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R. A. Bailey
Queen Mary and Westfield College, University of London

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Preface

The 1997 issue of the British Combinatorial Bulletin contains a short history, written by Norman Biggs, of the early years of the British Combinatorial Conference. The first one was held at Oxford in 1969. The sixth conference, held at Royal Holloway College in 1977, was the first at which a volume containing the invited talks was published in time to be available to participants at the conference. Peter Cameron was the pioneering editor of that volume. Such a volume has been produced for every conference thereafter.

The 1977 conference was also the first one that I attended. There I joined the British Combinatorial Committee, which was formally set up at that meeting although it had effectively existed for some years—the previous conferences didn't just organize themselves. As often happens, I found that being on the committee considerably widened my knowledge of the subject. I left the committee in 1981, but have never lost touch with combinatorial activity in Britain.

I was delighted when I was asked to edit the present volume. In spite of the work involved, I am still delighted. I have had a preview of nine magnificent papers, and come to know their subject matter much better than I would otherwise have done.

At the centre of this volume is a long paper by Bruce Reed about the tree width of graphs. This is a new measure of connectivity. It is intimately linked to the concept of a minor of a graph, which is obtained by erasing an edge or coalescing two vertices joined by an edge, or by a sequence of such operations. Although the idea of 'forbidden minors' was made famous by Kuratowski's characterization of planar graphs in 1930, the main theoretical development of graph minors has taken place over the last decade, led by Neil Robertson and Paul Seymour. At the 1985 British Combinatorial Conference in Glasgow, Seymour talked about the early stages of this work, hot off the press. Now Reed, who has himself been one of the contributors to the area, gives this splendid survey of exceptionally deep, interesting and valuable work, some of the most important work ever in graph theory. It shows that tree width and minors give a rich mathematical theory to graphs in a way that the more obvious concepts of k-connectivity and induced subgraphs do not. It is a long paper for a conference proceedings, but it is a magnificent exposition of a major piece of work. It is laid out in such a way that the non-specialist can read it with ease, while at the same time containing proofs of the important results. This should become the definitive paper on the topic.

Alexander Schrijver's paper is a natural accompaniment to Reed's, because it too is concerned in part with graph minors. He describes some new and intriguing graph parameters which are non-decreasing upon taking minors. One of these, introduced by Colin de Verdière in 1990, gives another characterization of planarity but also has wider applications. A related parameter was introduced by Schrijver and co-workers in 1995. Colin de Verdière's invariant was suggested by ideas from differential geometry; most of the related invariants are more obviously combinatorial. Schrijver gives simpler proofs than those in the literature and poses interesting open questions about the relationship between these parameters (and others).

Perhaps the most famous graph parameters are the chromatic number and chromatic index, the smallest number of colours with which the graph can be properly vertex-coloured and edge-coloured respectively. In a proper vertex-colouring, the colours on the the ends of an edge must be different. Natural restrictions on such a colouring are to demand that each unordered pair of colours is used on at most one edge or at least one edge. These give harmonious and complete colourings respectively. Keith Edwards gives a clear and interesting survey of work on the parameters associated with these two types of colouring, some of it extremely recent. The paper will form a very useful background for further research in this area.

One use of graphs is as tool to study partially ordered sets (posets). In a poset, for each pair x and y of distinct elements, either $x < y$ or $y < x$ or x and y are incomparable. The comparability graph of the poset is obtained by joining x to y whenever x and y are comparable. Some properties of posets are constant over all posets with the same comparability graph—comparability invariant.

In applications such as scheduling of tasks or dating archaeologicial finds, the basic objects are intervals on the real line. These can be turned into posets by considering when the whole of one interval is to the left of the whole of another. Thus begins the theory of interval orders. Or they can be turned into graphs—interval graphs—by joining any two intervals whose intersection is not empty. Tom Trotter has contributed extensively to the theory of posets and its links with graph theory. Here he gives an excellent survey of the topics of interval orders and interval graphs.

Although it is also about graphs, Cheryl Praeger's paper has quite a different focus. Her motivation is groups of automorphisms of graphs. She considers highly symmetric graphs, those which are at least vertex-transitive (i.e. which admit groups of automorphisms which are transitive on the vertices.) Often she demands that the graph be 2-arc transitive, which means that its automorphism group is transitive on ordered paths of length two. A well-studied stronger property than transitivity is primitivity: a group acts primitively if it preserves no non-trivial partition.

Praeger defines a quasiprimitive group action as a natural generalization of a primitive group action, and defines a graph to be quasiprimitive if it admits a quasiprimitive group of automorphisms. Surveying work in which she and her collaborators have played a major role, she gives a complete categorization of quasiprimitive groups into eight types, parallel to the categorization given for primitive groups by the celebrated O'Nan–Scott theorem. She summarizes what has been done so far towards the classification of quasiprimitive graphs, especially those that are 2-arc transitive. This is an excellent example of how advances in permutation group theory following the classification of finite simple groups have made feasible such classifications of graphs. This is a very clear account of work still in progress.

Of course, automorphisms are a natural tool for studying all kinds of combinatorial structures, not just graphs. Sometimes we want to find all structures admitting an automorphism group with certain properties, as in Praeger's paper. At the other extreme, sometimes we study one particular example of a combinatorial structure precisely because its automorphisms are so interesting. John Conway's paper takes this point of view. It presents a new construction of the Mathieu group M_{12}, one of the simplest available.

