

Index

- Abramov's formula, 58
 absolute continuity, **163**
 absolutely continuous, 163
 σ -algebra, **159**
 trivial, **45**
 Arzelà–Ascoli theorem, **76**, 157

 baker's map, **19**, 42
 Bernoulli measure, 8, 17, 19, **34**, 57,
 67, 150
 Bernoulli process, 18
 Bernoulli shift, 34, 57
 Boltzmann, Ludwig, 2, 8, 32, 169
 Borel σ -algebra, **159**
 Borel measure, **163**
 regularity, 163
 Borel–Cantelli lemma, **162**
 bounded distortion, 136, **136**, 137, 139

 circle map, r -fold covering, 125, 134,
 138, 139, 142
 concave function, **158**
 conditional expectation, 23, **164**
 conditional probability, 45, **164**
 conditional probability distribution, 36,
 165
 configuration, **1**, 2, 7, 13, 95
 $e^{-\psi}$ -conformal, **124**, 132, 137, 139
 continuous, **124**, 127, 139
 continued fraction transformation, 139,
 144
 contour, 113
 convex affine function, **158**
 convex function, **157**
 convex set, **157**

 cylinder set, **18**, 34, 50, 67, **159**

 (n, δ) -dense, **78**
 density, **162**, 163
 invariant, 132, 134, 135, 144
 derivative with respect to a measure,
 124
 disjointness condition, 150, **150**, 152
 dominated convergence theorem, **162**
 conditional, **164**
 dynamical system, **13**
 measure preserving, **21**
 topological, **61**
 Dynkin system, **159**

 elementary function, **161**
 empirical distribution, **115**, 116, **127**
 entropy
 dynamical, **54**, 56, 59
 of Bernoulli measures, 57
 w.r.t. a partition, **51**, 59
 Kolmogorov–Sinai, 55
 of a partition, **44**, 59
 conditional, **45**, 47, 48
 in shift spaces, 50
 of a probability vector, **2**
 thermodynamic, 6
 topological, **64**
 entropy function, **59**
 affinity, 59, 75
 semicontinuity, 67, 68, 70, 90
 equidistribution, 4, 6, 32, 126
 equilibrium state, 2, 18, **66**
 absolutely continuous, 139
 Bernoulli measure, 67, 150

- constructible, **82**, 89, 93, 94
- continuous dependence, 71
- ergodic decomposition, 77
- existence, 67
- on a finite set, **4**
- in iterated function systems, 150
- Markov measure, 108
- nonunique, 107, 110, 135, 150
- observable, 130
- unique, 73, 74, 103, 110
- equivalent metric, 85
- Ergode, **32**
- ergodic decomposition, **36**, 77, 126, 128
- ergodic theorem
 - Birkhoff's, **14**, **22**
 - for iterated function systems, 147
 - von Neumann's, 30
- ergodicity, **30**, 32, 41, 42
 - of the continued fraction transformation, 144
 - of Bernoulli shifts, 34
 - of circle maps, 134
 - of circle rotations, 34
 - of fibred systems, 137
 - of noninvariant measures, 137
 - of tent maps, 138
 - unique, **16**
- expansiveness, **89**
 - and semicontinuity of the entropy function, 90
 - of shift systems, 89
- expansiveness constant, **89**
- extension, **40**
 - Markov, 143
 - natural, **41**, 52, 58
- factor, **40**
- Fatou's lemma, **162**
- fibred system, **124**, 130
- finite range, **96**
- first return map, **38**
- first return time, **38**
- Fourier series, 34, **166**
- fractal set, **145**
- free energy, 5, 7, **126**, 130
- Frobenius–Perron theorem, 81, **158**
- full branches, **142**
- Gateaux differentiability, 74
- Gauss, Carl Friedrich, 144
- generator
 - \cap -stable, 30, 33, 34, 159, **159**, 160
 - for an action, **56**, 69, 132
 - for the shift action, 67
- m -generator, 132
- μ -generator for an action, **56**, 130, 133
- Gibbs distribution, *see* Gibbs measure
- Gibbs measure, **99**, 100
 - large deviations, 115
 - on a finite set, **4**, **4**, 9
 - unique, 101, 108, 149
- Gibbs state, *see* Gibbs measure
- Gibbs, John Willard, 2
- ground state, **6**, 10
- Hausdorff dimension
 - of a probability measure, **148**
 - of a set, **148**
- Hausdorff measure, 148
- Hölder-continuous, 96, 136, 149, **157**
- homomorphism, 40
- IFS, **145**, 147
 - conformal, **149**, 150
 - nondegenerate, **148**
- induced map, *see* first return map
- induced system, 40, 58
- information, **44**, 52
 - conditional, **45**, 46, 52
 - elementary, 3
- integrable, 161
 - uniformly, **162**
- T -invariant function, **22**

