ANALYTICAL MECHANICS

Analytical Mechanics provides a detailed introduction to the key analytical techniques of classical mechanics, one of the cornerstones of physics. It deals with all the important subjects encountered in an undergraduate course and prepares the reader thoroughly for further study at the graduate level.

The authors set out the fundamentals of Lagrangian and Hamiltonian mechanics early on in the book and go on to cover such topics as linear oscillators, planetary orbits, rigid-body motion, small vibrations, nonlinear dynamics, chaos, and special relativity. A special feature is the inclusion of many "e-mail questions," which are intended to facilitate dialogue between the student and instructor.

Many worked examples are given, and there are 250 homework exercises to help students gain confidence and proficiency in problem solving. It is an ideal textbook for undergraduate courses in classical mechanics and provides a sound foundation for graduate study.

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ANALYTICAL MECHANICS

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo, Mexico City

Cambridge University Press
32 Avenue of the Americas, New York, NY 10013-2473, USA

www.cambridge.org
Information on this title: www.cambridge.org/9780521575720

First published 1998
7th printing 2008

A catalog record for this publication is available from the British Library

Library of Congress Cataloging in Publication data
Hand, Louis N., 1933—
Analytical mechanics / Louis N. Hand, Janet D. Finch.
p. cm.
Includes bibliographical references and index.
ISBN 0-521-57327-0 — ISBN 0-521-57572-9 (pbk.)
1. Mechanics, Analytic. I. Finch, Janet D., 1969—
II. Title.
QA805.H26 1998
531′.′01′515352 – dc21 97-43334
CIP

ISBN 978-0-521-57327-6 Hardback
ISBN 978-0-521-57572-0 Paperback

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Cambridge University Press
978-0-521-57572-0 - Analytical Mechanics
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PREFACE

PREREQUISITES

The Physics Department at Cornell offers two intermediate-level undergraduate mechanics courses. This book evolved from lecture notes used in the more advanced of the two courses. Most of the students who took this course were considering postgraduate study leading to future careers in physics or astronomy. With a few exceptions, they had previously taken an introductory honors course in mechanics at the level of Kleppner and Kolenkow.* Many students also had an Advanced Placement physics course in high school. Since we can assume that a solid background in introductory college-level physics already exists, we have not included a systematic review of elementary mechanics in the book, other than the brief example at the beginning of Chapter 1.

Familiarity with a certain few basic mathematical concepts is essential. The student should understand Taylor series in more than one variable, partial derivatives, the chain rule, and elementary manipulations with complex variables.† Some elementary knowledge of matrices and determinants is also needed.‡ Almost all of the students who took the honors analytic mechanics course at Cornell have either completed, or were concurrently registered in, a mathematical physics course involving vector analysis, complex variable theory, and techniques for solving ordinary and partial differential equations. However, a thorough grounding in these subjects is not essential – in fact some of this material can be learned by taking a course based on this book.

INTRODUCTION

Our intention in writing this book is to reduce the gap between undergraduate and graduate physics training. Graduate students often complain that their undergraduate training did not prepare them for the rigors of graduate school. For that reason we have

* *An Introduction to Mechanics*, D. Kleppner and R. J. Kolenkow, McGraw-Hill, 1973.
† At the level of *Advanced Calculus*, 2d ed., W. Kaplan, Addison-Wesley, 1984.
‡ For linear algebra, we recommend a book on the level of *Linear Algebra with Applications*, 2d edition, S. J. Leon, Macmillan, 1986, or one of the many other suitable texts at the intermediate level.

written a text that emphasizes those concepts that will be useful to know later. We feel that only tradition stands in the way of teaching Lagrangian and Hamiltonian mechanics at an earlier stage than has been the case in the education of physicists. In addition to advancing the stage when these basic concepts are encountered, we have a second purpose in mind. In many colleges and universities, quantum mechanics is now taught at the intermediate undergraduate level. As a result of this recent trend, there often is a mismatch between these courses and the preparatory courses, and student preparation is often inadequate in the upper-level undergraduate courses. This places a heavy burden on the instructor in the courses introducing quantum mechanics and modern physics to juniors and seniors. Many important topics can be more easily visualized if taught in a classical mechanics course instead. The use of eigenvectors and eigenvalues to solve physical problems is a prime example.

