1

What are muons? What is muon science?

Scientific research using the fundamental particle known as the muon depends upon the muon's basic particle properties and also on the microscopic (atomic-level) interactions of muons with surrounding particles such as nuclei, electrons, atoms, and molecules. This chapter deals mainly with the fundamental properties of muons based on what is presently known from particle physics. Several relevant reference works exist, in particular regarding historical developments (Hughes and Kinoshita, 1977; Kinoshita, 1990).

1.1 Basic properties of the muon

In one sentence, the properties of muons can be summarized as follows:

Muons are unstable elementary particles of two charge types (positive μ^+ and negative μ^-) having a spin of 1/2, an unusual mass intermediate between the proton mass and the electron mass (1/9 m_p, 207 m_e), and 2.2 μs lifetime.

Over time, a deeper understanding of the above statement has been gained through the development of experimental methods and improvements in theoretical models. Some data relevant to muon science are summarized in Table 1.1.

The uniqueness of lifetime and mass can be understood by comparing muon values to those of other particles, as seen in Figure 1.1. These properties can be summarized as follows:

The muon has the second longest lifetime among all the fundamental unstable particles (that is, omitting particles believed to be stable, such as the proton, electron, and neutrino) after the neutron, and has the second smallest mass among all the fundamental particles after the electron.

The following paragraphs elaborate and clarify the contents of Table 1.1.

1.1.1 Mass of the muon

The most accurate determination of the mass of the muon (m_{μ^+}, m_{μ^-}) with reference to the electron mass (m_e), which is known to a precision of 10^{-8} (10 p.p.b.), can be made in the following two ways.

Table 1.1 Fundamental properties of muons

	μ^+	μ^-
Charge	$+1$	-1
Mass	206.768 277 (24) (m_e)[a]	105.659 (1) (MeV/c^2)[b]
Spin	1/2	1/2
Magnetic moment (μ_μ/μ_p)	3.183 345 13 (39)[a]	
Gyromagnetic ratio ($\mu_\mu/2\pi I$ in kH_z/G, $I = 1/2$)	13.553 42	
Gyromagnetic factor (g/2) ($\mu_\mu = g(e\hbar/2m_\mu)I$, $I = 1/2$)	1.001 165 920 3 (15)[c]	1.001 165 936 (12)
Free decay lifetime (10^{-6} s)	2.196 95 (6)[d] 2.197 078 (73)[e]	2.194 8 (10)[f] (in flight)
Decay mode	$e^+ + \overline{\nu}_\mu + \nu_e$	(100%)
	$e^+ + \gamma$	($<1.7 \times 10^{-10}$)
	$e^+ + e^- + e^+$	($<1.9 \times 10^{-9}$)
	$e^+ + \gamma + \gamma$	($<1.25 \times 10^{-8}$)

[a] Liu *et al.*, 1999; [b] Beltrami *et al.*, 1986; [c] Brown *et al.*, 2001; [d] Giovanetti *et al.*, 1984; [e] Bardin *et al.*, 1984; [f] Williams and Williams, 1972.

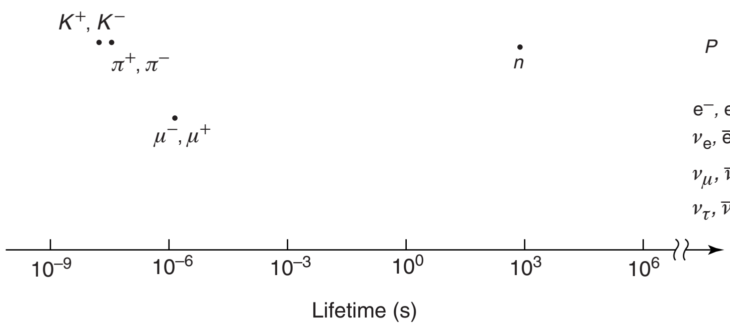
As will be described more fully later, the mass of the positive muon ($m_{\mu+}$) can be determined most accurately by measuring the energy interval between the 1s and 2s electronic quantum levels (ΔE_{1s-2s}) in muonium (the neutral bound state of μ^+ and e^-, closely analogous to the hydrogen atom, sometimes designated hereafter as Mu). Laser two-photon resonance spectroscopy gives an isotope shift in the ΔE_{1s-2s} absorption line for Mu with respect to H due to a change in the reduced mass correction.

