# Some Unsolved Problems

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During my long life I have written many papers on my favourite unsolved problems (see, for example, Baker *et al.* [2]). In the collection below, all the problems are either new ones, or they are problems about which there have been recent developments.

## Number theory

1. As usual, let us write $2 = p_1 < p_2 < \cdots$ for the sequence of consecutive primes. I proved in 1934 that there is a constant $c > 0$ such that for infinitely many $n$ we have

$$p_{n+1} - p_n > \frac{c \log n \log\log n}{(\log\log\log n)^2}.$$

Rankin [35] proved that for some $c > 0$ and infinitely many $n$ the following inequality holds:

$$p_{n+1} - p_n > \frac{c \log n \log\log n \log\log\log\log n}{(\log\log\log n)^2}. \tag{1}$$

I offered (perhaps somewhat rashly) \$10 000 for a proof that (1) holds for every $c$. The original value of $c$ was improved by Schönhage [38] and later by Rankin [36]. Rankin's result was recently improved by Maier and Pomerance [30].

2. Let $a_1 < a_2 < \cdots$ be an infinite sequence of integers. Denote by $f(n)$ the number of solutions of $n = a_i + a_j$. Assume that $f(n) > 0$ for all $n > n_0$, i.e. $(a_n)_{n=1}^{\infty}$ is an *asymptotic basis* of order 2. Turán and I conjectured that then

$$\varlimsup_{n\to\infty} f(n) = \infty \tag{2}$$

and probably $\varlimsup f(n)/\log n > 0$. I offer \$500 for a proof of (2). Perhaps (2) and $\varlimsup f(n)/\log n > 0$ already follow if we only assume $a_n < cn^2$ for all $n$.

Let $a_1 < a_2 < \cdots$ and $b_1 < b_2 < \cdots$ be two sequences of integers such that $a_n/b_n \to 1$ and let $g(n)$ be the number of solutions of $a_i + b_j = n$. Sárközy and I conjecture that if

$g(n) > 0$ for all $n > n_0$ then $\overline{\lim}\, g(n) = \infty$. The condition $a_n/b_n \to 1$ can not entirely be omitted but $1 - \epsilon < a_n/b_n < 1 + \epsilon$ ($\epsilon$ small) may suffice.

3. I proved that there is an asymptotic basis of order 2 for which

$$c_1 \log n < f(n) < c_2 \log n$$

(see Halberstam and Roth [26]). I conjecture that

$$\frac{f(n)}{\log n} \to C, \quad (0 < C < \infty),$$

is not possible and I offer \$500 for a proof or disproof of this conjecture. Sárközy and I proved that

$$\frac{|\, f(n) - \log n \,|}{\sqrt{\log n}} \to 0$$

cannot hold.

4. Is it true that

$$\sum \frac{1}{n! - 1}$$

is irrational? I conjectured that

$$\sum \frac{1}{2^n - 3}$$

is irrational. This assertion and its generalizations have been proved by Peter Borwein [6]. Denote by $\omega(n)$ the number of distinct prime factors of $n$. Is it true that

$$\sum \frac{\omega(n)}{2^n}$$

is irrational?

5. Is it true that if $n \not\equiv 0 \pmod 4$ then there is a squarefree natural number $\theta$ such that $n = 2^k + \theta$? I could only prove that almost all integers $n \not\equiv 0 \pmod 4$ can be written in the form $2^k + \theta$.

## Combinatorics

6. Let $m = m(n)$ be the smallest integer for which there are $n$-element sets $A_1, \ldots, A_m$ such that $A_i \cap A_j \neq \emptyset$ for all $1 \leqslant i < j \leqslant m$, and such that every set $S$ with at most $n-1$ elements is disjoint from some $A_i$. (Note that the lines of finite geometry have this property.) I conjectured with Lovász that $m(n)/n \to \infty$, but it is not even known whether $m(n) > 3n$ if $n$ sufficiently large. In the other direction, we could prove only that $m(n) < n^{\frac{3}{2}+\epsilon}$, but Jeff Kahn [27] very recently proved $m(n) < cn$.