Classical mechanics is an excellent way to introduce the basic tools of theoretical physics. Lagrangian methods can be used to simplify problems that would be difficult to solve by other means. Mechanics problems can be written and solved in a few lines using these powerful techniques. It is usually easier to work with more "advanced" techniques than with the more complicated "elementary" methods. Deeper insight into the motion of a mechanical system is obtained with these more sophisticated methods. We cite the role of conserved quantities derived from symmetries via Noether's theorem as one example.

A course in classical mechanics need not be justified entirely on the grounds that it provides a path to something else more glamorous and fashionable. To quote Gutzwiller:

> Elementary mechanics, both classical and quantum, has become a growth industry in the last decade. A newcomer to this flourishing field must get acquainted with some unfamiliar concepts and get rid of some cherished assumptions. The change in orientation is necessary because physicists have finally realized that most dynamical systems do not follow simple, regular, and predictable patterns, but run along a seemingly random, yet well-defined, trajectory. The generally accepted name for this phenomenon is *chaos*, a term that accurately suggests that we have failed to come to grips with the problem.*

Far from being a dead subject, classical mechanics has reemerged in the forefront of modern physics research! Deterministic chaos is a special topic discussed in Chapter 11. Since the 1960s chaos has developed as a new branch of classical mechanics. It has wide applicability to other fields outside of physics as well, although we do not consider this. Here we confine ourselves to treating two simple dynamical systems, one a conservative system and the other a dissipative system. This text is intended to give a purely introductory, rather than comprehensive, treatment of chaos. It is not a substitute for textbooks devoted entirely to this subject but might stimulate the student to investigate the subject more in future courses.

We have included a final chapter on special relativity. There are many excellent introductions to this subject, some of which are listed in the bibliography. However we felt that there may be value in having an introduction that applies the lessons of the previous

* Martin C. Gutzwiller, in *Chaos in Classical and Quantum Mechanics*.

chapters. The case of special relativity illustrates the modern viewpoint and shows the power of Lagrangian mechanics to transcend its original role, which was a reformulation of Newton's Laws of Motion.

One word of warning: We believe there is more material in this book than can be reasonably presented in a one-semester undergraduate course. The teacher must select those parts which he or she feels comprise a unified course in mechanics. In particular, there is a natural choice between teaching the material in Chapter 6 (advanced theory) and Chapter 11 (chaos) on the one hand and teaching special relativity (Chapter 12) on the other.

This book will be much more effective if it serves as part of an experience that directly involves the student in an active learning process. In a separate publication,* we will discuss the way in which the course was taught at Cornell, in particular the innovations made possible by the Sloan Foundation: the seminars and the e-mail questions.† Let us only say here that a dialogue with the individual students, separately and in groups, is essential for success in teaching the material. In our previous experience with teaching mechanics, it was found that relying only on standard lectures, with spontaneous student questions and discussion in lecture, did not stimulate the kind of individual thought process needed. The need to cover the material in the lecture tends to reduce thoughtful discussion to a minimum. Reading the text alone did not fill this gap. Students tended to read the text rather superficially, accepting what was said, rather than questioning it and working examples. Problem sets were done "just in time." This book was designed to provoke a more thoughtful reading experience, and more continuity in the study process, when it was used in combination with the seminar and the e-mail questions.

Support by the Sloan Foundation, which has an interest in acceleration of the undergraduate phase of education, allowed us the freedom to depart from the standard lecture format to develop the alternative ways to teach this material. Judging by the student reaction, we were successful in most cases. The use of undergraduate teaching assistants (TAs), students who had taken the course in the previous year, was particularly successful and helped provide role models.

ADVICE TO STUDENTS WHO USE THIS BOOK

Learning physics is an active experience.

If you were learning to ride a bicycle or to play the violin, you would expect to practice the technique until it became second nature. Falling off the bicycle gives you immediate feedback – it tells you that you need more practice. Exactly the same thing is true of learning physics. It must become part of you, something intuitive. Almost everyone has to work very hard to achieve this. Until you can solve problems, your understanding is not sufficiently deep. It is one thing to watch a lecturer solve a problem, where every step

* J. D. Finch and L. N. Hand, "Using an Email Tutorial and Student Seminars to Improve an Intermediate-Level Undergraduate Physics Course," *American Journal of Physics*, to be published in 1998.