The mass of the negative muon ($m_{\mu-}$), on the other hand, cannot be determined in this way, but can be obtained by measuring the energy intervals between atomic states formed by the μ^- around a nucleus. Such a system is called a muonic atom. Because the mass of the μ^- is 207 times that of the electron and thus much closer to the nuclear mass than those of the orbiting electrons in a conventional atom, specific atomic states should be carefully selected such that ambiguity related to the nuclear charge distribution (i.e. due to the fact that the nucleus is not point-like) is minimized. The actual determination was made by measuring 3d–2p transitions in muonic ^{28}Si with a careful correction (Beltrami *et al.*, 1986).

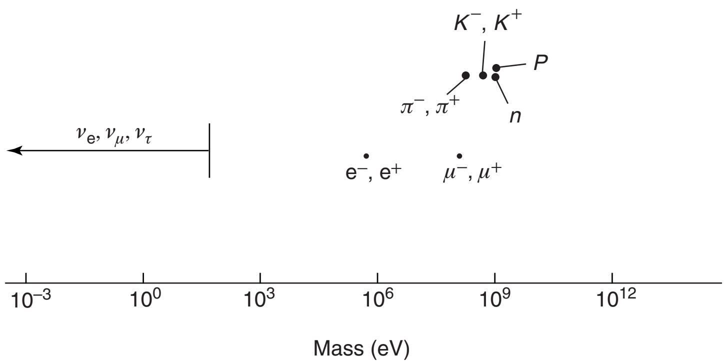
1.1.2 Lifetime of the muon

The μ^+ and μ^- in vacuum have the following major (100%) decay modes:

$$\mu^+ \to e^+ + \overline{\nu}_\mu + \nu_e$$
$$\mu^- \to e^- + \nu_\mu + \overline{\nu}_e$$



(*a*)



(*b*)

Figure 1.1 Lifetimes of various particles (*a*) and their masses (*b*).

where ν_e and ν_μ are the electron and muon neutrinos and $\bar{\nu}_e$ and $\bar{\nu}_\mu$ the corresponding antineutrinos.

The positive muon lifetime, τ_μ, can be measured from the shape of the time spectrum of the decay positrons with reference to the time of μ^+ stopping in some target material under the reasonable assumption that the decay mode of μ^+ in matter is not subject to any changes from that in vacuum. The time distribution of decay positrons $N_e(t)$ follows an exponential law:

$$N_e(t) = N_e(0)\, e^{-t/\tau_\mu}$$

After several attempts at various accelerator facilities worldwide, it was recognized that the removal of spin polarization-related effects is essential. One of the most reliable ways to do this is to measure the decay e^+ over the full 4π solid angle. Several subsequent trials have been conducted (Bardin *et al.*, 1984; Giovanetti *et al.*, 1984; Nakamura *et al.*, 1999).

The lifetime of μ^- must be measured in vacuum since that of bound μ^- in the 1s orbit of a muonic atom is significantly shortened by nuclear capture processes. An alternative method is to measure the lifetime of μ^- in flight compared to that of μ^+. Such a measurement was successfully carried out as a byproduct of the measurement of the muon anomalous magnetic moment $(g-2)_\mu$, assuming the validity of special relativity (Williams and Williams, 1972).

1.2 Muons in the current picture of particle physics

The size of the μ^+ and μ^- can be measured in high-energy collision experiments using e^+e^- colliders; the reaction $e^+ + e^- \rightarrow \mu^+ + \mu^-$, assuming quantum electrodynamics (QED) and with point-like e^+ and e^-, confirms that μ^+ and μ^-, too, are point-like, with $r_\mu \leq 10^{-16}$cm (Martyn, 1990).

The size of μ^+ and μ^- can also be estimated from high-precision measurements of muon properties such as the anomalous gyromagnetic ratio of muon, $(g-2)_\mu$, or from the upper limit upon flavor nonconserving decays such as $\mu^+ \rightarrow e^+ + \gamma$. These measurements also place a stringent upper limit on the existence of possible internal structure or excited states in the muon. These limits, with the aid of theoretical models, can be converted to give an upper limit on the size of the muon: $r_\mu \leq 10^{-17}$ cm (Brodsky and Drell, 1980).

All experimental results so far support a picture in which the muon is point-like, in contrast to the finite-sized nature of nuclei, nucleons, and mesons (π, K, others).

