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Nonparametric Econometrics

This book systematically and thoroughly covers a vast literature on the nonparametric and semiparametric statistics and econometrics that has evolved over the past five decades. Within this framework this is the first book to discuss the principles of the nonparametric approach to the topics covered in a first-year graduate course in econometrics, for example, regression function, heteroskedasticity, simultaneous equations models, logit-probit, and censored models. Nonparametric and semiparametric methods potentially offer considerable reward to applied researchers, owing to the methods' ability to adapt to many unknown features of the data. Professors Pagan and Ullah provide intuitive explanations of difficult concepts, heuristic developments of theory, and empirical examples emphasizing the usefulness of the modern nonparametric approach. The book should provide a new perspective on teaching and research in applied subjects in general and econometrics and statistics in particular.

Adrian Pagan is a Professor of Economics at the Institute of Advanced Studies, Australian National University. A Fellow of the Econometric Society, Australian Academy of Social Sciences, and Journal of Econometrics, he is the coauthor or author of several books and numerous articles in economics, econometrics, and public policy. Professor Pagan has been coeditor of the *Journal of Applied Econometrics and Econometric Theory* and associate editor of *Econometrica* and *Journal of Econometrics*. He is currently a member of the editorial boards of *Economic Record*, *Advances in Computational Economics*, and *Econometric Reviews* and is coeditor of the *Themes in Modern Econometrics* series for Cambridge University Press. He has also served as a visiting professor or scholar at UCLA, Johns Hopkins University, University of Rochester, Princeton and Yale Universities, and Institute of Advanced Studies, Vienna. Professor Pagan is also a member of the Board of Governors of the Reserve Bank of Australia.

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To my parents, Razia Begum and Ataullah Khan

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Preface

It took almost a decade to complete the work on this book. During this period Pagan moved from the University of Rochester to the Australian National University while Ullah moved to the University of California, Riverside from the University of Western Ontario. Such moves are always difficult and are made more so when the protagonists were trying to write about a field that at time seemed to be growing at an exponential rate. All of this disruption caused the book to be typed and retyped several times on two continents as well as ensured that the idea of completing it died and was reborn more than once. In the end our desire to see a completed text won out.

As anyone who performs applied work knows, many assumptions are made in coming up with answers. Mostly these involve assertions about the distributional features of variables and functional relations among them. It is not unusual for one to feel a little uncomfortable with some of these assumptions and to wonder what would happen if they could be relaxed. We were therefore rather intrigued to come across a literature in mathematical statistics that provided tools for doing some of the applied work with a minimal number of assumptions. This “nonparametrics” literature was found to be very useful in some of our early work on the risk premium, and that experience suggested that these methods could be quite useful more generally and that they should be more widely known to econometricians. Hence our objective became one of bringing this literature together in a systematic and integrated way within a single source. In the back of our mind was the idea that we might produce a book that paralleled traditional econometrics texts in the sense of explaining how one might perform the same analyses as was done parametrically in those books but in a nonparametric way. Principally, the idea was to provide a nonparametric treatment for the regression model along with its extensions to handle complications like heteroskedasticity and censoring. To some extent this vision faded under the weight of the rapidly expanding literature that we needed to cover, although traces of it are still evident in the selection of topics that are studied.

xviii **Preface**

The book is designed for those who might be beginning research in the area as well as those who are interested in appreciating some of the statistical theory that underlies the methods. Many of these techniques have seen increasing application as they have become more readily available in various computer packages. Generally the audience will be those taking graduate courses in econometrics and statistics, but the book may also be useful for students in other applied sciences, such as engineering, medicine, psychology, and sociology. Many techniques are discussed in the book. If it has a bias it is toward kernel-based methods of nonparametric estimation. The emphasis in the book is upon a discussion of the methods and the theory that underlies them. Some empirical examples and/or a summary of the empirical literature are presented to illustrate certain points, although the book is not an applied one in any sense.

It is a pleasure to express our appreciation toward those who have influenced this work. Michael Parkin was especially encouraging regarding this project. Joel Horowitz read the complete manuscript and made many valuable comments upon it. Given that we have sometimes not incorporated what he said he should not be blamed for it in any way. Many others have also made their mark on it. In particular, N. Roy, C. Chiu, J. Racine, P. Rilstone, Y. Fan, Q. Li, R. Breunig, M. Brenner, S. Kumar, K. Mundra, and S. Thompson have read and commented on several chapters. On the production side, B. Chatman and K. Lowney typed early drafts with skill and patience. Finally, as those closest to you are the ones who bear the greatest cost, we want to record our tremendous debt to our families who suffered through it all. For Aman this means his wife, Shobha, and daughter, Sushana. For Adrian it is Janet, Becky, and Mandy.