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978-0-521-57557-7 - Introduction to the Modern Theory of Dynamical Systems

Anatole Katok and Boris Hasselblatt

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This book provides the first self-contained comprehensive exposition of the theory of dynamical systems as a core mathematical discipline closely intertwined with most of the main areas of mathematics. The authors introduce and rigorously develop the theory while providing researchers interested in applications with fundamental tools and paradigms.

The book begins with a discussion of several elementary but fundamental examples. These are used to formulate a program for the general study of asymptotic properties and to introduce the principal theoretical concepts and methods. The main theme of the second part of the book is the interplay between local analysis near individual orbits and the global complexity of the orbit structure. The third and fourth parts develop in depth the theories of low-dimensional dynamical systems and hyperbolic dynamical systems.

The book is aimed at students and researchers in mathematics at all levels from advanced undergraduate up. Scientists and engineers working in applied dynamics, non-linear science, and chaos will also find many fresh insights in this concrete and clear presentation. It contains more than four hundred systematic exercises.

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Volume 54

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ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

- 18 H. O. Fattorini *The Cauchy problem*
- 19 G. G. Lorentz, K. Jetter, and S. D. Riemenschneider *Birkhoff interpolation*
- 21 W. T. Tutte *Graph theory*
- 22 J. R. Bastida *Field extensions and Galois theory*
- 23 J. R. Cannon *The one-dimensional heat equation*
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- 28 P. P. Petrushev and V. A. Popov *Rational approximation of real functions*
- 29 N. White (ed.) *Combinatorial geometries*
- 30 M. Pohst and H. Zassenhaus *Algorithmic algebraic number theory*
- 31 J. Aczel and J. Dhombres *Functional equations containing several variables*
- 32 M. Kuczma, B. Chozewski, and R. Ger *Iterative functional equations*
- 33 R. V. Ambartzumian *Factorization calculus and geometric probability*
- 34 G. Gripenberg, S.-O. Londen, and O. Staffans *Volterra integral and functional equations*
- 35 G. Gasper and M. Rahman *Basic hypergeometric series*
- 36 E. Torgersen *Comparison of statistical experiments*
- 37 A. Neumaier *Interval methods for systems of equations*
- 38 N. Korneichuk *Exact constants in approximation theory*
- 39 R. A. Brualdi and H. J. Ryser *Combinatorial matrix theory*
- 40 N. White (ed.) *Matroid applications*
- 41 S. Sakai *Operator algebras in dynamical systems*
- 42 W. Hodges *Model theory*
- 43 H. Stahl and V. Totik *General orthogonal polynomials*
- 44 R. Schneider *Convex bodies*
- 45 G. Da Prato and J. Zabczyk *Stochastic equations in infinite dimensions*
- 46 A. Björner, M. Las Vergnas, B. Sturmfels, N. White, and G. Ziegler *Oriented matroids*
- 47 E. A. Edgar and L. Sucheston *Stopping times and directed processes*
- 48 C. Sims *Computation with finitely presented groups*
- 49 T. Palmer *Banach algebras and the general theory of *-algebras*
- 50 F. Borceux *Handbook of categorical algebra I*
- 51 F. Borceux *Handbook of categorical algebra II*
- 52 F. Borceux *Handbook of categorical algebra III*

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*Introduction to the
Modern Theory of Dynamical Systems*

ANATOLE KATOK

Pennsylvania State University

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Tufts University

With a supplement by Anatole Katok and Leonardo Mendoza



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UNIVERSITY PRESS**

Cambridge University Press
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PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK www.cup.cam.ac.uk
40 West 20th Street, New York, NY 10011-4211, USA www.cup.org
10 Stamford Road, Oakleigh, Melbourne 3166, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain

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First published 1995
Reprinted 1996
First paperback edition 1997
Reprinted 1998, 1999

Printed in the United States of America

Typeset in Times

A catalog record for this book is available from the British Library

Library of Congress Cataloging in Publication Data is available

ISBN 0 521 34187 6 hardback
ISBN 0 521 57557 5 paperback

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Frontmatter

[More information](#)