Most constructions of sporadic simple groups such as M_{12} either produce some combinatorial structure (Steiner system, Hadamard matrix, code, ...), show that it is unique and deduce properties of its automorphism group (so that the actual automorphisms are not easy to construct), or else produce the permutations directly in such a way that it is hard to see the combinatorial structure. Conway's new construction takes the latter approach—all the permutations are produced at the outset—but the combinatorial properties are surprisingly easy to verify. Indeed, the exposition is skilfully constructed: once the descriptive parts of the paper have been read, the proofs are almost obvious.

A byproduct of this approach is the result that there exists a sharply 6-transitive set of permutations of 13 objects. They are the permutations of 12 counters and one hole in an analogue of the 15-puzzle played on the 13-point projective plane. This result will be interesting to researchers on extremal combinatorics, specifically, metric properties of sets of permutations.

Clement Lam's use of automorphisms is more conventional. The setting is 2-designs, also known as as balanced incompete-block designs, or just block designs—collections of k-subsets (called blocks) of a v-set with the property that every 2-subset is contained in the same number of blocks. Here is a very useful article about the use of the BDX program (demonstrated at the conference) to search for block designs. The reader is taken through an extended worked example—the search for designs which have a certain set of parameters and which admit an automorphism of order seven with no fixed points or blocks—and sees details of calculations which are not usually provided in published articles on mathematics. The reader is led so gently through the example that (s)he should have no difficulty in trying out BDX alone.

One of the classical sources of block designs is projective geometry: the points are the points of the geometry and the blocks are subspaces or conics, possibly with additional points. Thus we are led to questions in geometry such as: how may points are in the intersection of an object of one sort with an object of another sort? What size can a minimal blocking set be? (A blocking set is a set of points which meets every line.) Tamás Szőnyi's paper is a wide-ranging and detailed survey of such questions. It shows many important applications of two important results—Weil's theorem and Segre's lemma of tangents. It updates the material on blocking sets reported by Aart Blokhuis at the 1993 British Combinatorial Conference at Keele. A variety of results is described. The flavour of the proofs is given, rather than their technicalities, and further probable developments are discussed.

The papers by Edwards, Reed, Schrijver and Trotter all refer to problems in computational complexity. A typical problem is the following: given a graph G with n vertices, can we decide whether G is Hamiltonian? More specifically, is there an algorithm that will decide the answer within time that is linear in n, or polynomial in n, or otherwise bounded by some function of n? During the past decade or so a new type of complexity problem has been considered: not "can we?" but "how many?" and not "exactly" but "approximately". A typical example now is: given a graph G with n vertices, how many forests does G contain as induced subgraphs? Here "how many" means to within an order of magnitude. Once again one wants to know what sort of function of n bounds the running time for an algorithm that answers the question. Dominic Welsh's paper provides a fascinating survey of the very interesting material in this area, and thus rounds off this volume of papers.

In 1977 the task of the editor was to obtain typescripts from the speakers, by a deadline, and to submit camera-ready copy to the publisher. Over the twenty intervening years, what has changed? The deadlines are still there, and so is a publisher, even though Cambridge University Press has replaced Academic Press. Typescripts, with mathematical symbols either hand-written or typed from 'golf-balls', have vanished, being replaced first by word-processors then by mathematical type-setting packages such as TEX. As this happened, camera-ready copy at first became more diverse then converged as mathematicians came to agree on the use of a few systems. These proceedings are, I believe, the first in this series for which the editor has made a serious attempt at uniformity by asking all the authors to use the same system and giving them a style file to encourage them.

The authors have cooperated marvellously, often to tight schedules. Although they had not all used LATEX before, they all submitted their papers in fairly standard LATEX, using a rather simple style file which I had provided. My heartfelt thanks to them all.

None of this would have been possible without TEX, specifically the current version of LATEX, which is designed for people like me who are fussy about what their written mathematics looks like but do not want to program anything much more complicated than 'this is the end of the statement of the theorem'. So thanks to Donald Knuth for giving TEX to the world; to Leslie Lamport, for the original version of LATEX and for the manual which taught me to think in terms of generic mark-up; and to the LATEX3 team, led by Frank Mittelbach and Chris Rowley, for upgrading and maintaining LATEX, for incorporating features that I asked for, and for answering lots of my questions.

The other big change over the twenty years is the arrival of electronic mail. All the papers were submitted by email. Almost all of them were sent on to referees by email, and reports came back by the same route. During the editing phase the myriad queries such as 'Do you mean `\log` or `\ln` there?' were dispatched and answered promptly, all by email. Without email it would simply have been impossible to do the task within the given time-scale. I am grateful to the authors and the referees (as well as various people who helped me with the names of journals and books unfamiliar to me) for dealing with everything as quickly as they did.

In editing these proceedings I have tried to strike a fine balance between respecting the authors' wishes about how their mathematics is presented and providing the uniformity of layout that, by melting into the background, enables the reader to concentrate on the content and so read it more easily. If I have done my job well its effects should be invisible.

Finally, thanks to numerous people in the School of Mathematical Sciences at Queen Mary and Westfield College. Not only have they provided hardware, software, advice and moral support. They have been very patient with me while other jobs have been left on one side as I worked on these proceedings. This is particularly true of the other members of the local organizing team of this British Combinatorial Conference—Peter Cameron, Leonard Soicher and Shirley Wilkinson.

Rosemary A. Bailey
School of Mathematical Sciences
Queen Mary and Westfield College
Mile End Road
London E1 4NS
r.a.bailey@qmw.ac.uk

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