

Cambridge University Press

0521592852 - Synchronization: A Universal Concept in Nonlinear Sciences

Arkady Pikovsky, Michael Rosenblum and Jurgen Kurths

Frontmatter

[More information](#)

Synchronization

A universal concept in nonlinear sciences

First recognized in 1665 by Christiaan Huygens, synchronization phenomena are abundant in science, nature, engineering, and social life. Systems as diverse as clocks, singing crickets, cardiac pacemakers, firing neurons, and applauding audiences exhibit a tendency to operate in synchrony. These phenomena are universal and can be understood within a common framework based on modern nonlinear dynamics. The first half of this book describes synchronization without formulae, and is based on qualitative intuitive ideas. The main effects are illustrated with experimental examples and figures; the historical development is also outlined. The second half of the book presents the main effects of synchronization in a rigorous and systematic manner, describing both classical results on the synchronization of periodic oscillators and recent developments in chaotic systems, large ensembles, and oscillatory media. This comprehensive book will be of interest to a broad audience, from graduate students to specialist researchers in physics, applied mathematics, engineering, and natural sciences.

ARKADY PIKOVSKY was part of the Max-Planck research group on nonlinear dynamics before becoming Professor of Statistical Physics and Theory of Chaos at the University of Potsdam, Germany. A member of the German and American Physical Societies, he is also part of the Editorial Board of *Physical Review E*, for the term 2000–2002. Before this, he was a Humboldt fellow at the University of Wuppertal. His PhD focused on the theory of chaos and nonlinear dynamics, and was carried out at the Institute of Applied Physics of the USSR Academy of Sciences. Arkady Pikovsky studied radiophysics and physics at Gorky State University, and started to work on chaos in 1976, describing an electronic device generating chaos in his Diploma thesis and later proving it experimentally.

MICHAEL ROSENBLUM has been a research associate in the Department of Physics, University of Potsdam, since 1997. His main research interests are the application of oscillation theory and nonlinear dynamics to biological systems and time series analysis. He was a Humboldt fellow in the Max-Planck research group on nonlinear dynamics, and a visiting scientist at Boston University. Michael Rosenblum studied physics at Moscow Pedagogical University, and went on to work in the Mechanical Engineering Research Institute of the USSR Academy of Sciences, where he was awarded a PhD in physics and mathematics.

Cambridge University Press

0521592852 - Synchronization: A Universal Concept in Nonlinear Sciences

Arkady Pikovsky, Michael Rosenblum and Jurgen Kurths

Frontmatter

[More information](#)

JÜRGEN KURTHS has been Professor of Nonlinear Dynamics at the University of Potsdam and director of the Interdisciplinary Centre for Dynamics of Complex Systems since 1994. He is a fellow of the American Physical Society and fellow of the Fraunhofer Society (Germany), and is currently vice-president of the European Geophysical Society. He is also a member of the Editorial Board of the *International Journal of Bifurcation and Chaos*. Professor Kurths was director of the group for nonlinear dynamics of the Max-Planck Society from 1992 to 1996. He studied mathematics at Rostock University, and then went on to work at the Solar–Terrestrial Physics Institute, and later the Astrophysical Institute of the GDR Academy of Sciences. He obtained his PhD in physics, and started to work on nonlinear data analysis and chaos in 1984. His main research interests are nonlinear dynamics and their application to geophysics and physiology and to time series analysis.

Cambridge University Press
0521592852 - Synchronization: A Universal Concept in Nonlinear Sciences
Arkady Pikovsky, Michael Rosenblum and Jurgen Kurths
Frontmatter
[More information](#)

Cambridge Nonlinear Science Series 12

Editors

Professor Boris Chirikov Budker Institute of Nuclear Physics, Novosibirsk	Professor Frank Moss University of Missouri, St Louis
Professor Predrag Cvitanović Niels Bohr Institute, Copenhagen	Professor Harry Swinney Center for Nonlinear Dynamics, The University of Texas at Austin

The Cambridge Nonlinear Science Series contains books on all aspects of contemporary research in classical and quantum nonlinear dynamics, both deterministic and nondeterministic, at the level of graduate text and monograph. The intention is to have an approximately equal blend of experimental and theoretical works, with the emphasis in the latter on testable results. Specific subject areas suitable for consideration include: Hamiltonian and dissipative chaos; squeezed states and applications of quantum measurement theory; pattern selection; formation and recognition; networks and learning systems; complexity in low- and high-dimensional systems and random noise; cellular automata; fully developed, weak and phase turbulence; reaction–diffusion systems; bifurcation theory and applications; self-structured states leading to chaos; the physics of interfaces, including fractal and multifractal growth; and simulations used in studies of these topics.