Contents

PREFACE	xiii
0. INTRODUCTION	1
1. Principal branches of dynamics	1
2. Flows, vector fields, differential equations	6
3. Time-one map, section, suspension	8
4. Linearization and localization	10
Part 1 Examples and fundamental concepts	
1. FIRST EXAMPLES	15
1. Maps with stable asymptotic behavior	15
Contracting maps; Stability of contractions; Increasing interval maps	
2. Linear maps	19
3. Rotations of the circle	26
4. Translations on the torus	28
5. Linear flow on the torus and completely integrable systems	32
6. Gradient flows	35
7. Expanding maps	39
8. Hyperbolic toral automorphisms	42
9. Symbolic dynamical systems	47
Sequence spaces; The shift transformation; Topological Markov chains; The Perron–Frobenius operator for positive matrices	
2. EQUIVALENCE, CLASSIFICATION, AND INVARIANTS	57
1. Smooth conjugacy and moduli for maps	57
Equivalence and moduli; Local analytic linearization; Various types of moduli	
2. Smooth conjugacy and time change for flows	64
3. Topological conjugacy, factors, and structural stability	68
4. Topological classification of expanding maps on a circle	71
Expanding maps; Conjugacy via coding; The fixed-point method	
5. Coding, horseshoes, and Markov partitions	79
Markov partitions; Quadratic maps; Horseshoes; Coding of the toral automorphism	
6. Stability of hyperbolic toral automorphisms	87
7. The fast-converging iteration method (Newton method) for the conjugacy problem	90
Methods for finding conjugacies; Construction of the iteration process	
8. The Poincaré–Siegel Theorem	94
9. Cocycles and cohomological equations	100
3. PRINCIPAL CLASSES OF ASYMPTOTIC TOPOLOGICAL INVARIANTS	105
1. Growth of orbits	105
Periodic orbits and the ζ -function; Topological entropy; Volume growth; Topological complexity: Growth in the fundamental group; Homological growth	

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Anatole Katok and Boris Hasselblatt

Frontmatter

[More information](#)

viii	Contents	
	2. Examples of calculation of topological entropy	119
	Isometries; Gradient flows; Expanding maps; Shifts and topological Markov chains; The hyperbolic toral automorphism; Finiteness of entropy of Lipschitz maps; Expansive maps	
	3. Recurrence properties	128
4.	STATISTICAL BEHAVIOR OF ORBITS AND INTRODUCTION TO ERGODIC THEORY	133
	1. Asymptotic distribution and statistical behavior of orbits	133
	Asymptotic distribution, invariant measures; Existence of invariant measures; The Birkhoff Ergodic Theorem; Existence of asymptotic distribution; Ergodicity and unique ergodicity; Statistical behavior and recurrence; Measure-theoretic isomorphism and factors	
	2. Examples of ergodicity; mixing	146
	Rotations; Extensions of rotations; Expanding maps; Mixing; Hyperbolic toral automorphisms; Symbolic systems	
	3. Measure-theoretic entropy	161
	Entropy and conditional entropy of partitions; Entropy of a measure-preserving transformation; Properties of entropy	
	4. Examples of calculation of measure-theoretic entropy	173
	Rotations and translations; Expanding maps; Bernoulli and Markov measures; Hyperbolic toral automorphisms	
	5. The Variational Principle	179
5.	SYSTEMS WITH SMOOTH INVARIANT MEASURES AND MORE EXAMPLES	183
	1. Existence of smooth invariant measures	183
	The smooth measure class; The Perron–Frobenius operator and divergence; Criteria for existence of smooth invariant measures; Absolutely continuous invariant measures for expanding maps; The Moser Theorem	
	2. Examples of Newtonian systems	196
	The Newton equation; Free particle motion on the torus; The mathematical pendulum; Central forces	
	3. Lagrangian mechanics	200
	Uniqueness in the configuration space; The Lagrange equation; Lagrangian systems; Geodesic flows; The Legendre transform	
	4. Examples of geodesic flows	205
	Manifolds with many symmetries; The sphere and the torus; Isometries of the hyperbolic plane; Geodesics of the hyperbolic plane; Compact factors; The dynamics of the geodesic flow on compact hyperbolic surfaces	
	5. Hamiltonian systems	219
	Symplectic geometry; Cotangent bundles; Hamiltonian vector fields and flows; Poisson brackets; Integrable systems	
	6. Contact systems	229
	Hamiltonian systems preserving a 1-form; Contact forms	
	7. Algebraic dynamics: Homogeneous and affine systems	233
Part 2 Local analysis and orbit growth		
6.	LOCAL HYPERBOLIC THEORY AND ITS APPLICATIONS	237
	1. Introduction	237
	2. Stable and unstable manifolds	239
	Hyperbolic periodic orbits; Exponential splitting; The Hadamard–Perron Theorem; Proof of the Hadamard–Perron Theorem; The Inclination Lemma	

