pointing out many typographical errors in an earlier version of the book. Thanks are also owed to Brian Watts and Patty Phillips for their patient proof-reading skills. I would also thank George Scherer, David Clarke, David

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