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Para Javier, Gabo, Nacho e Isabel

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Preface

This set of notes grew out of a course that I gave at Ohio University in the spring of 1996. My aim was to give graduate students who were familiar with ordinary character theory an introduction to Brauer characters and blocks of finite groups.

To do that I chose an objective: the Glauberman Z^* -theorem. This theorem gives an excellent excuse for introducing modular representation theory to students interested in groups. Glauberman's outstanding result is one of the major applications of the theory to finite groups. However, to be able to prove it, one needs to proceed from the very basic facts to the three main theorems of R. Brauer.

In Chapter 1, I prove what is absolutely necessary to get started. Assuming that the students have already had a course on ordinary characters, I use this chapter to remind them of some familiar ideas while introducing some new ones.

In Chapter 2, I introduce Brauer characters (in the same spirit as in the book of M. Isaacs) and develop their basic properties.

In Chapter 3, I introduce blocks and, in Chapter 4, Brauer's first main theorem is given. The second main theorem is proven in Chapter 5 and its proof is a new "elementary" proof by Isaacs based on work by A. Juhász and Y. Tsushima. The third main theorem is given in a very general form and its proof is due to T. Okuyama. Once the third main theorem has been proven, we are ready for the Z^* -theorem.

After Glauberman's theorem is completed, I include Chapter 8 on the basic behaviour of Brauer characters. Blocks and Brauer characters of p -solvable groups are studied in Chapter 10.

The relationship between blocks and normal subgroups (which is needed in Chapter 10, but not for the Z^* -theorem) is covered in Chapter 9.

Finally, in Chapter 11, I develop one of the highlights of the whole theory: the description of the p -blocks of the groups with a Sylow p -subgroup of order p .

When writing down this set of notes, I could not resist introducing several topics which were not necessary to accomplish my objective, but which have interest on their own. Perhaps some of them may be taught if time allows it.

For many years, modular representation theory of finite groups was developed only through the incredible talent of Richard Brauer. I take this opportunity to express my deepest admiration for his work.

These notes would not have been possible without the help of Martin Isaacs, to whom I am very much indebted. Thomas Keller read the complete set of notes which have thus benefited from his comments. Chris Puin helped me with the English.

I also extend my thanks to G. Glauberman, M. Lewis, J. Muñoz, F. Pérez Monasor, L. Sanus, W. Willems and T. Wolf.

Valencia
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