Perhaps more is true: for every $C > 0$ there is an $\epsilon > 0$ such that if $n$ is sufficiently large and $m \leqslant Cn$ then for every $n$-element set $A_1, \ldots, A_m$ with $A_i \cap A_j \neq \emptyset$ there is a set S with $|\, S \,| < n(1 - \epsilon)$ which meets all $A_i$.

7. Is it true that in a finite geometry there is always a blocking set which meets every line in at most $c$ points where $c$ is an absolute constant?

More generally: Let $|\mathscr{S}| = n$, $A_1, \ldots, A_m$ be a family of subsets of $\mathscr{S}$, $|A_i| > c\sqrt{n}$, $c < 1$, $|A_i \cap A_j| \leqslant 1$. Is it then true that there is a set $B$ for which $B \cap A_i \neq \emptyset$ but $|B \cap A_i| < c\prime$ for all $i$? In other words, is there a set $B$ which meets all the $A_i$'s but none in many points?

8. Here is a problem of Jean Larson and myself [19]. Is it true that there is an absolute constant $c$ so that for every $n$ and $|\mathscr{S}| = n$ there is a family of subsets $A_1, \ldots, A_m$ of $\mathscr{S}$, $|A_i| > n^{1/2} - c$, $|A_i \cap A_j| \leqslant 1$ and every $x, y \in \mathscr{S}$ is contained in some $A_i$?

Shrikhande and Singhi [39] have proved that every pairwise balanced design on $n$ points in which each block is of size $\geqslant n^{\frac{1}{2}} - c$ can be embedded in a projective plane of order $n + i$ for some $i \leqslant c + 2$ if $n$ is sufficiently large. This implies that if the projective plane conjecture (that the order of every projective plane is a prime power) is true then the Erdős–Larson conjecture is false. But the problem remains for which functions $h(n)$ will the condition $|A_i| > n^{\frac{1}{2}} - h(n)$ make the conjecture true?

## Graph theory

9. I offer \$500 for a proof or disproof of the following conjecture of Faber, Lovász and myself. Let $G_1, \ldots, G_n$ be complete graphs (each on $n$ vertices), no two of which have an edge in common. Is it then true that $\chi(\bigcup_{i=1}^{n} G_i) \leqslant n$?

Jeff Kahn [27] recently proved that the chromatic number is $n + o(n)$.

10. Is it true that every triangle-free graph on $5n$ vertices can be made bipartite by the omission of at most $5n^2$ edges? Is it true that every triangle-free graph on $5n$ vertices can contain at most $n^5$ pentagons? Ervin Győri [25] proved this with $1.03n^5$.

Győri now proved $n^5$ for $n > n_0$. One could ask more generally: Assume that the number of vertices is $(2r+1)n$ and that the smallest odd cycle has size $2r+1$. Is it then true that the number of cycles of size $2r+1$ is at most $n^{2r+1}$?

11. Let $H$ be a graph and let $G^n$ be a graph on $n$ vertices which does not contain $H$ as an induced subgraph. Hajnal and I [13] asked whether there is an absolute constant $c = c(H)$ such that $G^n$ contains either a complete graph or an independent set on $n^c$ vertices? If $H$ is $C_4$ then $\frac{1}{3} \leqslant c < \frac{1}{2}$.

12. Let $Q^n$ be the graph of the $n$-dimensional cube $\{0,1\}^n$. I offered \$100 for a proof or disproof of the conjecture that for every $\epsilon > 0$ there is an $n_0$ such that, for $n > n_0$, every subgraph of $Q^n$ with at least $(\frac{1}{2} + \epsilon)e(Q^n)$ edges contains $C_4$. It is easy to find subgraphs with more than $\frac{1}{2}e(Q^n)$ edges and no $C_4$; Guan (see Chung [9]) has constructed an example with $(1 + o(1))(n+3)2^{n-2}$ edges. Chung has given an upper bound of $(\alpha + o(1))n2^{n-1}$, where $\alpha \approx 0.623$.

I also conjectured that every subgraph of $Q^n$ with $\epsilon e(Q^n)$ edges contains a $C_6$, for $n$ sufficiently large. Chung [9] and Brouwer, Dejter and Thomassen [7] disproved this by constructing an edge-partition of $Q^n$ into four subgraphs containing no $C_6$.