Titles in print in this series

G. M. Zaslavsky, R. Z. Sagdeev, D. A. Usikov and A. A. Chernikov 1. Weak chaos and quasi-regular patterns	H. Kantz and T. Schreiber 7. Nonlinear time series analysis
J. L. McCauley 2. Chaos, dynamics and fractals: an algorithmic approach to deterministic chaos	T. Bohr, M. H. Jensen, G. Paladin and A. Vulpiani 8. Dynamical systems approach to turbulence
C. Beck and F. Schlögl 4. Thermodynamics of chaotic systems: an introduction	P. Gaspard 9. Chaos, scattering and statistical mechanics
P. Meakin 5. Fractals, scaling and growth far from equilibrium	E. Schöll 10. Nonlinear spatio-temporal dynamics and chaos in semiconductors
R. Badii and A. Politi 6. Complexity – hierarchical structures and scaling in physics	J.-P. Rivet and J. P. Boon 11. Lattice gas hydrodynamics
	A. Pikovsky, M. Rosenblum and J. Kurths 12. Synchronization – a universal concept in nonlinear sciences

Cambridge University Press

0521592852 - Synchronization: A Universal Concept in Nonlinear Sciences

Arkady Pikovsky, Michael Rosenblum and Jürgen Kurths

Frontmatter

[More information](#)

Synchronization

A universal concept in nonlinear sciences

Arkady Pikovsky, Michael Rosenblum
and Jürgen Kurths

University of Potsdam, Germany



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
0521592852 - Synchronization: A Universal Concept in Nonlinear Sciences
Arkady Pikovsky, Michael Rosenblum and Jurgen Kurths
Frontmatter
[More information](#)

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014, Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa
<http://www.cambridge.org>

© A. Pikovsky, M. Rosenblum and J. Kurths 2001

This book is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without
the written permission of Cambridge University Press.

First published 2001
First paperback edition 2003
Reprinted 2003

Printed in the United Kingdom at the University Press, Cambridge

Typeface Times 10.25/13.5pt. *System* L^AT_EX 2_ε [DBD]

A catalogue record of this book is available from the British Library

Library of Congress Cataloguing in Publication data

Pikovsky, Arkady, 1956–
Synchronization: a universal concept in nonlinear sciences / Arkady Pikovsky,
Michael Rosenblum, Jürgen Kurths.
p. cm. – (The Cambridge nonlinear science series; 12)
Includes bibliographical references and index.
ISBN 0 521 59285 2
1. Synchronization. 2. Nonlinear theories. I. Rosenblum, Michael, 1958–
II. Kurths, J. (Jürgen), 1953– III. Title. IV. Series.

Q172.5.S96 P54 2001.
003'.75–dc21 2001018104

ISBN 0 521 59285 2 hardback
ISBN 0 521 53352 X paperback

Cambridge University Press
0521592852 - Synchronization: A Universal Concept in Nonlinear Sciences
Arkady Pikovsky, Michael Rosenblum and Jurgen Kurths
Frontmatter
[More information](#)

To my father Samuil AP
 To Sonya MR
To my father Herbert JK

Contents

Preface xvii

Chapter 1 Introduction 1

- 1.1 Synchronization in historical perspective 1
- 1.2 Synchronization: just a description 7
 - 1.2.1 *What is synchronization?* 8
 - 1.2.2 *What is NOT synchronization?* 14
- 1.3 Synchronization: an overview of different cases 18
 - 1.3.1 *Terminological remarks* 22
- 1.4 Main bibliography 23

