

## Contents

	<i>Editor's foreword</i>	<i>page</i> xvii
	<i>Preface</i>	xix
<b>Part I</b>	<b>Principles and elementary applications</b>	
	1 Plausible reasoning	3
	1.1 Deductive and plausible reasoning	3
	1.2 Analogies with physical theories	6
	1.3 The thinking computer	7
	1.4 Introducing the robot	8
	1.5 Boolean algebra	9
	1.6 Adequate sets of operations	12
	1.7 The basic desiderata	17
	1.8 Comments	19
	1.8.1 Common language vs. formal logic	21
	1.8.2 Nitpicking	23
	2 The quantitative rules	24
	2.1 The product rule	24
	2.2 The sum rule	30
	2.3 Qualitative properties	35
	2.4 Numerical values	37
	2.5 Notation and finite-sets policy	43
	2.6 Comments	44
	2.6.1 'Subjective' vs. 'objective'	44
	2.6.2 Gödel's theorem	45
	2.6.3 Venn diagrams	47
	2.6.4 The 'Kolmogorov axioms'	49
	3 Elementary sampling theory	51
	3.1 Sampling without replacement	52
	3.2 Logic vs. propensity	60
	3.3 Reasoning from less precise information	64
	3.4 Expectations	66
	3.5 Other forms and extensions	68

3.6	Probability as a mathematical tool	68
3.7	The binomial distribution	69
3.8	Sampling with replacement	72
3.8.1	Digression: a sermon on reality vs. models	73
3.9	Correction for correlations	75
3.10	Simplification	81
3.11	Comments	82
3.11.1	A look ahead	84
4	Elementary hypothesis testing	86
4.1	Prior probabilities	87
4.2	Testing binary hypotheses with binary data	90
4.3	Nonextensibility beyond the binary case	97
4.4	Multiple hypothesis testing	98
4.4.1	Digression on another derivation	101
4.5	Continuous probability distribution functions	107
4.6	Testing an infinite number of hypotheses	109
4.6.1	Historical digression	112
4.7	Simple and compound (or composite) hypotheses	115
4.8	Comments	116
4.8.1	Etymology	116
4.8.2	What have we accomplished?	117
5	Queer uses for probability theory	119
5.1	Extrasensory perception	119
5.2	Mrs Stewart's telepathic powers	120
5.2.1	Digression on the normal approximation	122
5.2.2	Back to Mrs Stewart	122
5.3	Converging and diverging views	126
5.4	Visual perception – evolution into Bayesianity?	132
5.5	The discovery of Neptune	133
5.5.1	Digression on alternative hypotheses	135
5.5.2	Back to Newton	137
5.6	Horse racing and weather forecasting	140
5.6.1	Discussion	142
5.7	Paradoxes of intuition	143
5.8	Bayesian jurisprudence	144
5.9	Comments	146
5.9.1	What is queer?	148
6	Elementary parameter estimation	149
6.1	Inversion of the urn distributions	149
6.2	Both $N$ and $R$ unknown	150
6.3	Uniform prior	152
6.4	Predictive distributions	154

<i>Contents</i>		ix
6.5	Truncated uniform priors	157
6.6	A concave prior	158
6.7	The binomial monkey prior	160
6.8	Metamorphosis into continuous parameter estimation	163
6.9	Estimation with a binomial sampling distribution	163
6.9.1	Digression on optional stopping	166
6.10	Compound estimation problems	167
6.11	A simple Bayesian estimate: quantitative prior information	168
6.11.1	From posterior distribution function to estimate	172
6.12	Effects of qualitative prior information	177
6.13	Choice of a prior	178
6.14	On with the calculation!	179
6.15	The Jeffreys prior	181
6.16	The point of it all	183
6.17	Interval estimation	186
6.18	Calculation of variance	186
6.19	Generalization and asymptotic forms	188
6.20	Rectangular sampling distribution	190
6.21	Small samples	192
6.22	Mathematical trickery	193
6.23	Comments	195
7	The central, Gaussian or normal distribution	198
7.1	The gravitating phenomenon	199
7.2	The Herschel–Maxwell derivation	200
7.3	The Gauss derivation	202
7.4	Historical importance of Gauss’s result	203
7.5	The Landon derivation	205
7.6	Why the ubiquitous use of Gaussian distributions?	207
7.7	Why the ubiquitous success?	210
7.8	What estimator should we use?	211
7.9	Error cancellation	213
7.10	The near irrelevance of sampling frequency distributions	215
7.11	The remarkable efficiency of information transfer	216
7.12	Other sampling distributions	218
7.13	Nuisance parameters as safety devices	219
7.14	More general properties	220
7.15	Convolution of Gaussians	221
7.16	The central limit theorem	222
7.17	Accuracy of computations	224
7.18	Galton’s discovery	227
7.19	Population dynamics and Darwinian evolution	229
7.20	Evolution of humming-birds and flowers	231