INDEX

177

- T -invariant measure, **14**
 T -invariant set, **14, 22**
 \mathcal{T} -invariant measure, **21, 59**
 \mathcal{T} -invariant set, **69**
 Ising model, **10, 96, 110**
 anti-ferromagnetic, **10**
 iterated function system, *see* IFS

 Jensen inequality, **18, 87, 161**
 conditional, **164**
 elementary, **158**

 Kac's recurrence theorem, **39**
 Kolmogorov–Sinai theorem, **56**
 Krylov–Bogolubov theorem, **15, 19, 61**

 large deviations
 for Gibbs measure, **115**
 for Bernoulli measures, **8**
 for fibred systems, **128**
 Lebesgue decomposition, **163**
 Lipschitz-continuous, **157**
 local energy function, **7, 62, 66**
 regular, **96, 115, 149**
 logistic map, **125, 144**
 Lyapunov exponent, **14, 17, 126**

 m.p.d.s., **21**
 magnetization
 average, **110**
 spontaneous, **110**
 Markov chain, **108**
 Markov matrix, **12**
 martingale convergence theorem, **164**
 measurable function, **161**
 measurable mapping, **161**
 measurable set, **159**
 measurable space, **159**
 measure
 finite, **160**
 of maximal entropy, **66, 142**
 mutually singular, **163**
 non-atomic, **160**
 observable, **126, 127, 130**
 purely atomic, **160**
 mixing, **32, 33, 41, 42**
 of Bernoulli shifts, **34**
 Möbius transformation, **149**
 monotone convergence theorem, **162**

 orthogonal complement, **165**

 partition, **43, 44, 54, 124**
 independent, **44, 48**
 μ -partition, **43**
 partition function, **3, 65**
 periodic orbit, **32**
 phase transition, **73, 110**
 in the Ising model, **11, 110**
 piecewise monotonic map, **125, 127**
 pigeon-hole principle, **67**
 pressure, **64**
 in finite systems, **7**
 in thermodynamics, **7**
 pressure function, **65**
 convexity, **75**
 derivative of, **73, 74**
 for IFS, **149, 150**
 tangent functionals of, **73, 73**
 probability measure, **160**
 probability space, **160**
 product measure, **160**
 product system, **35**

 Radon–Nikodym theorem, **163**
 Riesz representation theorem, **163**
 Rohlin formula, **139, 142**
 rotation, **34**
 Ruelle inequality, **139**

 SBR measure, **123, 126**
 self-similarity exponent, **151**
 (n, δ) -separated, **78, 79, 81, 84, 85, 90**
 in shift spaces, **78**

- maximal, **78**, 86, 96
- Shannon–McMillan–Breiman theorem, **52**, 141
- shift transformation, **7**, 17, 34
- Sierpinski gasket, 146, 150
- Sinai–Bowen–Ruelle measure, *see* SBR measure
- state, **2**, 12, 61
- Stirling’s formula, 9
- strictly positive matrix, 108

- t.d.s., **61**
- tail-field, **58**, 103
- tangent functional, **73**
- temperature, 5, 110
- tent map
 - full skew, 16
 - symmetric, 125, 138, 142, 144
- topological support, 135, **146**
- transfer operator, **130**

- uniformly expanding, **132**
- uniformly integrable, **162**
- uniquely ergodic, **16**
- upper semicontinuous, **62**, 63

- variational principle, 78, **85**, 123
 - on finite spaces, 4

- weak compactness, **163**
- weak convergence, **163**
- weak mixing, 35, **35**, 41, 42