As summarized in Figure 1.2, at the present state of knowledge, the elementary constituents of matter are quarks and leptons. The masses of these elementary particles are distributed as shown in the figure.

In this framework, the properties of the muon can be summarized as follows:

The muon belongs to the lepton family, along with the electron, the τ-particle, and their corresponding neutrinos (ν_e, ν_μ, and ν_τ). The muon interacts with other particles and matter through both electromagnetic and weak interactions.

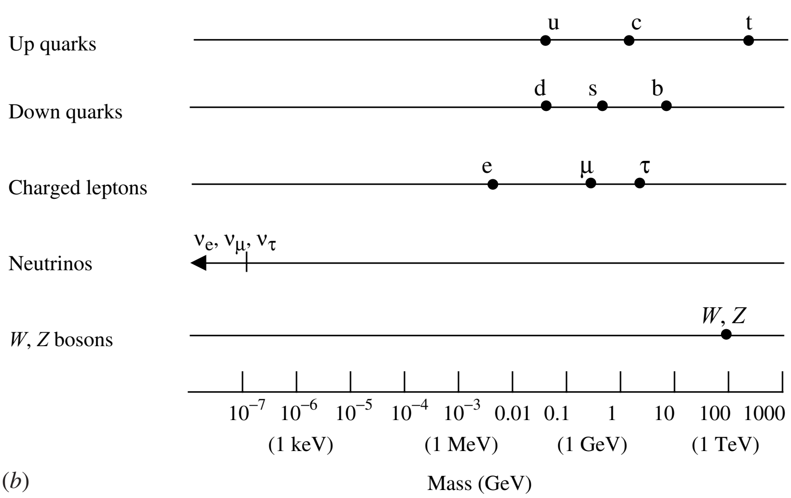
The classification of these elementary particles, as shown in Figure 1.2, is strictly determined by the conservation laws; generations cannot be mixed. Thus, reactions such as $\mu^+ \rightarrow e^+ + \gamma$ or $\mu^- + A \rightarrow e^- + A$ are forbidden. As described later, in the search for violation of generation conservation, the conservation or nonconservation of flavor is one of the central subjects in current particle physics.

1.3 Fundamental interactions of the muon

The μ^+ and μ^- are subject to electromagnetic and weak interactions. These two interactions are now unified into an electroweak interaction within the framework of the standard model. In the following section, these fundamental interactions, which appear again and again in muon science studies, are summarized.

		Generation			Charges			Interactions				
		1	2	3	Electric	Weak	Color	Electric	Weak	Strong	"Mass"	Gravity
"Matter" particles	Quarks	Up Up Up	Charm Charm Charm	Top Top Top	2/3	↑	Red Green Blue	○	○	○	○	○
		Up-type quarks										
		Down Down Down	Strange Strange Strange	Bottom Bottom Bottom	−1/3	↓	Red Green Blue	○	○	○	○	○
		Down-type quarks										
	Leptons	ν_e	ν_μ	ν_τ	0	↑	White	×	○	×	× ?	○ ?
		Neutrinos										
		Electrons	μ	τ	−1	↓	White	○	○	×	○	○ ?
		Charged leptons										
"Force" particles								γ (photon)	*W*, *Z*	Gluons	Higgs ?	Gravitons ?
								Gauge bosons				

(*a*)



(*b*)

Figure 1.2 Basic properties of quarks, leptons, and gauge particles (*a*), and their masses (*b*).

1.3.1 Electromagnetic (EM) interaction

Both charge types of the muon interact with other charged particles via the Coulomb interaction in which the potential energy is given by $-e^2 Z/r$, where Z is the charge of the other particle (the charge on the muon being ± 1). Several important atomic bound states are formed, including: muonium ($\mu^+ e^-$), muonic hydrogen ($\mu^- p$), and muonic Z-atoms ($\mu^- Z$).