Part I: Synchronization without formulae

Chapter 2 Basic notions: the self-sustained oscillator and its phase 27

- 2.1 Self-sustained oscillators: mathematical models of natural systems 28
 - 2.1.1 *Self-sustained oscillations are typical in nature* 28
 - 2.1.2 *Geometrical image of periodic self-sustained oscillations: limit cycle* 29
- 2.2 Phase: definition and properties 31
 - 2.2.1 *Phase and amplitude of a quasilinear oscillator* 31
 - 2.2.2 *Amplitude is stable, phase is free* 32
 - 2.2.3 *General case: limit cycle of arbitrary shape* 33
- 2.3 Self-sustained oscillators: main features 35
 - 2.3.1 *Dissipation, stability and nonlinearity* 35
 - 2.3.2 *Autonomous and forced systems: phase of a forced system is not free!* 38
- 2.4 Self-sustained oscillators: further examples and discussion 40
 - 2.4.1 *Typical self-sustained system: internal feedback loop* 40

2.4.2 *Relaxation oscillators* 41

Chapter 3 Synchronization of a periodic oscillator by external force 45

3.1 Weakly forced quasilinear oscillators 46

3.1.1 *The autonomous oscillator and the force in the rotating reference frame* 46

3.1.2 *Phase and frequency locking* 49

3.1.3 *Synchronization transition* 53

3.1.4 *An example: entrainment of respiration by a mechanical ventilator* 56

3.2 Synchronization by external force: extended discussion 59

3.2.1 *Stroboscopic observation* 59

3.2.2 *An example: periodically stimulated firefly* 61

3.2.3 *Entrainment by a pulse train* 62

3.2.4 *Synchronization of higher order: Arnold tongues* 65

3.2.5 *An example: periodic stimulation of atrial pacemaker cells* 67

3.2.6 *Phase and frequency locking: general formulation* 67

3.2.7 *An example: synchronization of a laser* 69

3.3 Synchronization of relaxation oscillators: special features 71

3.3.1 *Resetting by external pulses. An example: the cardiac pacemaker* 71

3.3.2 *Electrical model of the heart by van der Pol and van der Mark* 72

3.3.3 *Variation of the threshold. An example: the electronic relaxation oscillator* 73

3.3.4 *Variation of the natural frequency* 76

3.3.5 *Modulation vs. synchronization* 77

3.3.6 *An example: synchronization of the songs of snowy tree crickets* 78

3.4 Synchronization in the presence of noise 79

3.4.1 *Phase diffusion in a noisy oscillator* 80

3.4.2 *Forced noisy oscillators. Phase slips* 81

3.4.3 *An example: entrainment of respiration by mechanical ventilation* 85

3.4.4 *An example: entrainment of the cardiac rhythm by weak external stimuli* 85

3.5 Diverse examples 86

3.5.1 *Circadian rhythms* 86

3.5.2 *The menstrual cycle* 88

3.5.3 *Entrainment of pulsatile insulin secretion by oscillatory glucose infusion* 89