Contents	ix
3. Local stability of a hyperbolic periodic point The Hartman–Grobman Theorem; Local structural stability	260
4. Hyperbolic sets Definition and invariant cones; Stable and unstable manifolds; Closing Lemma and periodic orbits; Locally maximal hyperbolic sets	263
5. Homoclinic points and horseshoes General horseshoes; Homoclinic points; Horseshoes near homoclinic points	273
6. Local smooth linearization and normal forms Jets, formal power series, and smooth equivalence; General formal analysis; The hyperbolic smooth case	278
7. TRANSVERSALITY AND GENERICITY	287
1. Generic properties of dynamical systems Residual sets and sets of first category; Hyperbolicity and genericity	287
2. Genericity of systems with hyperbolic periodic points Transverse fixed points; The Kupka–Smale Theorem	290
3. Nontransversality and bifurcations Structurally stable bifurcations; Hopf bifurcations	298
4. The theorem of Artin and Mazur	304
8. ORBIT GROWTH ARISING FROM TOPOLOGY	307
1. Topological and fundamental-group entropies	308
2. A survey of degree theory Motivation; The degree of circle maps; Two definitions of degree for smooth maps; The topological definition of degree	310
3. Degree and topological entropy	316
4. Index theory for an isolated fixed point	318
5. The role of smoothness: The Shub–Sullivan Theorem	323
6. The Lefschetz Fixed-Point Formula and applications	326
7. Nielsen theory and periodic points for toral maps	330
9. VARIATIONAL ASPECTS OF DYNAMICS	335
1. Critical points of functions, Morse theory, and dynamics	336
2. The billiard problem	339
3. Twist maps Definition and examples; The generating function; Extensions; Birkhoff periodic orbits; Global minimality of Birkhoff periodic orbits	349
4. Variational description of Lagrangian systems	365
5. Local theory and the exponential map	367
6. Minimal geodesics	372
7. Minimal geodesics on compact surfaces	376
Part 3 Low-dimensional phenomena	
10. INTRODUCTION: WHAT IS LOW-DIMENSIONAL DYNAMICS? Motivation; The intermediate value property and conformality; Very low-dimensional and low-dimensional systems; Areas of low-dimensional dynamics	381
11. HOMEOMORPHISMS OF THE CIRCLE	387
1. Rotation number	387

