Synchronization

A universal concept in nonlinear sciences

First recognized in 1665 by Christiaan Huygens, synchronization phenomena are abundant in science, nature, engineering, and social life. Systems as diverse as clocks, singing crickets, cardiac pacemakers, firing neurons, and applauding audiences exhibit a tendency to operate in synchrony. These phenomena are universal and can be understood within a common framework based on modern nonlinear dynamics. The first half of this book describes synchronization without formulae, and is based on qualitative intuitive ideas. The main effects are illustrated with experimental examples and figures; the historical development is also outlined. The second half of the book presents the main effects of synchronization in a rigorous and systematic manner, describing both classical results on the synchronization of periodic oscillators and recent developments in chaotic systems, large ensembles, and oscillatory media. This comprehensive book will be of interest to a broad audience, from graduate students to specialist researchers in physics, applied mathematics, engineering, and natural sciences.

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Editors

The Cambridge Nonlinear Science Series contains books on all aspects of contemporary research in classical and quantum nonlinear dynamics, both deterministic and nondeterministic, at the level of graduate text and monograph. The intention is to have an approximately equal blend of experimental and theoretical works, with the emphasis in the latter on testable results. Specific subject areas suitable for consideration include: Hamiltonian and dissipative chaos; squeezed states and applications of quantum measurement theory; pattern selection; formation and recognition; networks and learning systems; complexity in low- and high-dimensional systems and random noise; cellular automata; fully developed, weak and phase turbulence; reaction–diffusion systems; bifurcation theory and applications; self-structured states leading to chaos; the physics of interfaces, including fractal and multifractal growth; and simulations used in studies of these topics.

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Synchronization

A universal concept in nonlinear sciences

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To my father Samuil AP

To Sonya MR

To my father Herbert JK

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Part III: Synchronization of chaotic systems

Preface

The word "synchronous" is often encountered in both scientific and everyday language. Originating from the Greek words $\chi\rho\acute{o}\nu o\varsigma$ (*chronos*, meaning time) and $\sigma\acute{\upsilon}\nu$ (*syn*, meaning the same, common), in a direct translation "synchronous" means "sharing the common time", "occurring in the same time". This term, as well as the related words "synchronization" and "synchronized", refers to a variety of phenomena in almost all branches of natural sciences, engineering and social life, phenomena that appear to be rather different but nevertheless often obey universal laws.

A search in any scientific data base for publication titles containing the words with the root "synchro" produces many hundreds (if not thousands) of entries. Initially, this effect was found and investigated in different man-made devices, from pendulum clocks to musical instruments, electronic generators, electric power systems, and lasers. It has found numerous practical applications in electrical and mechanical engineering. Nowadays the "center of gravity" of the research has moved towards biological systems, where synchronization is encountered on different levels. Synchronous variation of cell nuclei, synchronous firing of neurons, adjustment of heart rate with respiration and/or locomotory rhythms, different forms of cooperative behavior of insects, animals and even humans – these are only some examples of the fundamental natural phenomenon that is the subject of this book.

Our surroundings are full of oscillating objects. Radio communication and electrical equipment, violins in an orchestra, fireflies emitting sequences of light pulses, crickets producing chirps, birds flapping their wings, chemical systems exhibiting oscillatory variation of the concentration of reagents, a neural center that controls the contraction of the human heart and the heart itself, a center of pathological activity that causes involuntary shaking of limbs as a consequence of Parkinson's disease – all these and many other systems have a common feature: they produce rhythms. Usually these objects are not isolated from their environment, but interact with other objects, in other words they are open systems. Indeed, biological clocks that govern daily (circadian) cycles are subject to the day–night and seasonal variations of illuminance and temperature, a violinist hears the tones played by her/his neighbors, a firefly is influenced by the light emission of the whole population, different centers of rhythmic brain activity may influence each other, etc. This interaction can be very weak, sometimes hardly perceptible, but nevertheless it often causes a qualitative transition: an object adjusts its rhythm in conformity with the rhythms of other objects. As a result, violinists play in unison, insects in a population emit acoustic or light pulses with a common rate, birds in a flock flap their wings simultaneously, the heart of a rapidly galloping horse contracts once per locomotory cycle.

This adjustment of rhythms due to an interaction is the essence of synchronization, the phenomenon that is systematically studied in this book.

The aim of the book is to address a broad readership: physicists, chemists, biologists, engineers, as well as other scientists conducting interdisciplinary research;[1] it is intended for both theoreticians and experimentalists. Therefore, the presentation of experimental facts, of the main principles, and of the mathematical tools is not uniform and sometimes repetitive. The diversity of the audience is reflected in the structure of the book.

The first part of this book, Synchronization without formulae, is aimed at readers with minimal mathematical background (pre-calculus), or at least it was written with this intention. Although Part I contains almost no equations, it describes and explains the main ideas and effects at a qualitative level.[2] Here we illustrate synchronization phenomena with experiments and observations from various fields. Part I can be skipped by theoretically oriented specialists in physics and nonlinear dynamics, or it may be useful for examples and applications.

Parts II and III cover the same ideas, but on a quantitative level; the reader of these parts is assumed to be acquainted with the basics of nonlinear dynamics. We hope that the bulk of the presentation will be comprehensible for graduate students. Here we review classical results on the synchronization of periodic oscillators, both with and without noisy perturbations; consider synchronization phenomena in ensembles of oscillators as well as in spatially distributed systems; present different effects that occur due to the interaction of chaotic systems; provide the reader with an extensive bibliography.

[1] As the authors are physicists, the book is inevitably biased towards the physical description of the natural phenomena.

[2] To simplify the presentation, we omit in Part I citations to the original works where these ideas were introduced; one can find the relevant references in the bibliographic section of the Introduction as well as in the bibliographic notes of Parts II and III.

We hope that this book bridges a gap in the literature. Indeed, although almost every book on oscillation theory (or, in modern terms, on nonlinear dynamics) treats synchronization among other nonlinear effects, only the books by Blekhman [1971, 1981], written in the "pre-chaotic" era, are devoted especially to the subject. These books mainly deal with mechanical and electromechanical systems, but they also contain extensive reviews on the theory, natural phenomena and applications in various fields. In writing our book we made an attempt to combine a description of classical theory and a comprehensive review of recent results, with an emphasis on interdisciplinary applications.

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Book homepage

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