3.5.4 *Synchronization in protoplasmic strands of Physarum* 90

3.6 Phenomena around synchronization 90

3.6.1 *Related effects at strong external forcing* 91

3.6.2 *Stimulation of excitable systems* 93

3.6.3 *Stochastic resonance from the synchronization viewpoint* 94

3.6.4 *Entrainment of several oscillators by a common drive* 98

Chapter 4	Synchronization of two and many oscillators	102
4.1	Mutual synchronization of self-sustained oscillators	102
4.1.1	<i>Two interacting oscillators</i>	103
4.1.2	<i>An example: synchronization of triode generators</i>	105
4.1.3	<i>An example: respiratory and wing beat frequency of free-flying barnacle geese</i>	107
4.1.4	<i>An example: transition between in-phase and anti-phase motion</i>	108
4.1.5	<i>Concluding remarks and related effects</i>	110
4.1.6	<i>Relaxation oscillators. An example: true and latent pacemaker cells in the sino-atrial node</i>	111
4.1.7	<i>Synchronization of noisy systems. An example: brain and muscle activity of a Parkinsonian patient</i>	112
4.1.8	<i>Synchronization of rotators. An example: Josephson junctions</i>	114
4.1.9	<i>Several oscillators</i>	117
4.2	Chains, lattices and oscillatory media	119
4.2.1	<i>Synchronization in a lattice. An example: laser arrays</i>	119
4.2.2	<i>Formation of clusters. An example: electrical activity of mammalian intestine</i>	121
4.2.3	<i>Clusters and beats in a medium: extended discussion</i>	122
4.2.4	<i>Periodically forced oscillatory medium. An example: forced Belousov–Zhabotinsky reaction</i>	124
4.3	Globally coupled oscillators	126
4.3.1	<i>Kuramoto self-synchronization transition</i>	126
4.3.2	<i>An example: synchronization of menstrual cycles</i>	129
4.3.3	<i>An example: synchronization of glycolytic oscillations in a population of yeast cells</i>	130
4.3.4	<i>Experimental study of rhythmic hand clapping</i>	131
4.4	Diverse examples	131
4.4.1	<i>Running and breathing in mammals</i>	131
4.4.2	<i>Synchronization of two salt-water oscillators</i>	133
4.4.3	<i>Entrainment of tubular pressure oscillations in nephrons</i>	133
4.4.4	<i>Populations of cells</i>	133
4.4.5	<i>Synchronization of predator–prey cycles</i>	134
4.4.6	<i>Synchronization in neuronal systems</i>	134
Chapter 5	Synchronization of chaotic systems	137
5.1	Chaotic oscillators	137
5.1.1	<i>An exemplar: the Lorenz model</i>	138
5.1.2	<i>Sensitive dependence on initial conditions</i>	140

xii	Contents
5.2	Phase synchronization of chaotic oscillators 141
5.2.1	<i>Phase and average frequency of a chaotic oscillator</i> 142
5.2.2	<i>Entrainment by a periodic force. An example: forced chaotic plasma discharge</i> 144
5.3	Complete synchronization of chaotic oscillators 147
5.3.1	<i>Complete synchronization of identical systems. An example: synchronization of two lasers</i> 148
5.3.2	<i>Synchronization of nonidentical systems</i> 149
5.3.3	<i>Complete synchronization in a general context. An example: synchronization and clustering of globally coupled electrochemical oscillators</i> 150
5.3.4	<i>Chaos-destroying synchronization</i> 152
 Chapter 6 Detecting synchronization in experiments 153	
6.1	Estimating phases and frequencies from data 153
6.1.1	<i>Phase of a spike train. An example: electrocardiogram</i> 154
6.1.2	<i>Phase of a narrow-band signal. An example: respiration</i> 155
6.1.3	<i>Several practical remarks</i> 155
6.2	Data analysis in “active” and “passive” experiments 156
6.2.1	<i>“Active” experiment</i> 156
6.2.2	<i>“Passive” experiment</i> 157
6.3	Analyzing relations between the phases 160
6.3.1	<i>Straightforward analysis of the phase difference. An example: posture control in humans</i> 160
6.3.2	<i>High level of noise</i> 163
6.3.3	<i>Stroboscopic technique</i> 163
6.3.4	<i>Phase stroboscope in the case $n\Omega_1 \approx m\Omega_2$. An example: cardiorespiratory interaction</i> 164
6.3.5	<i>Phase relations in the case of strong modulation. An example: spiking of electroreceptors of a paddlefish</i> 166
6.4	Concluding remarks and bibliographic notes 168
6.4.1	<i>Several remarks on “passive” experiments</i> 168
6.4.2	<i>Quantification and significance of phase relation analysis</i> 170
6.4.3	<i>Some related references</i> 171
 Part II: Phase locking and frequency entrainment	
Chapter 7 Synchronization of periodic oscillators by periodic external action 175	
7.1	Phase dynamics 176
7.1.1	<i>A limit cycle and the phase of oscillations</i> 176

- 7.1.2 *Small perturbations and isochrones* 177
- 7.1.3 *An example: complex amplitude equation* 179
- 7.1.4 *The equation for the phase dynamics* 180
- 7.1.5 *An example: forced complex amplitude equations* 181
- 7.1.6 *Slow phase dynamics* 182
- 7.1.7 *Slow phase dynamics: phase locking and synchronization region* 184
- 7.1.8 *Summary of the phase dynamics* 187
- 7.2 *Weakly nonlinear oscillator* 189
- 7.2.1 *The amplitude equation* 189
- 7.2.2 *Synchronization properties: isochronous case* 192
- 7.2.3 *Synchronization properties: nonisochronous case* 198
- 7.3 *The circle and annulus map* 199
- 7.3.1 *The circle map: derivation and examples* 201
- 7.3.2 *The circle map: properties* 204
- 7.3.3 *The annulus map* 210
- 7.3.4 *Large force and transition to chaos* 213
- 7.4 *Synchronization of rotators and Josephson junctions* 215
- 7.4.1 *Dynamics of rotators and Josephson junctions* 215
- 7.4.2 *Overdamped rotator in an external field* 217
- 7.5 *Phase locked loops* 218
- 7.6 *Bibliographic notes* 221