x	Contents	
	2. The Poincaré classification	393
	Rational rotation number; Irrational rotation number; Orbit types and measurable classification	
12.	CIRCLE DIFFEOMORPHISMS	401
	1. The Denjoy Theorem	401
	2. The Denjoy example	403
	3. Local analytic conjugacies for Diophantine rotation number	405
	4. Invariant measures and regularity of conjugacies	410
	5. An example with singular conjugacy	412
	6. Fast-approximation methods	415
	Conjugacies of intermediate regularity; Smooth cocycles with wild coboundaries	
	7. Ergodicity with respect to Lebesgue measure	419
13.	TWIST MAPS	423
	1. The Regularity Lemma	424
	2. Existence of Aubry–Mather sets and homoclinic orbits	425
	Aubry–Mather sets; Invariant circles and regions of instability	
	3. Action functionals, minimal and ordered orbits	434
	Minimal action; Minimal orbits; Average action and minimal measures; Stable sets for Aubry–Mather sets	
	4. Orbits homoclinic to Aubry–Mather sets	441
	5. Nonexistence of invariant circles and localization of Aubry–Mather sets	447
14.	FLOWS ON SURFACES AND RELATED DYNAMICAL SYSTEMS	451
	1. Poincaré–Bendixson theory	452
	The Poincaré–Bendixson Theorem; Existence of transversals	
	2. Fixed-point-free flows on the torus	457
	Global transversals; Area-preserving flows	
	3. Minimal sets	460
	4. New phenomena	464
	The Cherry flow; Linear flow on the octagon	
	5. Interval exchange transformations	470
	Definitions and rigid intervals; Coding; Structure of orbit closures; Invariant measures; Minimal nonuniquely ergodic interval exchanges	
	6. Application to flows and billiards	479
	Classification of orbits; Parallel flows and billiards in polygons	
	7. Generalizations of rotation number	483
	Rotation vectors for flows on the torus; Asymptotic cycles; Fundamental class and smooth classification of area-preserving flows	
15.	CONTINUOUS MAPS OF THE INTERVAL	489
	1. Markov covers and partitions	489
	2. Entropy, periodic orbits, and horseshoes	493
	3. The Sharkovsky Theorem	500
	4. Maps with zero topological entropy	505
	5. The kneading theory	511
	6. The tent model	514

Contents	xi
16. SMOOTH MAPS OF THE INTERVAL	519
1. The structure of hyperbolic repellers	519
2. Hyperbolic sets for smooth maps	520
3. Continuity of entropy	525
4. Full families of unimodal maps	526
Part 4 Hyperbolic dynamical systems	
17. SURVEY OF EXAMPLES	531
1. The Smale attractor	532
2. The DA (derived from Anosov) map and the Plykin attractor The DA map; The Plykin attractor	537
3. Expanding maps and Anosov automorphisms of nilmanifolds	541
4. Definitions and basic properties of hyperbolic sets for flows	544
5. Geodesic flows on surfaces of constant negative curvature	549
6. Geodesic flows on compact Riemannian manifolds with negative sectional curvature	551
7. Geodesic flows on rank-one symmetric spaces	555
8. Hyperbolic Julia sets in the complex plane Rational maps of the Riemann sphere; Holomorphic dynamics	559
18. TOPOLOGICAL PROPERTIES OF HYPERBOLIC SETS	565
1. Shadowing of pseudo-orbits	565
2. Stability of hyperbolic sets and Markov approximation	571
3. Spectral decomposition and specification Spectral decomposition for maps; Spectral decomposition for flows; Specifica- tion	574
4. Local product structure	581
5. Density and growth of periodic orbits	583
6. Global classification of Anosov diffeomorphisms on tori	587
7. Markov partitions	591
19. METRIC STRUCTURE OF HYPERBOLIC SETS	597
1. Hölder structures The invariant class of Hölder-continuous functions; Hölder continuity of conju- gacies; Hölder continuity of orbit equivalence for flows; Hölder continuity and differentiability of the unstable distribution; Hölder continuity of the Jacobian	597
2. Cohomological equations over hyperbolic dynamical systems The Livschitz Theorem; Smooth invariant measures for Anosov diffeomor- phisms; Time change and orbit equivalence for hyperbolic flows; Equivalence of torus extensions	608
20. EQUILIBRIUM STATES AND SMOOTH INVARIANT MEASURES	615
1. Bowen measure	615
2. Pressure and the variational principle	623
3. Uniqueness and classification of equilibrium states Uniqueness of equilibrium states; Classification of equilibrium states	628