13. Suppose that $G$ is a graph of order $n$ with the property that every set of $p$ vertices spans at least $q$ edges. We let $H(n; p, q)$ be the largest integer such that $G$ necessarily contains a clique of that order. In the case where $q = 1$ this corresponds to the standard

finite Ramsey problem: the condition is precisely that $G$ contains no independent set of size $p$.

Faudree, Rousseau, Schelp and I investigated the behaviour of $H(n;p,q)$ as a function of $n$. We set

$$c(p,q) = \varliminf_{n\to\infty} \left(\frac{\log H(n;p,q)}{\log n}\right).$$

Standard bounds on Ramsey numbers (see, for example, Bollobás [5]) tell us that

$$1/(p-1) \leqslant c(p,1) \leqslant 2/(p+1).$$

We conjecture that with $p$ fixed, $c(p,q)$ is a strictly increasing function of $q$ for $1 \leqslant q \leqslant \binom{p-1}{2}+1$. It is easy to see that if $q = \binom{p-1}{2}+1$ then $c(p,q) = 1$, which is as large as possible. For in this case, the complement of $G$ cannot contain any connected subgraphs of size $p$, so all components of the complement have size less than $p$. Hence the complement contains at least $n/(p-1)$ independent vertices so $G$ contains a clique of size at least $n/(p-1)$. On the other hand, we have shown that $H(n;p,\binom{p-1}{2}) \leqslant cn^{\frac{1}{2}}$, so $c(p,\binom{p-1}{2}) \leqslant 1/2$.

14. For $\epsilon > 0$, Rödl [37] constructed graphs with chromatic number $\aleph_0$ such that every subgraph of order $n$ can be made bipartite by omitting $\epsilon n$ edges, for every $n$; another construction was given by Lovász. Now let $f(n) \to \infty$ as slowly as we please. Is there a graph of chromatic number $\aleph_0$ such that every subgraph of $n$ vertices can be made bipartite by omitting $f(n)$ edges?

Perhaps for every $\epsilon > 0$, there is a graph with chromatic number $\aleph_1$ for which every subgraph of order $n$ can be made bipartite by omitting $\epsilon n$ edges, but this seems unlikely and I would guess that there is a subgraph of size $n$ which cannot be made bipartite by omitting $nh(n)$ edges, where $h(n) \to \infty$. But perhaps $h(n)$ does not have to tend to infinity fast. See also the paper with Hajnal and Szemerédi [17].

Hajnal, Shelah and I [16] proved that if $G$ has chromatic number $\aleph_1$ then for some $n_0$ it contains a cycle of length $n$ for every $n > n_0$. Now if $F(n)$ tends to infinity sufficiently fast, then is it true that every graph of chromatic number $\aleph_1$ has a subgraph on at most $F(n)$ vertices with chromatic number $n$, for all $n$ sufficiently large?

## Geometry

15. Let $x_1, \ldots, x_n$ be $n$ distinct points in the plane, and let $s_1 \geqslant s_2 \geqslant \cdots \geqslant s_k$ be the multiplicities of the distances they determine, so

$$\sum_{i=1}^{k} s_i = \binom{n}{2}.$$

I conjectured [12] that

$$\sum_{i=1}^{k} s_i^2 < cn^3(\log n)^\alpha \tag{3}$$

for some $\alpha > 0$. The lattice points show that we must have $\alpha \geqslant 1$.

In forthcoming papers Fishburn and I conjecture that if $x_1, \ldots, x_n$ form a convex set

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then (3) can be improved to

$$\sum_{i=1}^{k} s_i^2 < cn^3, \tag{4}$$

and that $\sum s_i^2$ is maximal for the regular $n$-gon, for $n \geqslant 8$.

A weaker inequality than (3) would follow easily from the following conjecture. Let $A(x_1, \ldots, x_n)$ be the number of pairs $x_i, x_j$ whose distance is 1, and let $f(n)$ be the maximum $A(x_1, \ldots, x_n)$ over all sets of $n$ distinct points in the plane. I conjecture that

$$f(n) < n^{1+c/\log\log n}. \tag{5}$$

The best bound found to date is due to Spencer, Szemerédi and Trotter [40], who proved $f(n) < cn^{\frac{4}{3}}$. It would follow from (5) that $\sum s_i^2 < cn^{3+c/\log\log n}$.

