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978-0-521-59075-4 - An Introduction to the Mathematics of Neurons: Modeling in the Frequency Domain, Second Edition

Frank C. Hoppensteadt

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