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PREFACE TO THE SECOND EDITION

This book has evolved from a course of lectures for final-year undergraduates and first-year postgraduates. This was given at the University of Nottingham, first in the session 1972–73 and many times since, and at Pusan National University and the University of Adelaide. The material is accessible to anyone with the mathematical maturity consistent with first courses in linear algebra, group theory and ring theory, and is primarily intended as an introduction to the subject known as Combinatorial Group Theory. This abuts on other branches of group theory: finite, infinite, homological, and computational. A secondary aim is to introduce a wide variety of examples of groups and types of group.

No attempt at completeness is feasible in a work of this size and scope. Major omissions, or taking-off points, include small cancellation theory, decision problems and embedding theorems as well as such other topics of historical and current importance as commutator calculus, Fuchsian groups, braid and knot groups, one-relator groups, free products with amalgamation, HNN-extensions, and geometric methods.

This edition is an extension of the first, with errors corrected, diagrams improved and new material added: several exercises and Chapter 16.

My thanks are due to a host of people (not least the students who have attended the course) whose names are to be found scattered through the pages of the text. It is a pleasure to acknowledge a special debt of gratitude to Sandy Green for first introducing me to research mathematics, and to Roger Lyndon, Bernhard Neumann, Jim Wiegold, Mike Newman, Edmund Robertson, Rick Thomas and Geoff Smith for their support and encouragement over the years. Thanks also to Roger Astley and all at C.U.P. for their efficient handling of the production, promotion and distribution.