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978-0-521-58401-2 - Control Theory for Partial Differential Equations: Continuous and Approximation Theories, Volume II - Abstract Hyperbolic-like Systems over a Finite Time Horizon

Irena Lasiecka and Roberto Triggiani

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