Chapter 8 Mutual synchronization of two interacting periodic oscillators 222

- 8.1 *Phase dynamics* 222
- 8.1.1 *Averaged equations for the phase* 224
- 8.1.2 *Circle map* 226
- 8.2 *Weakly nonlinear oscillators* 227
- 8.2.1 *General equations* 227
- 8.2.2 *Oscillation death, or quenching* 229
- 8.2.3 *Attractive and repulsive interaction* 230
- 8.3 *Relaxation oscillators* 232
- 8.4 *Bibliographic notes* 235

Chapter 9 Synchronization in the presence of noise 236

- 9.1 *Self-sustained oscillator in the presence of noise* 236
- 9.2 *Synchronization in the presence of noise* 237
- 9.2.1 *Qualitative picture of the Langevin dynamics* 237
- 9.2.2 *Quantitative description for white noise* 240

9.2.3 *Synchronization by a quasiharmonic fluctuating force* 244
9.2.4 *Mutual synchronization of noisy oscillators* 245
9.3 Bibliographic notes 246

Chapter 10 Phase synchronization of chaotic systems 247

10.1 Phase of a chaotic oscillator 248
10.1.1 *Notion of the phase* 248
10.1.2 *Phase dynamics of chaotic oscillators* 254
10.2 Synchronization of chaotic oscillators 255
10.2.1 *Phase synchronization by external force* 256
10.2.2 *Indirect characterization of synchronization* 258
10.2.3 *Synchronization in terms of unstable periodic orbits* 260
10.2.4 *Mutual synchronization of two coupled oscillators* 262
10.3 Bibliographic notes 263

Chapter 11 Synchronization in oscillatory media 266

11.1 Oscillator lattices 266
11.2 Spatially continuous phase profiles 269
11.2.1 *Plane waves and targets* 269
11.2.2 *Effect of noise: roughening vs. synchronization* 271
11.3 Weakly nonlinear oscillatory medium 273
11.3.1 *Complex Ginzburg–Landau equation* 273
11.3.2 *Forcing oscillatory media* 276
11.4 Bibliographic notes 278

Chapter 12 Populations of globally coupled oscillators 279

12.1 The Kuramoto transition 279
12.2 Noisy oscillators 283
12.3 Generalizations 286
12.3.1 *Models based on phase approximation* 286
12.3.2 *Globally coupled weakly nonlinear oscillators* 289
12.3.3 *Coupled relaxation oscillators* 290
12.3.4 *Coupled Josephson junctions* 291
12.3.5 *Finite-size effects* 294
12.3.6 *Ensemble of chaotic oscillators* 294
12.4 Bibliographic notes 296

Part III: Synchronization of chaotic systems

Chapter 13 Complete synchronization I: basic concepts 301

13.1	The simplest model: two coupled maps	302
13.2	Stability of the synchronous state	304
13.3	Onset of synchronization: statistical theory	307
13.3.1	<i>Perturbation is a random walk process</i>	307
13.3.2	<i>The statistics of finite-time Lyapunov exponents determine diffusion</i>	308
13.3.3	<i>Modulational intermittency: power-law distributions</i>	310
13.3.4	<i>Modulational intermittency: correlation properties</i>	316
13.4	Onset of synchronization: topological aspects	318
13.4.1	<i>Transverse bifurcations of periodic orbits</i>	318
13.4.2	<i>Weak vs. strong synchronization</i>	319
13.4.3	<i>Local and global riddling</i>	322
13.5	Bibliographic notes	323