xii	Contents	
	4. Smooth invariant measures	637
	Properties of smooth invariant measures; Smooth classification of Anosov diffeomorphisms on the torus; Smooth classification of contact Anosov flows on 3-manifolds	
	5. Margulis measure	643
	6. Multiplicative asymptotic for growth of periodic points	651
	Local product flow boxes; The multiplicative asymptotic of orbit growth	
	Supplement	
S.	DYNAMICAL SYSTEMS WITH NONUNIFORMLY HYPERBOLIC BEHAVIOR BY ANATOLE KATOK AND LEONARDO MENDOZA	659
	1. Introduction	659
	2. Lyapunov exponents	660
	Cocycles over dynamical systems; Examples of cocycles; The Multiplicative Ergodic Theorem; Osedelec–Pesin ϵ -Reduction Theorem; The Ruelle inequality	
	3. Regular neighborhoods	672
	Existence of regular neighborhoods; Hyperbolic points, admissible manifolds, and the graph transform	
	4. Hyperbolic measures	678
	Preliminaries; The Closing Lemma; The Shadowing Lemma; Pseudo-Markov covers; The Livschitz Theorem	
	5. Entropy and dynamics of hyperbolic measures	693
	Hyperbolic measures and hyperbolic periodic points; Continuous measures and transverse homoclinic points; The Spectral Decomposition Theorem; Entropy, horseshoes, and periodic points for hyperbolic measures	
	Appendix	
A.	BACKGROUND MATERIAL	703
	1. Basic topology	703
	Topological spaces; Homotopy theory; Metric spaces	
	2. Functional analysis	711
	3. Differentiable manifolds	715
	Differentiable manifolds; Tensor bundles; Exterior calculus; Transversality	
	4. Differential geometry	727
	5. Topology and geometry of surfaces	730
	6. Measure theory	731
	Basic notions; Measure and topology	
	7. Homology theory	735
	8. Locally compact groups and Lie groups	738
	NOTES	741
	HINTS AND ANSWERS TO THE EXERCISES	765
	REFERENCES	781
	INDEX	793

Preface

The theory of dynamical systems is a major mathematical discipline closely intertwined with most of the main areas of mathematics. Its mathematical core is the study of the global orbit structure of maps and flows with emphasis on properties invariant under coordinate changes. Its concepts, methods, and paradigms greatly stimulate research in many sciences and have given rise to the vast new area of applied dynamics (also called nonlinear science or chaos theory). The field of dynamical systems comprises several major disciplines, but we are interested mainly in finite-dimensional differentiable dynamics. This theory is inseparably connected with several other areas, primarily ergodic theory, symbolic dynamics, and topological dynamics. So far there has been no account that treats differentiable dynamics from a sufficiently comprehensive point of view encompassing the relations with these areas. This book attempts to fill this gap. It provides a self-contained coherent comprehensive exposition of the fundamentals of the theory of smooth dynamical systems together with the related areas of other fields of dynamics as a core mathematical discipline while providing researchers interested in applications with fundamental tools and paradigms. It introduces and rigorously develops the central concepts and methods in dynamical systems and their applications to a wide variety of topics.

What this book contains. We begin with a detailed discussion of a series of elementary but fundamental examples. These are used to formulate the general program of the study of asymptotic properties as well as to introduce the principal notions (differentiable and topological equivalence, moduli, structural stability, asymptotic orbit growth, entropies, ergodicity, etc.) and, in a simplified way, a number of important methods (fixed-point methods, coding, KAM-type Newton method, local normal forms, homotopy trick, etc.).

The main theme of the second part is the interplay between local analysis near individual (e.g., periodic) orbits and the global complexity of the orbit structure. This is achieved by exploring hyperbolicity, transversality, global topological invariants, and variational methods. The methods include the study of stable and unstable manifolds, bifurcations, index and degree, and construction of orbits as minima and maximaxes of action functionals.

