

CONTENTS

<i>Preface</i>	xi
1 LAGRANGIAN MECHANICS	1
1.1 Example and Review of Newton's Mechanics: A Block Sliding on an Inclined Plane	1
1.2 Using Virtual Work to Solve the Same Problem	3
1.3 Solving for the Motion of a Heavy Bead Sliding on a Rotating Wire	7
1.4 Toward a General Formula: Degrees of Freedom and Types of Constraints	10
1.5 Generalized Velocities: How to "Cancel the Dots"	14
1.6 Virtual Displacements and Virtual Work – Generalized Forces	14
1.7 Kinetic Energy as a Function of the Generalized Coordinates and Velocities	16
1.8 Conservative Forces: Definition of the Lagrangian L	18
1.9 Reference Frames	20
1.10 Definition of the Hamiltonian	21
1.11 How to Get Rid of Ignorable Coordinates	22
1.12 Discussion and Conclusions – What's Next after You Get the EOM?	23
1.13 An Example of a Solved Problem	24
Summary of Chapter 1	25
Problems	26
Appendix A. About Nonholonomic Constraints	36
Appendix B. More about Conservative Forces	41
2 VARIATIONAL CALCULUS AND ITS APPLICATION TO MECHANICS	44
2.1 History	44
2.2 The Euler Equation	46
2.3 Relevance to Mechanics	51
2.4 Systems with Several Degrees of Freedom	53
2.5 Why Use the Variational Approach in Mechanics?	54
2.6 Lagrange Multipliers	56

2.7	Solving Problems with Explicit Holonomic Constraints	57
2.8	Nonintegrable Nonholonomic Constraints – A Method that Works	62
2.9	Postscript on the Euler Equation with More Than One Independent Variable	65
	Summary of Chapter 2	65
	Problems	66
	Appendix. About Maupertuis and What Came to Be Called “Maupertuis’ Principle”	75
3	LINEAR OSCILLATORS	81
3.1	Stable or Unstable Equilibrium?	82
3.2	Simple Harmonic Oscillator	87
3.3	Damped Simple Harmonic Oscillator (DSHO)	90
3.4	An Oscillator Driven by an External Force	94
3.5	Driving Force Is a Step Function	96
3.6	Finding the Green’s Function for the SHO	99
3.7	Adding up the Delta Functions – Solving the Arbitrary Force	103
3.8	Driving an Oscillator in Resonance	105
3.9	Relative Phase of the DSHO Oscillator with Sinusoidal Drive	110
	Summary of Chapter 3	113
	Problems	114
4	ONE-DIMENSIONAL SYSTEMS: CENTRAL FORCES AND THE KEPLER PROBLEM	123
4.1	The Motion of a “Generic” One-Dimensional System	123
4.2	The Grandfather’s Clock	125
4.3	The History of the Kepler Problem	130
4.4	Solving the Central Force Problem	133
4.5	The Special Case of Gravitational Attraction	141
4.6	Interpretation of Orbits	143
4.7	Repulsive $\frac{1}{r^2}$ Forces	151
	Summary of Chapter 4	156
	Problems	156
	Appendix. Tables of Astrophysical Data	167
5	NOETHER’S THEOREM AND HAMILTONIAN DYNAMICS	170
5.1	Discovering Angular Momentum Conservation from Rotational Invariance	170
5.2	Noether’s Theorem	172
5.3	Hamiltonian Dynamics	175
5.4	The Legendre Transformation	175
5.5	Hamilton’s Equations of Motion	180
5.6	Liouville’s Theorem	184
5.7	Momentum Space	189

