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Im Wald und auf der Heide,
da such ich meine Freude,
ich bin ein Jaegersmann.

Wilhelm Bornemann

CONTENTS

FOREWORD

As I believe is well known, I did not anticipate the wealth of mathematics that has resulted from the introduction of the spaces J and JT. In fact, the discovery of J was somewhat accidental. I had proved that a Banach space X is reflexive if each linear functional attains its supremum on the unit ball for any equivalent norm and if X has a basis with certain properties. This theorem lost interest when Victor Klee proved it without any assumption about a basis (Klee's theorem lost interest when I proved it with only the assumption that each linear functional attains its supremum on the given unit ball of X, but this theorem did not come easily!). However, the use of properties that had been assumed for the basis led to the realization that X^{**} could be described explicitly if X has a basis with a certain property (later called *shrinking* by M.M. Day). The definition of the space J isomorphic to J^{**} then came very easily. At a research conference about 24 years later, Charles Stegall asked what I thought about the conjecture that X has a subspace isomorphic with ℓ_1 if X is separable and X^{**} is not separable. He had asked this question the year before, but I had no ideas at that time. But this time I had been working on some other things that made the idea for JT come rather easily.

One always feels great pleasure when others discover applications of something one has done. Thus I feel deep gratitude for the work done by Helga Fetter Nathansky and Berta Gamboa de Buen in preparing this account of the mathematics that has developed from J and JT.

Robert C. James

PROLOGUE

When Stefan Banach introduced in the 30's the spaces which now carry his name, his aim was to provide a convenient framework for the solution of equations in infinitely many variables. Few examples of such spaces were known at that time: sequence spaces, function spaces.

The structure of Banach spaces was not as rich as that of Hilbert spaces (the inner product was missing), but it was general enough to handle a large variety of situations. The distinction was made, at an early stage, between reflexive and non-reflexive Banach spaces. The former enjoyed weak compactness properties, a tool which in many cases could replace inner products.

So Banach spaces developed smoothly, and many general theorems were proved, first by Banach himself, then by many others, for instance Steinhaus, Saks, and later Grothendieck (1950), Dvoretzky (1963).

The last open question, at this stage of the theory, was the existence of a basis, for separable spaces: it would have been nice to have some replacement tool for the so-convenient hilbertian basis, and the easy expansions it allows. The question was not too embarrassing, however: all known spaces had bases, and, despite the lack of success of Grothendieck on this question, one thought that some young and talented guy would soon come to settle the matter.

Unfortunately, before anyone could do it, and before the logicians could prove it undecidable, Per Enflo, in 1972, constructed an example of a separable Banach space with no basis. The opening of Pandora's box had awful consequences, and a lot of unexpected devils flew away: even the most ordinary spaces showed signs of disease, with pathological topologies and strange subspaces. Then the disease started spreading, and strange spaces started to show up: spaces with too few subspaces, or conversely too many, or just not the right ones, those which any civilized person would have expected.

Among the most horrible constructions, we cite those of R.C. James - the topic of the present book - who built a space isomorphic to its second dual, without being reflexive, and of B.S. Tsirelson, who created a space with only strange subspaces: none of them contained ℓ_p or c_0. The

present author contributed to the general hysteria, by creating a space which had all the bad properties of both James's and Tsirelson's spaces, without enjoying any of the good ones.

Is the box going to close, and shall we see – as the legend wants – Hope leaving last? We don't know: we have not seen it yet.

Such a considerable flourish of examples had at least one consequence: everyone got lost. Nobody knew any longer what to expect, and even the most impetuous newcomers could hardly make any conjecture, which, for a mathematician, is a sad situation. The only general structure theorem which has been proved since then was Rosenthal's, dealing with ℓ_1 and weak Cauchy subsequences.

So, in order to describe all these strange things, have a look at the past and a guess at the future, a book was needed. Here it is. It has a major quality: around a single example, James' space and its variations, it presents almost all the deep tools introduced by the Geometry of Banach spaces. It will, moreover, have another benefit: to help the diffusion of the results.

This tremendous activity was confined to a small circle of specialists, and had very little impact on other branches of mathematics. This is unfortunate: the powerful tools which have been created over the last fifty years should have more applications to other fields, such as, for instance, Operator Theory, Harmonic Analysis, Numerical Analysis, Economics; some applications are already presented by Pelczynski [1], and, more recently, by Wojtaszczyk [1]. The topics of the present book, pathological subspaces, should find general applications to Harmonic Analysis (see Varopoulos [1] for a first step in this direction) and to Approximation Theory.

If such a confinement were to last too long, the net effect would be harmful. All the patiently developed material might be forgotten by the next generation, unless it had to use it. As Thomas Gray said in 1742:

Full many a gem of purest ray serene,
The dark unfathomed caves of ocean bear;
Full many a flower is born to blush unseen,
And waste its sweetness in the desert air.

Bernard Beauzamy