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Preface

The designers and users of complex systems have an interest in knowing how those systems behave under different conditions. This is true in all engineering domains, from transport and manufacturing to computing and communications. It is necessary to have a clear understanding, both qualitative and quantitative, of the factors that influence the performance and reliability of a system. Such understanding may be obtained by constructing and analysing mathematical models. The purpose of this book is to provide the necessary background, methods and techniques.

A model is inevitably an approximation of reality: a number of simplifying assumptions are usually made. However, that need not diminish the value of the insights that can be gained. A mathematical model can capture all the essential features of a system, display underlying trends and provide quantitative relations between input parameters and performance characteristics. Moreover, analysis is cheap, whereas experimentation is expensive. A few simple calculations carried out on the back of an envelope can often yield as much information as hours of observations or simulations.

The systems in which we are interested are subjected to demands of random character. The processes that take place in response to those demands are also random. Accordingly, the modelling tools that are needed to study such systems come from the domains of probability theory, stochastic processes and queueing theory.

Probabilities, random variables and distributions are introduced in chapter 1. Much of the theory concerned with arrivals, services, queues, scheduling policies, optimization and networks can be developed without any more sophisticated mathematical apparatus. This is done in chapters 2, 3 and 4, where the emphasis is on average performance. Chapter 5 deals with the important topic of Markov chains and processes, and

their applications. Finally, chapter 6 is devoted to a particular class of models—queues in Markovian environments—and the methods available for their solution.

Throughout the text, an effort has been made to make the material easily accessible. There is much emphasis on explaining ideas and providing intuition along with the formal derivations and proofs. Some of the more difficult results are, in fact, stated without proofs. Generality is sometimes sacrificed to clarity. For instance, the mean value treatment of closed queueing networks was chosen in preference to an approach based on the product-form solution because it is simpler, albeit somewhat less powerful.

The book is intended for operations research and computer science undergraduates and postgraduates, and for practitioners in the field. Some mathematical background is assumed, including first-year calculus. Readers familiar with probability may wish to skip some, or all, of chapter 1.

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