In the third and fourth parts the general program outlined in the first part is carried out to considerable depth for low-dimensional and hyperbolic dynamical systems which are particularly amenable to such analysis. Hyperbolic systems are the prime example of well-understood complexity. This manifests itself in an orbit structure that is rich both from the topological and statistical point of view and stable under perturbation. At the same time the principal features can be described qualitatively and quantitatively with great precision. In low-dimensional dynamical systems on the other hand there are two situations. In the “very low-dimensional” case the orbit structure is simplified and admits only a limited amount of complexity. In the “low-dimensional” case some complexity is possible, yet additional major aspects of the orbit structure can be understood via hyperbolicity or related types of behavior.

Although we develop most themes related to differentiable dynamics in some depth we have not tried to write an encyclopedia of differentiable dynamics. Even if this were possible, the resulting work would be strictly a reference source and not useful as an introduction or a text. Consequently we also do not strive to present the most definitive results available but rather to provide organizing principles for methods and results. This is also not a book on applied dynamics and the examples are not chosen from those models that are widely studied in various disciplines. Instead our examples arise naturally from the internal structure of the subject and contribute to its understanding. The emphasis placed on various areas in the field is not dictated by the relative amount of published work or research activity in those areas, but reflects our understanding of what is basic and fundamental in the subject. An obvious disparity appears in the area of one-dimensional (real and especially complex) dynamics, which witnessed a great surge of activity in the past 15 years producing a number of brilliant results. It plays a relatively modest role in this book. Real one-dimensional dynamics is used mainly as an easy model situation in which various methods can be applied with considerable success. Complex dynamics, which is in our view a fascinating but rather specialized area, appears only as a source of examples of hyperbolic sets. On the other hand we try to point out and emphasize the interactions of dynamics with other areas of mathematics (probability theory, algebraic and differential topology, geometry, calculus of variations, etc.) even in some situations where the current state of knowledge is somewhat tentative.

How to use this book. This book can be used both as a text for a course or for self-study and as a reference book. As a text it would most naturally be used as the primary source for graduate students with background equivalent to one year of graduate study at a major U.S. university who are interested in becoming specialists in dynamical systems or want to acquire solid general knowledge of the field. Some portions of this book do not assume as much background and can be used by advanced undergraduate students or graduate students in science and engineering who want to learn about the subject without becoming experts. Those portions include Chapter 1, most of Chapters 2, 3, and 5, parts of Chapters 4, 6, 8, and 9, Chapters 10 and 11, and most of Chapters 12, 14, 15, and 16. The 472 exercises are a very important part of the book. They fall into several categories. Some of them directly illustrate the use of results or methods from the text; others explore examples that are not discussed in the text or indicate further developments. Sometimes an important side topic is developed in a series of exercises. Those 317 that we do not consider routine have been provided with hints or brief solutions in the back of the book. An asterisk indicates our subjective assessment of higher difficulty, due to the need for either inventiveness or familiarity with material not obviously related to the subject at hand.

Each of the four parts of the book can be the basis of a course roughly at the second-year graduate level running one semester or longer. From this book one can tailor many courses dedicated to more specialized topics, such

as variational methods in classical mechanics, hyperbolic dynamical systems, twist maps and applications, an introduction to ergodic theory and smooth ergodic theory, and the mathematical theory of entropy. In order to assist both students and teachers in selecting material for a course we summarize the principal interrelations between the chapters in Figure F.1. A solid arrow $A \rightarrow B$ indicates that a major portion of the material from Chapter A is used in Chapter B (this relation is transitive). A dashed arrow $A \dashrightarrow B$ indicates that material from Chapter A is used in some parts of Chapter B.

