
Contents

<i>Preface</i>	<i>page</i> ix
1 Introduction to probability theory	1
1.1 Sample points, events and probabilities	1
1.1.1 An algebra of events	3
1.1.2 Probabilities	4
1.1.3 Conditional probability	7
1.2 Random variables	10
1.2.1 Distribution functions	11
1.2.2 Probability density functions	13
1.2.3 Joint distributions	15
1.2.4 Largest and smallest	18
1.3 Expectation and other moments	20
1.3.1 Properties of expectation	23
1.3.2 Variance and covariance	24
1.4 Bernoulli trials and related random variables	28
1.4.1 The geometric distribution	28
1.4.2 The binomial distribution	31
1.5 Sums, transforms and limits	36
1.5.1 Generating functions	37
1.5.2 Laplace transforms and characteristic functions	39
1.5.3 The normal distribution and central limit theorem	41
1.6 Literature	46
2 Arrivals and services	47
2.1 Renewal processes	47
2.1.1 Forward and backward renewal times	50
2.1.2 The ‘paradox’ of residual life	53
2.2 The exponential distribution and its properties	55

vi	<i>Contents</i>	
2.2.1	First and last	56
2.3	The Poisson process	59
2.3.1	Properties of the Poisson process	61
2.3.2	Arrivals during a random interval	67
2.4	Application: ALOHA and CSMA	69
2.5	Literature	73
3	Queueing systems: average performance	74
3.1	Little's theorem and applications	75
3.1.1	Utilization and response time laws	77
3.2	The single-class M/G/1 queue	81
3.2.1	Busy periods	86
3.2.2	The M/M/1 queue	88
3.3	Different scheduling policies	92
3.3.1	The Processor-Sharing policy	94
3.3.2	Symmetric policies and multiclass queues	96
3.3.3	Which policy is better?	98
3.4	Priority scheduling	99
3.4.1	Preemptive-resume priorities	104
3.4.2	Averaging and lumping job types	105
3.5	Optimal scheduling policies	107
3.5.1	A conservation law for waiting times	108
3.5.2	The c/ρ rule	110
3.5.3	Optimization with preemptions	112
3.5.4	Characterization of achievable performance	116
3.6	Literature	119
4	Queueing networks	122
4.1	Open networks	124
4.1.1	Traffic equations	125
4.1.2	Performance measures	128
4.2	Closed networks	136
4.2.1	Mean value analysis	138
4.3	Multiclass networks	146
4.3.1	Closed multiclass networks	150
4.4	Literature	154
5	Markov chains and processes	156
5.1	Markov chains	157
5.1.1	Steady state	161
5.2	A Markov chain embedded in the M/G/1 model	168
5.3	Markov processes	172

<i>Contents</i>	vii
5.3.1 Transient behaviour	178
5.3.2 Steady state	180
5.4 First passages and absorptions	184
5.5 Birth-and-Death queueing models	186
5.5.1 The M/M/n model	188
5.5.2 The M/M/n/n and M/M/∞ systems	192
5.5.3 The M/M/1/N queue	194
5.5.4 The M/M/n/·/K model	195
5.6 Literature	199
6 Queues in Markovian environments	201
6.1 Quasi-Birth-and-Death models	202
6.1.1 The MMPP/M/1 queue	204
6.1.2 A multiserver queue with breakdowns and repairs	205
6.1.3 Manufacturing blocking	207
6.1.4 Phase-type distributions	208
6.2 Solution methods	210
6.2.1 Spectral expansion method	211
6.2.2 Matrix-geometric solution	215
6.3 Generalizations	216
6.4 Literature	219
<i>Index</i>	221