Is it true that the number of incongruent sets of $n$ points with $f(n)$ unit distances exceeds one for $n > 3$ and tends to infinity with $n$?

Leo Moser and I conjectured that if $x_1, \ldots, x_n$ is a convex $n$-gon then

$$A(x_1, \ldots, x_n) < cn. \tag{6}$$

Füredi [22] proved that $A(x_1, \ldots, x_n) < cn \log n$; this gives an upper bound of $cn^3 \log n$ in (4). The inequality (4) would follow from (6).

16. Let $x_1, \ldots, x_n$ be $n$ distinct points in the plane. Denote by $F_k(n)$ the maximum number of distinct lines passing through at least $k$ of our points and by $f_k(n)$ the maximum number of lines passing through exactly $k$ of our points. Clearly $f_k(n) \leqslant F_k(n)$. Determine or estimate $f_k(n)$ and $F_k(n)$ as well as possible. Trivially $f_2(n) = F_2(n) = \binom{n}{2}$. The problem with $k = 3$ is the Orchard problem, and really goes back to Sylvester. Burr, Grünbaum and Sloane [8] proved that

$$f_3(n) = \frac{n^2}{6} - O(n) \quad \text{and} \quad F_3(n) = \frac{n^2}{6} - O(n).$$

Determine $\lim_{n\to\infty} F_k(n)/n^2$ and $\lim_{n\to\infty} f_k(n)/n^2$, if they exist. The upper bound $F_k(n) \leqslant \binom{n}{2}/\binom{k}{2}$ follows from an obvious counting argument; a lower bound can be obtained by considering a rectangle of $k$ by $n/k$ lattice points. Are the limits attained by the lattice points?

17. Let $f(n)$ denote the minimum number of distinct distances among a set $\mathscr{C} = (x_i)_1^n$ of points in the plane. In 1946 [12] I proved that

$$\sqrt{n - \frac{3}{4}} - \frac{1}{2} \leqslant f(n) \leqslant cn/\sqrt{\log n},$$

and conjectured that the upper bound gave the true order of $f(n)$. So far, the best lower bound is $n^{\frac{4}{5}}(\log n)^{-c}$, due to Chung, Szemerédi and Trotter [10].

The question also arises whether, in general, a particular point of the configuration is associated with a large number of distances. I conjecture that in any configuration there is some point with at least $cn/\sqrt{\log n}$ distinct distances to other points. In fact this may be true for all but a few of the points.

Altman [1] showed that if $\mathscr{C}$ is convex then there are at least $n/2$ distinct distances between the points; I conjecture that there is some point associated with at least $n/2$ distinct distances. Szemerédi conjectured there are at least $n/2$ distinct distances among the points of $\mathscr{C}$ provided only that $\mathscr{C}$ has no three points collinear, but could only prove this with a bound of $n/3$.

18. Consider two configurations $\mathscr{C} = (x_i)_1^n$, $\mathscr{C}' = (y_i)_1^n$, and define $F(2n)$ to be the minimum over all $\mathscr{C}$ and $\mathscr{C}'$ of the number of distinct distances $\| x_i - y_i \|$. Is $F$ identically 1 in four dimensions (in this case $f(2n) > n^\epsilon$)? How does $f/F$ behave at dimension 2 or 3? Do we have $f/F \to \infty$ or is the ratio bounded? Possibly it is unbounded in $\mathbb{R}^3$ but bounded in $\mathbb{R}^2$.

19. Let $\mathscr{C}$ be a set of points in the plane such that distinct distances between the points always differ by at least 1. I conjecture that the diameter of $\mathscr{C}$ is at least $n-1$ provided $n$ is large enough. Note that if $\mathscr{C} = ((i,0))_{i=1}^n$ we obtain equality. However for $n < 10$ some configurations have diameter less than $n-1$.

The best result in this direction so far is due to Kanold [29], who proved that $diam\,\mathscr{C} \geqslant 0.366 n^{\frac{3}{4}}$.

20. Let $\mathscr{C}$ be a set of $n$ points in Euclidean space among which all distances differ by at least 1. A conjecture independent of dimension is that $diam\,\mathscr{C} \geqslant (1+o(1))n^2$. Clearly $diam\,\mathscr{C}$ is always at least $\binom{n}{2}$.