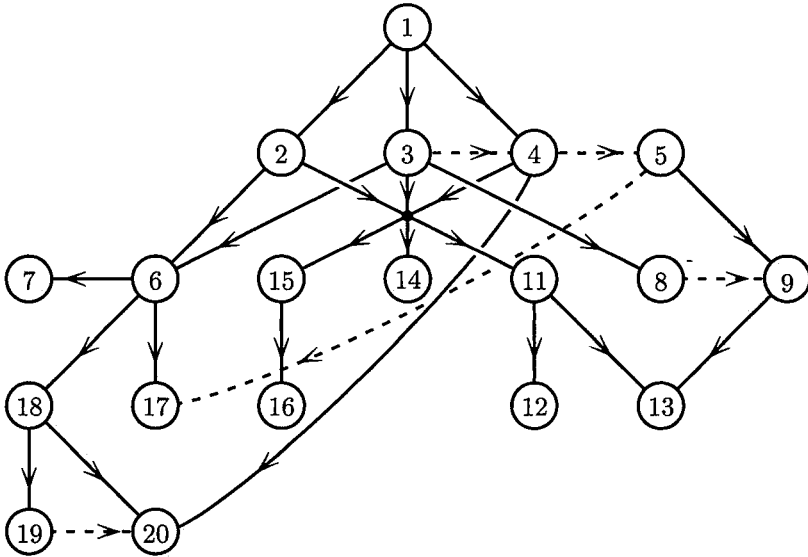


FIGURE F.1.

With the exception of Chapters 1–4, which form a common basis for the rest of the book, generally the material in the left part of the diagram deals with hyperbolic dynamics, that in the middle with low-dimensional dynamics, and that in the right with aspects of differentiable dynamics related to topology and classical mechanics.

There are various kinds of material used in this book. First of all we tacitly use, and assume familiarity with, the results of linear algebra (including Jordan normal forms), calculus of several variables, the basic theory of ordinary differential equations (including systems), elementary complex analysis, basic set theory, elementary Lebesgue integration, basic group theory, and some Fourier series. There is a next higher level of essential background material which is reviewed in the appendix. Most of the material in the appendix is of this nature, namely, the standard theory of topological, metric, and Banach spaces, elementary homotopy theory, the basic theory of differentiable manifolds including

vector fields, bundles and differential forms, and the definition and basic properties of Riemannian metrics. Some topics are used on isolated occasions only. This last level of material includes the basic topology and geometry of surfaces and the general theory of measures, σ -algebras, and Lebesgue spaces, homology theory, material related to Lie groups and symmetric spaces, curvature and connections on manifolds, transversality, and normal families of complex functions. Most, but not all, of this material is also reviewed in the appendix, usually in a less detailed fashion. Either such material can be taken on faith without loss to the application in the text, or otherwise the pertinent portion of the text can be skipped without great loss.

On several occasions we include an important background fact without proof in the text. This happens when a certain result is organically related to a particular section. The Lefschetz Fixed-Point Formula is a good example of such a result.

Sources. Most of the material in this book does not consist of original results. Nevertheless the presentation of most of the material is our own and consists of original or considerably modified proofs of known results, explanations of the structure and interconnectedness of the subject, and so forth. Some portions of the text, roughly a sixth of it, mostly in Parts 3 and 4, closely follow other published sources, the majority of these being original research articles. An outstanding example is the presentation of portions of the hyperbolic theory in Chapters 18 and 20 which was given such a clear treatment by R. Bowen in his articles in the seventies that it could hardly be improved. On several occasions we follow the exposition of a subject in existing books. With the exception of some basic subjects such as Hamiltonian formalism or variational calculus this occurs only in Part 3. The reason for this is that low-dimensional dynamics has a much better developed expository literature than the field as a whole. We acknowledge all borrowings of proofs and presentations of which we are aware in the notes near the end of the book.

Since we aim to present the subject by developing it from first principles and in a self-contained way, rather than to give an exhaustive account of the development and current state of the field, we do not attempt a comprehensive listing of relevant references which would easily increase our bibliography to a thousand items or more. In particular, not all theorems are attributed to the original authors, especially if the results are part of the broader developments in the field, rather than landmark results or of a rather special nature. Most of the attributions are relegated to the notes. These consist of general comments arranged by section and some numbered remarks to particular points in the main text. Furthermore, in order not to interrupt the logical flow of the text all bibliographical references for the main part of the book are also relegated to the notes. Our historical comments, in both the introduction and the notes, do not aim to present a coherent account of the development of the subject, but to subjectively select some of its major moments.