The magnetic moments of μ^+ and μ^- (μ_μ) interact with magnetic fields either intrinsic to the atoms themselves or externally applied. The hyperfine splittings in the atomic bound states and the spin precession frequencies around the external field (H_{ext}) are thus determined by the relevant parameters:

$$\Delta E_{hfs}(\mathrm{Mu, 1s}) = \mu_\mu \times \mu_e <1/r^3>, \qquad f_\mu = \gamma_{Mu}(= \mu_\mu/2\pi)H_{ext}$$

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Table 1.2 Hyperfine splitting of the muonium ground state $\Delta\nu = \Delta E_{\mathrm{hfs}}(\mathrm{Mu}, 1s)/h$; terms of the theoretical predictions and experiment (Sapirstein and Yennie, 1990)

Theory	Experiment
4 463 303.11(1.33)(0.40)(1.0) kHz	4 463 302.765(53) kHz
Fermi splitting; E_{F}	(Liu *et al.*, 1999)
$\frac{16}{3}\alpha^2 \frac{m_{\mathrm{r}}^3}{m_{\mathrm{e}}^2 m_\mu} hcR_\infty$	
where $m_{\mathrm{r}} = m_{\mathrm{e}}/(1 + m_{\mathrm{e}}/m_\mu)$	
R_∞ = Rydberg constant	
$\frac{8}{3}\times$ [1 233 690 735.4(1) MHz]	
QED correction: ΔE_{hfs}(QED)	
$E_F\left(1 + a_\mu\right)\left\{1 + \frac{3}{2}(Z\alpha)^2 + a_{\mathrm{e}} + \alpha(Z\alpha)\left(\ln 2 - \frac{5}{2}\right)\right.$	
$-\frac{8\alpha(Z\alpha)^2}{3\pi}\ln Z\alpha\left(\ln Z\alpha - \ln 4 + \frac{281}{480}\right)$	
$\left.+\frac{\alpha(Z\alpha)^2}{\pi}(15.88 \pm 0.29) + \frac{\alpha^2(Z\alpha)}{\pi}D_1\right\}$	
where a_μ, a_{e} = anomalous magnetic moment of muon and electron	
D_1 = uncalculated radiative corrections involving two virtual photons	
Recoil correction: ΔE_{hfs}(rec)	
$E_{\mathrm{F}}\left\{-\frac{3\alpha}{\pi}\frac{m_{\mathrm{e}}m_\mu}{m_\mu{}^2 - m_{\mathrm{e}}{}^2}\ln\frac{m_\mu}{m_{\mathrm{e}}}\right.$	
$\left.+\frac{\gamma^2}{m_{\mathrm{e}}m_\mu}\left[2\ln\frac{m_{\mathrm{r}}}{2\gamma} - 6\ln 2 + 3\frac{11}{18}\right]\right\}$	
where $\gamma = m_{\mathrm{r}}\alpha$	
Radiative-recoil correction: ΔE_{hfs} (rad-rec)	
$E_{\mathrm{F}}\left(\frac{\alpha}{\pi}\right)^2\frac{m_{\mathrm{e}}}{m_\mu}\left[-2\ln^2\frac{m_\mu}{m_{\mathrm{e}}} + \frac{13}{12}\ln\frac{m_\mu}{m_{\mathrm{e}}}\right.$	
$\left.+\frac{21}{2}\zeta(3) + \frac{\pi^2}{6} + \frac{35}{9} + (1.9 \pm 0.3)\right]$	

QED, quantum electrodynamics.

The energy levels of the bound states of Mu, a centrosymmetric two-particle system very similar to H, have been the objects of several precise measurements. The energy levels of the excited states with reference to the ground state are seen in Figure 1.3. In contrast to H, the core in Mu is truly a single structureless particle, and so the fundamental EM interaction in the two-body bound state can be studied without corrections for core structure; thus, at least in principle, the experimental values of the fundamental parameters of the EM interaction can be obtained more straightforwardly through studies of Mu. The present status of experiment and theory on the Mu ground-state hyperfine energy splitting of the Mu in vacuum is summarized in Table 1.2. The result is presented in terms of $\Delta\nu$ ($\equiv \Delta E_{\mathrm{hfs}}(\mathrm{Mu}/1s)/h$), while, in some other cases, the following expression is used: $\Delta E_{\mathrm{hfs}} = \hbar\omega_0$. The ground state of Mu is subject to energy splitting in an applied external