The conjecture is settled only for $\mathscr{C} \subset \mathbb{R}$ (not even for $\mathbb{R}^2$). To prove it for $\mathbb{R}$, let $\mathscr{C} = (x_1)_1^n$ with $0 = x_1 < x_2 < \ldots < x_n$. Further, let $y_{k,i} = x_{i+k} - x_i$ and let $Y_k = \sum_{i=1}^{n-k} y_{k,i} = x_n + x_{n-1} + \ldots + x_{n-k+1} < kx_n$. Because the $y_{k,i}$ are all distinct (even over $k$), we have

$$x_n = Y_1 \geqslant \sum_{i=1}^{n-1} i = \binom{n}{2}$$

$$3x_n > Y_1 + Y_2 \geqslant \binom{2n-2}{2}$$

and for $k \leqslant n/2$,

$$\binom{k+1}{2} x_n > Y_1 + \ldots + Y_K \geqslant \binom{(n-1) + \ldots + (n-k) + 1}{2}$$

Now let $k = \lceil \sqrt{n} \rceil$. Roughly, we get

$$\frac{n}{2} x_n > \binom{\sqrt{n}(n-\sqrt{n})}{2} \approx \frac{n}{2}\left(n - \sqrt{n}\right)^2$$

so $x_n \geqslant n^2(1+o(1))$ as desired.

## Analysis

21. We let $I = [-1,1]$ and supose $f : I \to \mathbb{R}$ is a continuous function which we wish to approximate by a polynomial. Suppose we are given, for each $1 \leqslant n < \infty$,

$$-1 \leqslant x_1^{(n)} < x_2^{(n)} < \ldots < x_n^{(n)} \leqslant 1,$$

so we have a triangular matrix $X = (x_i^{(n)})$. Let $l_i^{(n)}$ be the unique polynomial of degree

$n-1$ satisfying

$$l_i^{(n)}(x_i^{(n)}) = 1 \quad \text{and} \quad l_i^{(n)}(x_j^{(n)}) = 0 \quad \text{if} \quad j \neq i$$

so

$$l_i^{(n)}(x) = \prod_{j\neq i}(x - x_j) \Big/ \prod_{j\neq i}(x_i - x_j).$$

Then we denote by $\mathscr{L}_n(f, X)$, or simply $\mathscr{L}_n(f)$, the polynomial given by

$$\mathscr{L}_n(f)(x) = \sum_{i=1}^{n} f(x_i^{(n)}) l_i^{(n)}(x),$$

so this is the unique polynomial of degree $n-1$ agreeing with $f$ on $x_1^{(n)} \dots x_n^{(n)}$.

It is known that for certain choices of $X$, if $f$ is of bounded variation then $\mathscr{L}_n(f)(x) \to f(x)$ uniformly. However, for more general continuous $f$ the behaviour is not so good and, as we now describe, a number of authors have examined how bad this behaviour can be.

With a fixed choice of $X$, we can regard $\mathscr{L}_n$ as a linear map from $C(I)$ to itself. Let us write down its norm. Let

$$\lambda_n(x) = \sum_{i=1}^{n} |l_i^{(n)}(x)|.$$

Then we easily see that

$$\max_{\|f\|=1} \|\mathscr{L}_n(f)(x)\| = \lambda_n(x),$$

so if we let

$$\lambda_n = \max_{-1 \leqslant x \leqslant 1} \lambda_n(x),$$

then $\|\mathscr{L}_n\| = \lambda_n$.

Faber [21] proved that for any choice of $X$, $\overline{\lim}_{n\to\infty} \lambda_n = \infty$. It therefore follows from the Principle of Uniform Boundedness that there exists an $f$ with $\overline{\lim}_{n\to\infty} \|\mathscr{L}_n(f)\| = \infty$.