Chapter 14 Complete synchronization II: generalizations and complex systems 324

14.1	Identical maps, general coupling operator	324
14.1.1	<i>Unidirectional coupling</i>	325
14.1.2	<i>Asymmetric local coupling</i>	327
14.1.3	<i>Global (mean field) coupling</i>	328
14.2	Continuous-time systems	329
14.3	Spatially distributed systems	331
14.3.1	<i>Spatially homogeneous chaos</i>	331
14.3.2	<i>Transverse synchronization of space–time chaos</i>	332
14.3.3	<i>Synchronization of coupled cellular automata</i>	334
14.4	Synchronization as a general symmetric state	335
14.4.1	<i>Replica-symmetric systems</i>	336
14.5	Bibliographic notes	337

Chapter 15 Synchronization of complex dynamics by external forces 340

15.1	Synchronization by periodic forcing	341
15.2	Synchronization by noisy forcing	341
15.2.1	<i>Noisy forced periodic oscillations</i>	343
15.2.2	<i>Synchronization of chaotic oscillations by noisy forcing</i>	345
15.3	Synchronization of chaotic oscillations by chaotic forcing	346
15.3.1	<i>Complete synchronization</i>	346
15.3.2	<i>Generalized synchronization</i>	347

Cambridge University Press
0521592852 - Synchronization: A Universal Concept in Nonlinear Sciences
Arkady Pikovsky, Michael Rosenblum and Jurgen Kurths
Frontmatter
[More information](#)

15.3.3 *Generalized synchronization by quasiperiodic driving* 352
15.4 Bibliographic notes 353

Appendices

Appendix A1: Discovery of synchronization by Christiaan Huygens 357

A1.1 A letter from Christiaan Huygens to his father, Constantyn Huygens 357
A1.2 Sea clocks (sympathy of clocks). Part V 358

Appendix A2: Instantaneous phase and frequency of a signal 362

A2.1 Analytic signal and the Hilbert transform 362
A2.2 Examples 363
A2.3 Numerics: practical hints and know-hows 366
A2.4 Computation of the instantaneous frequency 369

References 371
Index 405

Preface

The word “synchronous” is often encountered in both scientific and everyday language. Originating from the Greek words *χρόνος* (*chronos*, meaning time) and *σύν* (*syn*, meaning the same, common), in a direct translation “synchronous” means “sharing the common time”, “occurring in the same time”. This term, as well as the related words “synchronization” and “synchronized”, refers to a variety of phenomena in almost all branches of natural sciences, engineering and social life, phenomena that appear to be rather different but nevertheless often obey universal laws.

A search in any scientific data base for publication titles containing the words with the root “synchro” produces many hundreds (if not thousands) of entries. Initially, this effect was found and investigated in different man-made devices, from pendulum clocks to musical instruments, electronic generators, electric power systems, and lasers. It has found numerous practical applications in electrical and mechanical engineering. Nowadays the “center of gravity” of the research has moved towards biological systems, where synchronization is encountered on different levels. Synchronous variation of cell nuclei, synchronous firing of neurons, adjustment of heart rate with respiration and/or locomotory rhythms, different forms of cooperative behavior of insects, animals and even humans – these are only some examples of the fundamental natural phenomenon that is the subject of this book.

Our surroundings are full of oscillating objects. Radio communication and electrical equipment, violins in an orchestra, fireflies emitting sequences of light pulses, crickets producing chirps, birds flapping their wings, chemical systems exhibiting oscillatory variation of the concentration of reagents, a neural center that controls the

contraction of the human heart and the heart itself, a center of pathological activity that causes involuntary shaking of limbs as a consequence of Parkinson’s disease – all these and many other systems have a common feature: they produce rhythms. Usually these objects are not isolated from their environment, but interact with other objects, in other words they are open systems. Indeed, biological clocks that govern daily (circadian) cycles are subject to the day–night and seasonal variations of illuminance and temperature, a violinist hears the tones played by her/his neighbors, a firefly is influenced by the light emission of the whole population, different centers of rhythmic brain activity may influence each other, etc. This interaction can be very weak, sometimes hardly perceptible, but nevertheless it often causes a qualitative transition: an object adjusts its rhythm in conformity with the rhythms of other objects. As a result, violinists play in unison, insects in a population emit acoustic or light pulses with a common rate, birds in a flock flap their wings simultaneously, the heart of a rapidly galloping horse contracts once per locomotory cycle.