7.21	Application to economics	233
7.22	The great inequality of Jupiter and Saturn	234
7.23	Resolution of distributions into Gaussians	235
7.24	Hermite polynomial solutions	236
7.25	Fourier transform relations	238
7.26	There is hope after all	239
7.27	Comments	240
	7.27.1 Terminology again	240
8	Sufficiency, ancillarity, and all that	243
8.1	Sufficiency	243
8.2	Fisher sufficiency	245
	8.2.1 Examples	246
	8.2.2 The Blackwell–Rao theorem	247
8.3	Generalized sufficiency	248
8.4	Sufficiency plus nuisance parameters	249
8.5	The likelihood principle	250
8.6	Ancillarity	253
8.7	Generalized ancillary information	254
8.8	Asymptotic likelihood: Fisher information	256
8.9	Combining evidence from different sources	257
8.10	Pooling the data	260
	8.10.1 Fine-grained propositions	261
8.11	Sam’s broken thermometer	262
8.12	Comments	264
	8.12.1 The fallacy of sample re-use	264
	8.12.2 A folk theorem	266
	8.12.3 Effect of prior information	267
	8.12.4 Clever tricks and gamesmanship	267
9	Repetitive experiments: probability and frequency	270
9.1	Physical experiments	271
9.2	The poorly informed robot	274
9.3	Induction	276
9.4	Are there general inductive rules?	277
9.5	Multiplicity factors	280
9.6	Partition function algorithms	281
	9.6.1 Solution by inspection	282
9.7	Entropy algorithms	285
9.8	Another way of looking at it	289
9.9	Entropy maximization	290
9.10	Probability and frequency	292
9.11	Significance tests	293
	9.11.1 Implied alternatives	296

<i>Contents</i>		xi
9.12	Comparison of psi and chi-squared	300
9.13	The chi-squared test	302
9.14	Generalization	304
9.15	Halley's mortality table	305
9.16	Comments	310
9.16.1	The irrationalists	310
9.16.2	Superstitions	312
10	Physics of 'random experiments'	314
10.1	An interesting correlation	314
10.2	Historical background	315
10.3	How to cheat at coin and die tossing	317
10.3.1	Experimental evidence	320
10.4	Bridge hands	321
10.5	General random experiments	324
10.6	Induction revisited	326
10.7	But what about quantum theory?	327
10.8	Mechanics under the clouds	329
10.9	More on coins and symmetry	331
10.10	Independence of tosses	335
10.11	The arrogance of the uninformed	338
<b>Part II Advanced applications</b>		
11	Discrete prior probabilities: the entropy principle	343
11.1	A new kind of prior information	343
11.2	Minimum $\sum p_i^2$	345
11.3	Entropy: Shannon's theorem	346
11.4	The Wallis derivation	351
11.5	An example	354
11.6	Generalization: a more rigorous proof	355
11.7	Formal properties of maximum entropy distributions	358
11.8	Conceptual problems – frequency correspondence	365
11.9	Comments	370
12	Ignorance priors and transformation groups	372
12.1	What are we trying to do?	372
12.2	Ignorance priors	374
12.3	Continuous distributions	374
12.4	Transformation groups	378
12.4.1	Location and scale parameters	378
12.4.2	A Poisson rate	382
12.4.3	Unknown probability for success	382
12.4.4	Bertrand's problem	386
12.5	Comments	394

13	Decision theory, historical background	397
13.1	Inference vs. decision	397
13.2	Daniel Bernoulli's suggestion	398
13.3	The rationale of insurance	400
13.4	Entropy and utility	402
13.5	The honest weatherman	402
13.6	Reactions to Daniel Bernoulli and Laplace	404
13.7	Wald's decision theory	406
13.8	Parameter estimation for minimum loss	410
13.9	Reformulation of the problem	412
13.10	Effect of varying loss functions	415
13.11	General decision theory	417
13.12	Comments	418
13.12.1	'Objectivity' of decision theory	418
13.12.2	Loss functions in human society	421
13.12.3	A new look at the Jeffreys prior	423
13.12.4	Decision theory is not fundamental	423
13.12.5	Another dimension?	424
14	Simple applications of decision theory	426
14.1	Definitions and preliminaries	426
14.2	Sufficiency and information	428
14.3	Loss functions and criteria of optimum performance	430
14.4	A discrete example	432
14.5	How would our robot do it?	437
14.6	Historical remarks	438
14.6.1	The classical matched filter	439
14.7	The widget problem	440
14.7.1	Solution for Stage 2	443
14.7.2	Solution for Stage 3	445
14.7.3	Solution for Stage 4	449
14.8	Comments	450
15	Paradoxes of probability theory	451
15.1	How do paradoxes survive and grow?	451
15.2	Summing a series the easy way	452
15.3	Nonconglomerability	453
15.4	The tumbling tetrahedra	456
15.5	Solution for a finite number of tosses	459
15.6	Finite vs. countable additivity	464
15.7	The Borel–Kolmogorov paradox	467
15.8	The marginalization paradox	470
15.8.1	On to greater disasters	474