This result was strengthened by Bernstein [4] who showed that for any $X$, there exist $f \in C[-1, 1]$ and $x_0 \in [-1, 1]$ such that

$$\overline{\lim_{n\to\infty}} \|\mathscr{L}_n(f)(x_0)\| = \infty, \qquad \text{i.e.} \quad \overline{\lim_{n\to\infty}} \lambda_n(x_0) = \infty.$$

In several papers (Bernstein [3], Grünwald ([23], [24]), Marcinkiewicz [31] and Privalov ([32], [33])) it was shown that for particular choices of $X$, this kind of bad behaviour can occur almost everywhere and, in certain cases, everywhere. In 1980 Vértesi and I [20] showed that given any $X$, there exists an $f$ with

$$\overline{\lim_{n\to\infty}} \|\mathscr{L}_n(f)(x)\| = \infty \quad \text{for almost all } x.$$

Certainly this result cannot be extended from almost all $x$ to *all* $x$. For example, if $x_0$ appears in all but finitely many rows of $X$ – i.e. is equal to some $x_i^{(n)}$ for all $n \geqslant n_0$ – then we have $\mathscr{L}_n(f)(x_0) = f(x_0)$ for $n \geqslant n_0$. Does there, however, exist an $X$, such that for every $f$, there is some point $x_0$ where divergence would be possible, i.e. where

$$\overline{\lim_{n\to\infty}} \lambda(x_0) = \infty \qquad \text{yet} \quad \mathscr{L}_n(f)(x_0) \to f(x_0)?$$

22. Let $f(z) = z^n + \dots$ be a monic polynomial of degree $n$.

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Is it true that the length of $\{z \in \mathbb{C} : | f(z) |= 1\}$ is maximal in the case when $f(z) = z^n - 1$?

This problem was posed, along with many others, in my paper with Herzog and Piranian [18].

23. Let $|z_n| = 1 (1 \leqslant n < \infty)$. Put

$$f_n(z) = \prod_{k=1}^{n} (z - z_k)$$

and

$$M_n = \max_{|z|=1} |f_n(z)|.$$

Is it true that $\overline{\lim} M_n = \infty$? This conjecture was settled by Wagner: he proved that there is a $c > 0$ such that $M_n > (\log n)^c$ holds for infinitely many values of $n$. I further conjectured that $M_n > n^c$ for some $c > 0$ and infinitely many $n$ and, in fact, for every $n$ we have

$$\sum_{k=1}^{n} M_k > n^{1+c}. \tag{7}$$

Inequality (7), if true, may very well be difficult, so I offer \$100 for a solution.

24. Let $x_1, x_2, \ldots$ be a sequence of real numbers tending to 0. We call $(y_n)_{n=1}^{\infty}$ *similar* to $(x_n)_{n=1}^{\infty}$ if $y_n = ax_n + b$ for some $a, b \in \mathbb{R}$ and all $n$. Is it true that there is a set $E \subseteq \mathbb{R}$ of positive measure which contains no subsequence $(y_n)_{n=1}^{\infty}$ similar to $(x_n)_{n=1}^{\infty}$?

Komjáth proved that if $x_n \to 0$ slowly ($x_n > c/n$) then there is a set of positive measure which contains no subsequence similar to $(x_n)_{n=1}^{\infty}$.

## Set theory

25. I have not included our many problems on set theory with Hajnal since undecidability raises its ugly head everywhere and many of our problems have been proved or disproved or shown to be undecidable (this happened most often). However, I think that the following simple problem is still open. Let $\alpha$ be a cardinal or ordinal number or an order type. Assume $\alpha \to (\alpha, 3)^2$. Is it then true that, for every finite $n$, $\alpha \to (\alpha, n)^2$ also holds? Here $\alpha \to (\alpha, n)^2$ is the well-known arrow symbol of Rado and myself: if $G$ is a graph whose vertices form a set of type $\alpha$ then either $G$ contains a complete graph $K_n$ or an independent set of type $\alpha$. See Erdős, Hajnal and Milner [15] and Erdős, Hajnal, Máté and Rado [14].

## Group theory

26. Let $G$ be a group. Assume that it has at most $n$ elements which do not commute pairwise. Denote by $h(n)$ the smallest integer for which any such $G$ can be covered by $h(n)$ Abelian subgroups. Determine or estimate $h(n)$ as well as possible. Pyber [34] proved that

$$(1 + c_1)^n < h(n) < (1 + C_2)^n,$$

for some positive constants $c_1$ and $c_2$. The lower bound was already known to Isaacs.

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