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978-0-521-58631-3 - Elements of the Representation Theory of Associative Algebras: Volume 1 Techniques of Representation Theory

Ibrahim Assem, Andrzej Skowronski and Daniel Simson

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