CONTENTS

vii

5.8	Hamiltonian Dynamics in Accelerated Systems	190
	Summary of Chapter 5	195
	Problems	196
	Appendix A. A General Proof of Liouville's Theorem	
	Using the Jacobian	202
	Appendix B. Poincaré Recurrence Theorem	204
6	THEORETICAL MECHANICS: FROM CANONICAL TRANSFORMATIONS TO ACTION-ANGLE VARIABLES	207
6.1	Canonical Transformations	208
6.2	Discovering Three New Forms of the Generating Function	213
6.3	Poisson Brackets	217
6.4	Hamilton–Jacobi Equation	218
6.5	Action–Angle Variables for 1-D Systems	230
6.6	Integrable Systems	235
6.7	Invariant Tori and Winding Numbers	237
	Summary of Chapter 6	239
	Problems	240
	Appendix. What Does “Symplectic” Mean?	248
7	ROTATING COORDINATE SYSTEMS	252
7.1	What Is a Vector?	253
7.2	Review: Infinitesimal Rotations and Angular Velocity	254
7.3	Finite Three-Dimensional Rotations	259
7.4	Rotated Reference Frames	259
7.5	Rotating Reference Frames	263
7.6	The Instantaneous Angular Velocity $\vec{\omega}$	264
7.7	Fictitious Forces	267
7.8	The Tower of Pisa Problem	267
7.9	Why Do Hurricane Winds Rotate?	271
7.10	Foucault Pendulum	272
	Summary of Chapter 7	275
	Problems	276
8	THE DYNAMICS OF RIGID BODIES	283
8.1	Kinetic Energy of a Rigid Body	284
8.2	The Moment of Inertia Tensor	286
8.3	Angular Momentum of a Rigid Body	291
8.4	The Euler Equations for Force-Free Rigid Body Motion	292
8.5	Motion of a Torque-Free Symmetric Top	293
8.6	Force-Free Precession of the Earth: The “Chandler Wobble”	299
8.7	Definition of Euler Angles	300
8.8	Finding the Angular Velocity	304
8.9	Motion of Torque-Free Asymmetric Tops: Poinsot Construction	305

8.10	The Heavy Symmetric Top	313
8.11	Precession of the Equinoxes	317
8.12	Mach's Principle	323
	Summary of Chapter 8	325
	Problems	326
	Appendix A. What Is a Tensor?	333
	Appendix B. Symmetric Matrices Can Always Be Diagonalized by "Rotating the Coordinates"	336
	Appendix C. Understanding the Earth's Equatorial Bulge	339
9	THE THEORY OF SMALL VIBRATIONS	343
9.1	Two Coupled Pendulums	344
9.2	Exact Lagrangian for the Double Pendulum	348
9.3	Single Frequency Solutions to Equations of Motion	352
9.4	Superimposing Different Modes; Complex Mode Amplitudes	355
9.5	Linear Triatomic Molecule	360
9.6	Why the Method Always Works	363
9.7	<i>N</i> Point Masses Connected by a String	367
	Summary of Chapter 9	371
	Problems	373
	Appendix. What Is a Cofactor?	380
10	APPROXIMATE SOLUTIONS TO NONANALYTIC PROBLEMS	383
10.1	Stability of Mechanical Systems	384
10.2	Parametric Resonance	388
10.3	Lindstedt–Poincaré Perturbation Theory	398
10.4	Driven Anharmonic Oscillator	401
	Summary of Chapter 10	411
	Problems	413
11	CHAOTIC DYNAMICS	423
11.1	Conservative Chaos – The Double Pendulum: A Hamiltonian System with Two Degrees of Freedom	426
11.2	The Poincaré Section	428
11.3	KAM Tori: The Importance of Winding Number	433
11.4	Irrational Winding Numbers	436
11.5	Poincaré–Birkhoff Theorem	439
11.6	Linearizing Near a Fixed Point: The Tangent Map and the Stability Matrix	442
11.7	Following Unstable Manifolds: Homoclinic Tangles	446
11.8	Lyapunov Exponents	449
11.9	Global Chaos for the Double Pendulum	451
11.10	Effect of Dissipation	452
11.11	Damped Driven Pendulum	453

CONTENTS

ix

11.12 Fractals	463
11.13 Chaos in the Solar System	468
Student Projects	474
Appendix. The Logistic Map: Period-Doubling Route to Chaos; Renormalization	481
12 SPECIAL RELATIVITY	493
12.1 Space–Time Diagrams	495
12.2 The Lorentz Transformation	498
12.3 Simultaneity Is Relative	501
12.4 What Happens to y and z if We Move Parallel to the X Axis?	503
12.5 Velocity Transformation Rules	504
12.6 Observing Light Waves	505
12.7 What Is Mass?	512
12.8 Rest Mass Is a Form of Energy	513
12.9 How Does Momentum Transform?	517
12.10 More Theoretical “Evidence” for the Equivalence of Mass and Energy	519
12.11 Mathematics of Relativity: Invariants and Four-Vectors	521
12.12 A Second Look at the Energy–Momentum Four-Vector	526
12.13 Why Are There Both Upper and Lower Greek Indices?	529
12.14 Relativistic Lagrangian Mechanics	530
12.15 What Is the Lagrangian in an Electromagnetic Field?	533
12.16 Does a Constant Force Cause Constant Acceleration?	535
12.17 Derivation of the Lorentz Force from the Lagrangian	537
12.18 Relativistic Circular Motion	539
Summary of Chapter 12	540
Problems	541
Appendix. The Twin Paradox	554
<i>Bibliography</i>	559
<i>References</i>	563
<i>Index</i>	565