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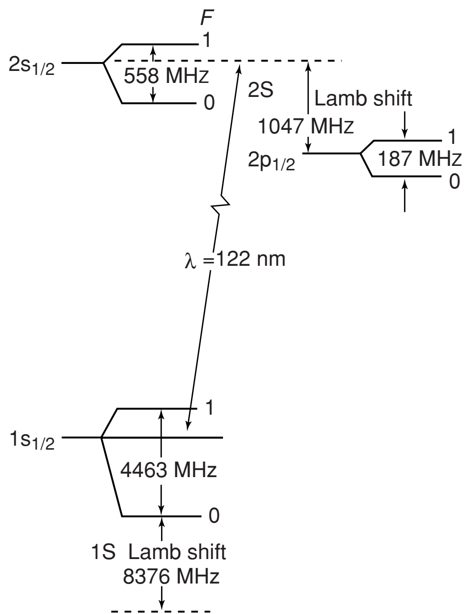

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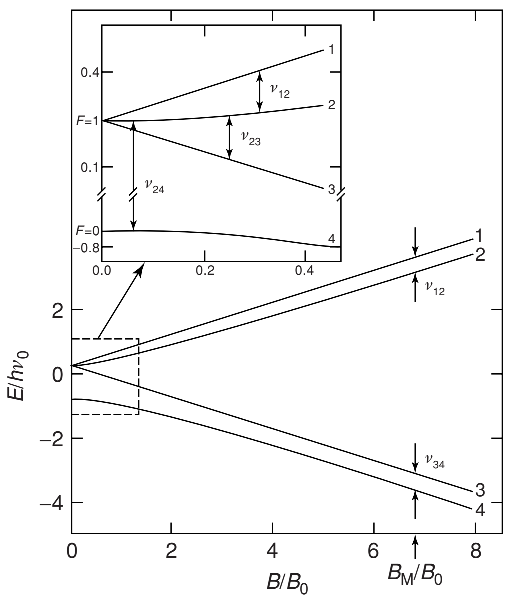


(*b*)

$$E_1 = \hbar\omega_0\left(\frac{1}{4} + \frac{\omega_-}{\omega_0}\right)$$

$$E_2 = \hbar\omega_0\left[-\frac{1}{4} + \sqrt{\frac{1}{4} + \left(\frac{\omega_+}{\omega_0}\right)}\right]$$

$$E_3 = \hbar\omega_0\left(\frac{1}{4} - \frac{\omega_-}{\omega_0}\right)$$

$$E_4 = \hbar\omega_0\left[-\frac{1}{4} - \sqrt{\frac{1}{4} \quad \left(\frac{\omega_+}{\omega_0}\right)^2}\right]$$

$$\omega_0 = \Delta E_{hfs}/\hbar = \gamma_e B_0$$

$$\omega_\pm = \frac{\omega_e \pm \omega_\mu}{2}$$

$$\omega_e = \gamma_e B, \ \omega_\mu = \gamma_\mu B$$

For $B << B_0$

$$E_1 - E_2 \sim \hbar\left(\omega_- - \frac{\omega_+^2}{\omega_0}\right)$$

$$E_2 - E_3 \sim \hbar\left(\omega_- + \frac{\omega_+^2}{\omega_0}\right)$$

$$(E_1 - E_2) - (E_2 - E_3) \sim \hbar\,\frac{2\omega_+^2}{\omega_0}$$

Figure 1.3 Energy levels of Mu, where the lifetime of each state is $\tau_{Mu}(2p) = 1.6$ ns and $\tau_{Mu}(2s) = 0.145$ s (*a*) and energy diagram of the ground state of Mu against applied external field, so called Breit–Rabi diagram, where energy is in units of $h\nu_0$ ($\nu_0 = 4.463.302$ MHz) and magnetic field is in units of B_0 (0.1585 T) (*b*).

field; this is expressed in the Breit–Rabi formula. The energy levels and related formulae are summarized in Figure 1.3.

Tests of the validity of the fundamental theory of QED have been carried out through high-precision spectroscopy making use of μ^+- and μ^--containing atoms. These experiments also yield values for the fundamental constants of the muon itself, such as the mass of the muon m_μ and the magnetic moment of the muon μ_μ. To give some examples:

1. ΔE (Mu, 1s–2s): Two-photon laser resonance was carried out for the first time at High Energy Accelerator Research Organization (KEK) (Chu *et al.*, 1988), and subsequently extended at ISIS–Rutherford Appleton Laboratory (ISIS–RAL) (Maas *et al.*, 1996; Meyer *et al.*, 2000). The most updated value of the muon–electron mass ratio obtained from ΔE (Mu, 1s–2s) is $m_{\mu^+}/m_{e^-} = 206.768\,38(17)$, which is consistent with the most accurate value to be mentioned later. This measurement is now known to provide potentially the most accurate determination of m_μ.
2. ΔE_{hfs} (Mu, 1s): Microwave resonance spectroscopy under high magnetic field simultaneously yielded ΔE_{hfs} to provide m_{μ^+}/m_{e^-} and μ_μ/μ_p (Mariam *et al.*, 1982). Theoretical progress towards the understanding of the experimental results (Sapirstein and Yennie, 1990) is summarized in Table 1.2. This experimental method requires a narrowing of the measured line in order to obtain an improved value. There, in order to overcome a limitation due to natural line width of 145 kHz ($\cong 1/\pi\tau_\mu$), a resonance line-narrowing technique is employed by interacting microwave with Mu atoms which have lived several times τ_μ. The most updated measurement (Liu *et al.*, 1999) provided $m_{\mu^+}/m_{e^-} = 206.768\,277(24)$ and $\mu_\mu/\mu_p = 3.183\,345\,13(39)$.

The other type of high-precision measurement of the EM interaction is the measurement of the anomalous magnetic moment of the muon by storing muon motion in a high magnetic field. Anomalous magnetic moment of the muon $a_\mu (= (g-2)/2)$ can be measured from the so-called $(g-2)$ precession. The $(g-2)$ precession which corresponds to the angular frequency difference between the spin precession frequency and the cyclotron frequency in a uniform magnetic field perpendicular to both the muon spin direction and the plane of the orbit has been measured using a muon storage ring, where, by selecting a muon momentum of 1.5 GeV/c, any effects due to the electric confinement field were removed. A precision of 10 p.p.m. was obtained in experiments conducted at the European Organization for Nuclear Research (CERN) (Bailey *et al.*, 1975). Improved measurements with the aim of obtaining $(g-2)_\mu$ to a precision of 0.5 p.p.m. are currently in progress at Brookhaven National Laboratory (BNL). For this level of precision, however, improvements over the present level in both m_μ and μ_μ are required. Currently, the weighted mean of all the experimental results agrees with the standard model with 3.6 ± 4.0 p.p.m. (Brown *et al.*, 2000). The latest report (Brown *et al.*, 2001) provides the comparison between the world-average experimental data and theoretical prediction based on the standard model $a_\mu(\mathrm{exp}) - a_\mu(\mathrm{theory}) = 43(16) \times 10^{-10}$, suggesting an existence of physics beyond the standard model. The more updated report is available (Bennett *et al.*, 2002).

1.3.2 Weak interaction

The weak interaction of the muon is the phenomenon underlying both the decay of μ^+ and μ^- and the nuclear capture of μ^- in muonic atoms. The fundamental law of flavor conservation has been confirmed through observations setting an upper limit on flavor conservation-violating processes such as $\mu^+ \to e^+ + \gamma$ or $\mu^- Z \to e^- Z$.

In addition to lepton number conservation, another important weak-interaction experiment involving muon, muonium, and muonic atom is to search for a conversion of muonium (Mu, μ^+e^-) to antimuonium ($\overline{\mathrm{Mu}}$, μ^-e^+). This is related to the mixing of lepton numbers, including multiplicative or additive schemes; the standard model in particle physics assumes an additive scheme. Various types of experiments have been done after establishing the experimental method of thermal Mu production in a vacuum. The present experiment gives an upper limit of the conversion probability of $P_{\mathrm{Mu}\overline{\mathrm{Mu}}} \leq 8.3 \times 10^{-11}$ (90% CL) (Willmann *et al.*, 1999).

At the same time, the detailed properties of the normal decay process of the μ^+ yielding an e^+ and two neutrinos have been studied to a high degree of precision. For a purely weak process, muon decay can be written using the four Michel parameters, ρ, η, ε, and δ, as summarized in Figure 1.4 (Michel, 1949; Kinoshita and Sirlin, 1957), where in some cases nonzero parameter values correspond to violation of a fundamental conservation law, again summarized in Figure 1.4.

1.4 Production and decay of polarized muons

Given the present upper limit for flavor nonconserving rare decay processes and the high-precision determination of the normal decay process, the μ^+ production and decay processes can be characterized as follows.

