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Geometric Galois Actions

2. The Inverse Galois Problem, Moduli Spaces and Mapping Class Groups

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Introduction

This volume grew out of the conference which was held at Luminy in August 1995 on the theme "Geometry and Arithmetic of Moduli Spaces". In some sense, it was conceived as a sequel to the 1993 Luminy conference on "The Grothendieck Theory of Dessins d'Enfants", which gave rise to proceedings bearing the same title (this series, number 200). The second conference revolved mostly around some "multidimensional" versions of the themes considered in the first one. All these themes are developments of ideas expressed in Grothendieck's *Esquisse d'un programme*. This seminal text has now been published in the companion volume to this one (this series, number 242).

In the second section of the *Esquisse*, Grothendieck sketches out what he terms "Galois-Teichmüller theory". This theory is an approach to the study of the absolute Galois group $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$, via the action of this group on the –profinite– Teichmüller modular groups (alias mapping class groups), which are the fundamental groups of the moduli spaces of Riemann surfaces with marked points. By Grothendieck's theory of the fundamental group, if X is a scheme defined over $\mathbb{Q}$, then $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ acts on the algebraic fundamental group $\hat{\pi}_1(X \otimes \bar{\mathbb{Q}})$. In the *Esquisse* Grothendieck singles out the case of the moduli spaces as being of particular interest. Note that the "fine" moduli spaces are not schemes but stacks, however a similar theory applies (one can consult the article by T. Oda in the companion volume to this one).

In the third section of the *Esquisse*, Grothendieck points out that "even" the simplest case is already extremely interesting. This case concerns the smallest non-trivial moduli space, that of spheres with four ordered marked points, which is isomorphic to the sphere with three points removed. The study of the Galois action on the fundamental group of this space (the profinite completion of the free group on two generators) is equivalent to the study of the Galois action on finite covers of the projective line ramified over at most three points. By Belyi's famous theorem, every algebraic curve defined over a number field can be realized as such a cover. The study of this action is known as the theory of "dessins d'enfants", and formed the subject of the 1993 conference. The 1995 conference dealt with the generalization of this theme to all moduli spaces, as discussed in the second section of the *Esquisse*.

During the conference, we learned that Jean Malgoire had obtained from Grothendieck permission to publish some of his mathematical manuscripts, among which was the *Esquisse*. We are very grateful to him for suggesting to us that it appear as part of a volume of proceedings. It took some thought to decide on the best way of including the *Esquisse* in such a volume; moreover we were agreeably surprised by the quantity of papers submitted both by participants at the conference and also by certain non-participants who were unable to come but would have liked to. The result is the two-volume series *Geometric Galois Actions*, of which the first volume contains the *Esquisse* itself, a letter from Grothendieck to Faltings on the subject of anabelian geometry, and various contributions all of which play a role in clarifying some of the specific themes introduced by Grothendieck – or some of the themes raised by these clarifications! – whereas the second, present, volume contains the original research papers submitted at the conference.

Let us briefly survey the contents of this volume. To begin with we include the abstracts of the talks which were actually given at Luminy; they were divided into five short courses of two lectures each, a series of individual talks, and an evening seminar given by graduate students, aimed at understanding the basics of Teichmüller and moduli spaces. The rest of the volume is divided into four separate but related sections. The first part, *Dessins d'enfants*, contains five articles forming a bridge with the previous conference. The first one, by N. Adrianov and G. Shabat, considers a special subclass of genus zero dessins, with the aim of determining finer Galois invariants than the well-known valency lists. The paper by G. Jones and M. Streit contains some introductory sections to dessins which are basically self-contained, and also gives a flavour of what has been happening recently in the field. The next paper by T. Hsu illustrates how group theoretic and combinatorial techniques can be used to bear on arithmetic problems, thanks to the very visual nature of dessins. Lastly, the paper by L.Zapponi concentrates on the genus one case, exploiting some of the specific techniques pertaining to the theory of elliptic curves.

The projective line with three points removed can be viewed, as Grothendieck saw it, as the simplest moduli space, but of course it can also be generalised to the projective line minus n points for arbitrary n, $n \geq 3$. Then one can go on to study families of unramified coverings of these objects, and this leads to the theory of Hurwitz spaces, which plays an important role in the investigation of the *Inverse Galois problem*. A recent aspect of this story can be found in the article by P. Dèbes and B. Deschamps, which surveys the state of affairs of the regular inverse Galois problem over so-called "large" fields, a notion introduced by Florian Pop. The paper by K. Strambach and H. Völklein deals with the regular realization of finite groups as Galois groups via the existence of rational points on Hurwitz spaces; it introduces the new idea of considering certain subvarieties of these spaces, rather than just its irreducible components. The third paper in this section, by M. Fried and Y. Kopeliovich, uses Hurwitz spaces to study the ramification properties of realizations as Galois groups of the characteristic finite quotients of the universal p-Frattini cover of a finite group.

The third part, *Galois actions and mapping class groups*, contains two papers which exemplify the kind of techniques that are currently being used in the study of the Galois action on fundamental groups. In his contribution, M. Matsumoto determines the Galois action on the mapping class groups of genus 3, via a remarkable connection between the genus 3 moduli space and the deformation space of E_7-singularity. The paper by Z. Wojtkowiak studies the monodromy of iterated integrals, making ample use of Hodge theory for fundamental groups, which makes it possible to use Lie algebras and differential techniques.

In the fourth section, *Universal Teichmüller Theory*, we have gathered three closely related papers. R. Penner first gives a gentle survey (in his own words) of a recent and promising theory of his which grew out from his previous studies of the "classical" decorated Teichmüller and moduli spaces. His paper serves as an introduction to this theory, but it also contains new features. It investigates the applications and completions of his "universal Ptolemy group" G, a group which was already known to group theorists as Richard Thompson's group, and which is in fact isomorphic to the group $PPSL_2(\mathbb{Z})$ of piecewise $PSL_2(\mathbb{Z})$ transformations. The identification of G with the Thompson group is not apparent at first sight; first noted by M. Kontsevitch, it was proved by M. Imbert and forms the subject of his contribution. In the last paper, by P. Lochak and L. Schneps, it is shown that by suitably enlarging and completing the Ptolemy group into a profinite Ptolemy-Teichmüller groupoid, one can let the Galois group act on this new object. This action actually occurs naturally as the restriction to $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ of an action of the Grothendieck-Teichmüller group (cf. the survey on this group in the companion volume to this one).