<i>Contents</i>		xiii
15.9	Discussion	478
	15.9.1 The DSZ Example #5	480
	15.9.2 Summary	483
15.10	A useful result after all?	484
15.11	How to mass-produce paradoxes	485
15.12	Comments	486
16	Orthodox methods: historical background	490
16.1	The early problems	490
16.2	Sociology of orthodox statistics	492
16.3	Ronald Fisher, Harold Jeffreys, and Jerzy Neyman	493
16.4	Pre-data and post-data considerations	499
16.5	The sampling distribution for an estimator	500
16.6	Pro-causal and anti-causal bias	503
16.7	What is real, the probability or the phenomenon?	505
16.8	Comments	506
	16.8.1 Communication difficulties	507
17	Principles and pathology of orthodox statistics	509
17.1	Information loss	510
17.2	Unbiased estimators	511
17.3	Pathology of an unbiased estimate	516
17.4	The fundamental inequality of the sampling variance	518
17.5	Periodicity: the weather in Central Park	520
	17.5.1 The folly of pre-filtering data	521
17.6	A Bayesian analysis	527
17.7	The folly of randomization	531
17.8	Fisher: common sense at Rothamsted	532
	17.8.1 The Bayesian safety device	532
17.9	Missing data	533
17.10	Trend and seasonality in time series	534
	17.10.1 Orthodox methods	535
	17.10.2 The Bayesian method	536
	17.10.3 Comparison of Bayesian and orthodox estimates	540
	17.10.4 An improved orthodox estimate	541
	17.10.5 The orthodox criterion of performance	544
17.11	The general case	545
17.12	Comments	550
18	The $A_p$ distribution and rule of succession	553
18.1	Memory storage for old robots	553
18.2	Relevance	555
18.3	A surprising consequence	557
18.4	Outer and inner robots	559

18.5	An application	561
18.6	Laplace's rule of succession	563
18.7	Jeffreys' objection	566
18.8	Bass or carp?	567
18.9	So where does this leave the rule?	568
18.10	Generalization	568
18.11	Confirmation and weight of evidence	571
	18.11.1 Is indifference based on knowledge or ignorance?	573
18.12	Carnap's inductive methods	574
18.13	Probability and frequency in exchangeable sequences	576
18.14	Prediction of frequencies	576
18.15	One-dimensional neutron multiplication	579
	18.15.1 The frequentist solution	579
	18.15.2 The Laplace solution	581
18.16	The de Finetti theorem	586
18.17	Comments	588
19	Physical measurements	589
	19.1 Reduction of equations of condition	589
	19.2 Reformulation as a decision problem	592
	19.2.1 Sermon on Gaussian error distributions	592
	19.3 The underdetermined case: $K$ is singular	594
	19.4 The overdetermined case: $K$ can be made nonsingular	595
	19.5 Numerical evaluation of the result	596
	19.6 Accuracy of the estimates	597
	19.7 Comments	599
	19.7.1 A paradox	599
20	Model comparison	601
	20.1 Formulation of the problem	602
	20.2 The fair judge and the cruel realist	603
	20.2.1 Parameters known in advance	604
	20.2.2 Parameters unknown	604
	20.3 But where is the idea of simplicity?	605
	20.4 An example: linear response models	607
	20.4.1 Digression: the old sermon still another time	608
	20.5 Comments	613
	20.5.1 Final causes	614
21	Outliers and robustness	615
	21.1 The experimenter's dilemma	615
	21.2 Robustness	617
	21.3 The two-model model	619
	21.4 Exchangeable selection	620
	21.5 The general Bayesian solution	622



<i>Contents</i>		xv
21.6	Pure outliers	624
21.7	One receding datum	625
22	Introduction to communication theory	627
22.1	Origins of the theory	627
22.2	The noiseless channel	628
22.3	The information source	634
22.4	Does the English language have statistical properties?	636
22.5	Optimum encoding: letter frequencies known	638
22.6	Better encoding from knowledge of digram frequencies	641
22.7	Relation to a stochastic model	644
22.8	The noisy channel	648
Appendix A	Other approaches to probability theory	651
A.1	The Kolmogorov system of probability	651
A.2	The de Finetti system of probability	655
A.3	Comparative probability	656
A.4	Holdouts against universal comparability	658
A.5	Speculations about lattice theories	659
Appendix B	Mathematical formalities and style	661
B.1	Notation and logical hierarchy	661
B.2	Our 'cautious approach' policy	662
B.3	Willy Feller on measure theory	663
B.4	Kronecker vs. Weierstrasz	665
B.5	What is a legitimate mathematical function?	666
B.5.1	Delta-functions	668
B.5.2	Nondifferentiable functions	668
B.5.3	Bogus nondifferentiable functions	669
B.6	Counting infinite sets?	671
B.7	The Hausdorff sphere paradox and mathematical diseases	672
B.8	What am I supposed to publish?	674
B.9	Mathematical courtesies	675
Appendix C	Convolutions and cumulants	677
C.1	Relation of cumulants and moments	679
C.2	Examples	680
	<i>References</i>	683
	<i>Bibliography</i>	705
	<i>Author index</i>	721
	<i>Subject index</i>	724