† The seminar problems are denoted by a "*" in the homework problem sections. The e-mail questions are distributed throughout the text.

seems to be a logical one, and quite another to tackle a real problem on your own. Do not think of learning physics as "art appreciation." It is a "do-it-yourself" activity.

The key to success is how one studies the subject outside of class. A last minute "all-nighter" to solve a problem set is an exercise in self-delusion. You are strongly advised against trying to learn physics in this way, because it inhibits the crucial transition from short-term to long-term memory. The new concepts have to soak into your consciousness. Remember that it took about 150 years to develop Hamiltonian dynamics. It can't be learned adequately in one night. You should put aside a regular time for studying this material and concentrate on it without distraction. Do the reading early in the week it is assigned. Think about the problems more than one day before they are due. Try to isolate the points you don't understand. Read the material again. Most important: *Discuss it with other students*. Don't hesitate to ask others for an explanation and don't be satisfied until you get one. Another tip: Make the effort to memorize what the notation means. By experience, we have often observed that lack of familiarity with the symbols and what they stand for is one difference between strong and weak students. Memorizing the meaning of symbols becomes automatic with trained physicists. Acquiring the skill of learning physics will serve you well in later years, when most learning must be self-taught.

If you don't fully understand what your are reading, try to construct a simple example for yourself. But don't let the lack of understanding remain. Pester someone – your teaching assistant, your colleague, or your professor – until your questions are answered. And don't assume that it is clear to everyone except yourself.

The questions scattered throughout the text are intended to test your comprehension as you read the material. Some of them were assigned to our students, and answers were given and graded by e-mail. Repeated improvements in the answer, following comments by the teaching assistant, led to repeated improvements in the grade for that particular question. Most people ended up with nearly perfect scores after a useful dialogue with the TA via e-mail.

ACKNOWLEDGMENTS

First and foremost, we would like to thank the students who took this course over a five-year period while the content and format were being developed. The course was taught by one of us (L. H.). The other (J. F.) joined for the last three years, first as an e-mail tutor and then, later, as an editor and coauthor on the several versions of the lecture notes that preceded this book.

Without the generous encouragement and support of Dr. Frank Mayadas of the Sloan Foundation, this book would not have been possible, nor would it have been possible to try the principal innovations in teaching classical mechanics at Cornell: a weekly two-hour problem-solving seminar and the e-mail tutorial which engages the students in a dialogue while they are doing the assigned reading. The importance of the Sloan support cannot be overstated. Nowdays, there are very few remaining sources of support for improving the quality of undergraduate physics education.

Thanks are also due to Prof. Kurt Gottfried of Cornell, who was Department Chairman at the time this course made the transition into a teaching experiment. His leadership and constant encouragement played a major role in bringing the project to fruition.

Special thanks should go to Dr. Steven Townsend, who not only did a superb job as an e-mail tutor, but later helped us revise the quiz questions and select those which had been helpful to the students. His insightful comments on the physics of these questions were of very great pedagogical value. We have tried to incorporate them here.

Paul Shocklee helped with proofreading the first edition of these notes. Alex Khein contributed many problems as well as a set of notes connecting Hamilton–Jacobi theory with quantum mechanics. Thomas Haeusser helped proofread, finding several errors which we corrected. The graduate student TAs: Alex Khein, Ard Louis, Thomas Haeusser, and Katrin Schenk provided solutions to assigned homework problems. The undergraduate TAs have been essential to the success of the weekly problem solving seminars we have conducted at Cornell. Thanks are due to Paul Shocklee, Robin Madryk, Abe Stroock, Smitha Vishveshwara and Jonathan Levine for their contributions as Teaching Assistants in these seminars.

We also would like to thank Kevin Hodgson of Triple C, who did a superb job of translating our rough sketches into figures which add immeasurably to the quality and professionalism of the book.

Prof. Saul Teukolsky provided some valuable information about the most recent numerical studies of the stability of the solar system. Dr. Mark Scheel contributed a critical reading of Chapter 12 on special relativity. We have adopted several of his excellent suggestions. Prof. Eberhardt Bodenshatz kindly allowed us to become part of his network of NEXT computers. Without his support, and the help of his graduate student Brendan Plapp, we would have found writing the book far more difficult.

The original suggestion that the lecture notes be turned into a book came from Dr. Robert Lieberman, who has been our agent. The book would not have been possible without him and his constant encouragement. We also wish to thank Dr. Philip Meyler of Cambridge University Press for his patience and constant support.