This adjustment of rhythms due to an interaction is the essence of synchronization, the phenomenon that is systematically studied in this book.

The aim of the book is to address a broad readership: physicists, chemists, biologists, engineers, as well as other scientists conducting interdisciplinary research;¹ it is intended for both theoreticians and experimentalists. Therefore, the presentation of experimental facts, of the main principles, and of the mathematical tools is not uniform and sometimes repetitive. The diversity of the audience is reflected in the structure of the book.

The first part of this book, Synchronization without formulae, is aimed at readers with minimal mathematical background (pre-calculus), or at least it was written with this intention. Although Part I contains almost no equations, it describes and explains the main ideas and effects at a qualitative level.² Here we illustrate synchronization phenomena with experiments and observations from various fields. Part I can be skipped by theoretically oriented specialists in physics and nonlinear dynamics, or it may be useful for examples and applications.

Parts II and III cover the same ideas, but on a quantitative level; the reader of these parts is assumed to be acquainted with the basics of nonlinear dynamics. We hope that the bulk of the presentation will be comprehensible for graduate students. Here we review classical results on the synchronization of periodic oscillators, both with and without noisy perturbations; consider synchronization phenomena in ensembles of oscillators as well as in spatially distributed systems; present different effects that occur due to the interaction of chaotic systems; provide the reader with an extensive bibliography.

¹ As the authors are physicists, the book is inevitably biased towards the physical description of the natural phenomena.
² To simplify the presentation, we omit in Part I citations to the original works where these ideas were introduced; one can find the relevant references in the bibliographic section of the Introduction as well as in the bibliographic notes of Parts II and III.

Cambridge University Press

0521592852 - Synchronization: A Universal Concept in Nonlinear Sciences

Arkady Pikovsky, Michael Rosenblum and Jurgen Kurths

Frontmatter

[More information](#)

We hope that this book bridges a gap in the literature. Indeed, although almost every book on oscillation theory (or, in modern terms, on nonlinear dynamics) treats synchronization among other nonlinear effects, only the books by Blekhman [1971, 1981], written in the “pre-chaotic” era, are devoted especially to the subject. These books mainly deal with mechanical and electromechanical systems, but they also contain extensive reviews on the theory, natural phenomena and applications in various fields. In writing our book we made an attempt to combine a description of classical theory and a comprehensive review of recent results, with an emphasis on interdisciplinary applications.

Acknowledgments

In the course of our studies of synchronization we enjoyed collaborations and discussions with V. S. Afraimovich, V. S. Anishchenko, B. Blasius, I. I. Blekhman, H. Chaté, U. Feudel, P. Glendinning, P. Grassberger, C. Grebogi, J. Hudson, S. P. Kuznetsov, P. S. Landa, A. Lichtenberg, R. Livi, Ph. Marcq, Yu. Maistrenko, E. Mosekilde, F. Moss, A. B. Neiman, G. V. Osipov, E.-H. Park, U. Parlitz, K. Piragas, A. Politi, O. Popovich, R. Roy, O. Rudzick, S. Ruffo, N. Rulkov, C. Schäfer, L. Schimansky-Geier, L. Stone, H. Swinney, P. Tass, E. Toledo, and A. Zaikin.

The comments of A. Nepomnyashchy, A. A. Pikovski, A. Politi, and C. Ziehmann, who read parts of the book, are highly appreciated.

O. Futer, N. B. Igosheva, and R. Mrowka patiently answered our numerous questions regarding medical and biological problems.

We would like to express our special gratitude to Michael Zaks, who supported our endeavor at all stages.

We also thank Philips International B.V., Company Archives, Eindhoven, the Netherlands for sending photographs and biography of Balthasar van der Pol and A. Kurths for her help in the preparation of the bibliography.

Finally, we acknowledge the kind assistance of the Cambridge University Press staff. We are especially thankful to S. Capelin for his encouragement and patience, and to F. Chapman for her excellent work on improving the manuscript.

Book homepage

We encourage all who wish to comment on the book to send e-mails to:

pikovsky@stat.physik.uni-potsdam.de;

mros@agnld.uni-potsdam.de;

jkurths@agnld.uni-potsdam.de.

All misprints and errors will be posted on the book homepage

(URL: <http://www.agnld.uni-potsdam.de/~syn-book/>).