1.4.1 Muon polarization in $\pi\mu$ decay

The muon is produced in the decay of the pion according to:

$$\pi^+ \to \mu^+ + \nu_\mu$$
$$\pi^- \to \mu^- + \overline{\nu}_\mu$$

Since the spin of the pion is zero and the muon neutrino has a definite helicity (h) such that $h = -1$ for $\overline{\nu}_\mu$ and $h = +1$ for ν_μ, the muon is 100% polarized in the center-of-mass system, as shown in Figure 1.5.

1.4.2 Asymmetry of electron/positron emission in muon decay

The muon decays into an electron and two neutrinos as follows:

$$\mu^+ \to e^+ + \nu_e + \overline{\nu}_\mu$$
$$\mu^- \to e^- + \overline{\nu}_e + \nu_\mu$$

$$dN(x,\theta) = \frac{d^3p}{(2\pi)^4}\frac{m_\mu E_0}{12}A\left\{6(1-x)+4\rho\left[\frac{4}{3}x-1-\frac{1}{3}\frac{m_e^2}{E_0^2 x}\right] + 6\eta\frac{m_e}{E_0}\frac{(1-x)}{x} \pm \beta\xi\cos\theta\left[2(1-x) + 4\delta\left(\frac{4}{3}x-1-\frac{1}{3}\frac{m_e^2}{m_\mu E_0}\right)\right]\right\}$$

upper sign: μ^+ decay, lower sign: μ^- decay

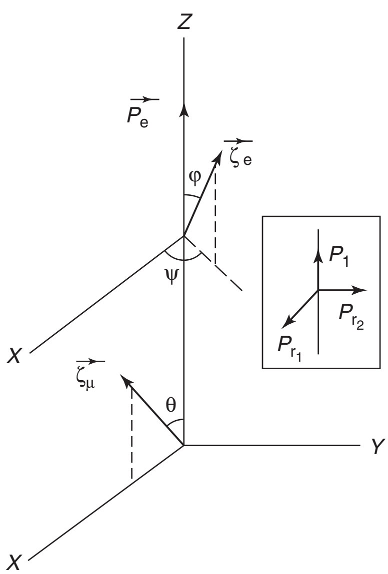
$$A = a + 4b + 6c$$

$$\rho = \frac{3b+6c}{a+4b+6c}$$

$$\eta = \frac{\alpha - 2\beta}{a+4b+6c}$$

$$\xi = \frac{-3a' - 4b' + 14c'}{a+4b+6c}$$

$$\delta = \frac{3b' - 6c'}{3a' + 4b' - 14c'}$$



where

$a = |g_S|^2 + |g_{S'}|^2 + |g_P|^2 + |g_{P'}|^2$

$b = |g_V|^2 + |g_{V'}|^2 + |g_A|^2 + |g_{A'}|^2$

$c = |g_T|^2 + |g_{T'}|^2$

$\alpha = |g_S|^2 + |g_{S'}|^2 - |g_P|^2 - |g_{P'}|^2$

$\beta = |g_V|^2 + |g_{V'}|^2 - |g_A|^2 - |g_{A'}|^2$

$a' = 2\mathrm{Re}(g_S g_{P'}{}^* + g_P g_{S'}{}^*)$

$b' = 2\mathrm{Re}(g_V g_{A'}{}^* + g_A g_{V'}{}^*)$

$c' = 2\mathrm{Re}(g_T g_{T'}{}^*)$

$x = E/E_0$

$E_0 = (m_\mu{}^2 + m_e{}^2)/2m_\mu$

$\quad \cong m_\mu/2$

Experimental data on decay parameters	
$\rho = -0.7518\ (26)$,	$\eta = -0.007\ (13)$
$\xi p_\mu \delta/\rho > 0.99682$,	$\delta = 0.779\ (4)$

Figure 1.4 The details of the general formula of normal muon decay where all the possible terms in four Fermion interaction are considered with interaction constants of g_S (scalar), g_V (vector), g_T (tensor), g_A (axial vector), and g_P (pseudoscalar) for the Hamiltonian of $\Sigma_i[(\psi_e\Gamma_i\psi_\mu)(\psi_\nu(g_i + g_i'\gamma_5)\psi_\nu) + \mathrm{h.c.}]$ and θ is the angle between electron momentum and muon spin. Experimental data on each parameter are also shown; data from Hagiwara, K. *et al.* (2002). *Phys. Rev.*, **D66**, 010001.