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Symmetries and Integrability of Difference Equations

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Contents

6 Symmetries of Difference Equations

7 Numerical Methods and Miscellaneous

8 Cellular Automata

Preface

The second international conference on "Symmetries and Integrability of Difference Equations" (SIDE II) was held at the University of Kent at Canterbury, July 1-5, 1996. It was the successor of a first meeting on the same topic held in Estérel (Québec, Canada), under the auspices of the Centre de Recherches Mathématiques (CRM) of the University of Montréal in 1994, cf. the Proceedings of that meeting, [1]. Like in Estérel, this SIDEII meeting aimed at bringing together researchers working in the general field of discrete systems and difference equations with emphasis on symmetries and integrability.

The subject area of the meeting is relatively young: in the last decade, and particularly during the last few years, a great deal of progress has been made on the mathematical aspects of discrete integrable systems, including integrable dynamical mappings, ordinary and partial difference equations, lattice solitons, discrete versions of the Painlevé equations, symmetry approaches and singularity analysis, and applications to numerical analysis, computer science and Physics. The two SIDE meetings have brought together many leading experts in the various aspects of this field, coming from quite different backgrounds. As such the interdisciplinary nature of the meeting is reflected in the present volume, marking contributions in a large variety of fields.

It is important to give some explanation as to why we believe that combining the efforts in the fields of discrete systems is significant, especially in this day and age. From a general perspective, research into discrete systems has greatly lagged behind some other fields, notably the far more developed theories of ordinary and partial differential equations. One should not forget, of course, the ambitious programme of the Birkhoff school at the beginning of the century, to develop the analytic aspects of linear difference equations (cf. [8]). The great tradition of the early 1900's, with the works of Painlevé and his school [19, 20, 21, 22], dealing with the classification of ordinary differential equations, was in line with the spirit of developing analogous theories for difference equations. Furthermore, a vast amount of knowledge, dating back to the classic works of the 19th century and earlier, has been accumulated, mostly relating to finite-element methods and the classic techniques of numerical analysis (cf. [9]). It is somehow tragic that this tradition doesn't seem to have been continued in recent years, with the notable exceptions of the fields of q-special functions and orthogonal polynomials (cf. [23, 24, 25, 26]).

From the point of view of algebra and geometry, symmetry approaches have been at the heart of the development of effective methods for integrating differential equations (cf. [12, 13, 14]). However, no comprehensive theory of symmetry methods exists to date for difference equations, although new impetus has been given to such a quest in recent years (cf. [15]). It is only now that symmetry reduction techniques, which have nowadays become so very much a standard tool in the analysis of partial differential equations, are being developed for difference and differential-difference equations.

One may ask what is the reason for this difference in development between the comparable theories for differential equations on the one hand and difference equations on the other hand. One explanation is probably the lack of motivation. Whereas in the "classical era", mathematicians such as Birkhoff naturally considered difference equations alongside the "allied" theories of (continuous) differential equations, after the second world war mathematical theories were dominated by continuum models. Thus there was no longer as much interest in studying the admittedly more difficult theory of difference equations. Also in physics, there has been an overall domination of continuum models to describe the processes of nature. It is understandable from one perspective: the continuum is the state of matter as it appears to us in our daily observation of nature, and in order to capture the smoothness of all movement and the coherence of all material we need the mathematical tools of differential calculus. This point of view reached its cumulation in theoretical high energy physics, especially in the fifties and sixties, with the significant developments of relativistic quantum field theory, a continuum theory *in extremis*, which was expected to provide the basis for the description for the fundamental forces in nature. However, it is known now that this theory has severe limitations exactly because of the difficulties of uniting on the one hand quantum mechanics, which is inherently an algebraic and discrete theory, with the notions of continuum functions on the other. One step has been the development of string theory, and more recently the theory of membranes, where in a partial fashion the pointwise structure of quantum fields has been generalised by allowing extended objects as our fundamental building blocks of nature. But even this may not be enough and we should contemplate taking the bold step of acknowledging the assumption that at the quantum mechanical level the continuum description of space-time actually is no longer valid and that an inherently discrete description should take precedence. This point of view, revolutionary as it might seem, has actually been proposed on many occasions (cf. [2, 3]), and there have been recent attempts to put these ideas into practice (cf. [7]). From a general perspective, one needs a much more developed theory of difference equations to be able to classify ordinary and partial difference equations, to develop the analytical tools to study their solutions and to get control over the intrinsically nonlocal features of their behaviour. For this one needs examples that can be rigorously treated. Thus we are led to study exactly integrable discrete systems in all their appearances.

The SIDE meetings, the second of which the present volume gives an account, try to bring together and unite the various developments in the different areas of research where discrete systems play a role. In such a way, we have seen in recent years on the discrete level the unification of the theory of ordinary differential equations, notably the Painlevé equations, with the theory of orthogonal polynomials, [24, 25], the unification of numerical analysis [26] and convergence algorithms with the theory of integrable discrete lattices, the unification of difference geometry [16], with graph theory and combinatorics, unification of quantum field theory with the theory of q-special functions and the unification of soliton equations and cellular automata viz. neural networks. This is the aim of the present volume: to demonstrate the linking elements between the various disciplines dealing with discrete systems. The contributions are written in such a way that they give a brief overview of the state of the art whilst reporting some original research in the subject area. In this way, we hope that the volume can on the one hand assist to familiarise young researcher with this relatively new field of research, and on the other hand serve as a benchmark for the present-day understanding in the various fields. Hopefully, the volume forms an inspiration for further research and so help to establish the links between the various communities working on discrete systems.

The contributions to the present volume cover roughly the following topics:

1. **Special Functions and Difference Equations.**
   Discrete Painlevé equations, difference- and q-difference orthogonal polynomials, separation of variables.

2. **Algebraic aspects.**
   Quantum algebras and representation theory, associated special functions.

3. **Computational and Numerical Aspects.**
   Formal theory of orthogonal polynomials, soliton cellular automata, symplectic and volume-preserving integrators, integrable versus non-integrable discretisations in computation.

4. Symmetry aspects.
Symmetries of difference equations, similarity reductions, integration techniques through symmetries.

5. Analytic aspects.
Analysis of difference equations, isomonodromic deformation theory for discrete systems, asymptotics of orthogonal polynomials.

6. Geometry.
Discrete curves and surfaces, connections with discrete soliton systems, visualisation techniques.

7. Applications.
Neural networks, coding theory and cryptology, data compression, mathematical biology and economics, integrable algorithms.

All these subjects are highly interconnected, and it is the objective of the present volume to demonstrate clearly to the readers how much all these issues are intertwined.

We would like to thank Dr Elizabeth Mansfield for her considerable assistance in the organisation of the meeting and doing most of the "running around", which was especially appreciated as one of us couldonly get around in a wheelchair at the time of the meeting. We also thank Pilar Gordoa, Koryn Grant, Andrew Hicks, Michael Ody, Andrew Pickering, Thomas Priestley, Barry Vowden and Helen Webster for their help with the meeting.

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Frontmatter
More information

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