We have included several types of literature in the bibliography. First, we have tried to list all major monographs and representative textbooks and sur-

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Frontmatter

[More information](#)

veys covering the principal branches of dynamics. Next there are landmark papers that introduce and develop various branches of our subject, define principal notions, or contain proofs of major results. We try to list all the sources on which the presentation in various parts of the book is based, or that inspired our presentation in other places and many (but not all) of the original sources for specific results presented in the text. Finally there is a sample of references to important work, both original and surveys, in some areas touched upon but not treated in the text. According to our principle of selecting models for their intrinsic interest rather than their value for concrete scientific problems we omit works by nonmathematicians (even important ones) that are dedicated to the study of models motivated by scientific problems so long as these contain only hypotheses and numerical results. References to such works are widely available in many of the books and surveys that we quote.

History and acknowledgments. The general idea of writing a broad introduction to the theory of dynamical systems first occurred to the first author when he taught a graduate course at the California Institute of Technology in 1984–5. This course resulted in two sets of lecture notes prepared by the second author and by his fellow graduate student John Lindner to whom we are deeply grateful. The key idea of introducing the principal notions and methods via a presentation of a series of basic examples crystallized when the first author was preparing and teaching an intensive four-week course in July 1986 at the Summer Mathematics Institute for graduate students at Fudan University in Shanghai. The summary and notes from that course became the germ for major parts of Chapters 1–4. Further progress was made during another graduate course at the California Institute of Technology in 1986–7 after which it became clear that the original project of a book of 300–350 pages would result in too sketchy and incomplete an account of the subject. In the summer of 1989 we developed a detailed plan of the book which has been carried out with some substantial later modification. Another graduate course during the first author's first year at the Pennsylvania State University (1990–1) helped to test some existing parts of the book and develop some new material.

We feel deep gratitude to the California Institute of Technology, Tufts University, and the Pennsylvania State University for providing excellent working conditions and supporting several mutual visits. Special thanks are due to the Mathematical Sciences Research Institute in Berkeley, California, where we worked together on major portions of the book in the summer of 1992. During this period our project was transformed from a collection of drafts into an incomplete but coherent product.

We also owe thanks and gratitude to numerous individuals for providing various kinds of help and inspiration during this project. We apologize for any omissions of people whose comments and suggestions may have been incorporated and forgotten.

Jessica Madow, the technical typist at the California Institute of Technology, typed major portions of the then existing manuscript in Exp. Kathy Wyland and Pat Snare at the Pennsylvania State University typed the first drafts of

Cambridge University Press

978-0-521-57557-7 - Introduction to the Modern Theory of Dynamical Systems

Anatole Katok and Boris Hasselblatt

Frontmatter

[More information](#)

xviii

Preface

many chapters in \TeX . Several people helped with computer support or typesetting advice. David Glaubman at the Mathematical Sciences Research Institute was very helpful, Michael Downes at the Technical Support Department of the American Mathematical Society helped to make the running heads come out right on every single page, and our colleague Uwe Schmock at the ETH wrote the overbar macro for \TeX and made other useful comments. Boris Katok made the majority of the illustrations for the book. Bill Schlesinger gave us the initial tutoring that enabled us to make numerous pictures using Matlab. We are deeply grateful to the editors of Cambridge University Press: David Tranah who encouraged and prodded us during the earlier stages of the project and Lauren Cowles who patiently guided us through the process of finishing the book and getting it ready for production. This book was typeset in \TeX using $\mathcal{A}\mathcal{M}\mathcal{S}\text{-}\text{\TeX}$, the \TeX macro package of the American Mathematical Society.

Viorel Nițică and Alexej Kononenko wrote solutions to the majority of the exercises. Their work helped to correct some flawed exercises, and we used their solutions to write many of our hints.

The following people made numerous suggestions, including pointing out mathematical and stylistical errors, misprints, and the need for better explanations: The greatest amount of this kind of help came from Howie Weiss at the Pennsylvania State University. Further comments were given to us by Luis Barreira, Misha Brin, Mirko Degli-Esposti, David DeLatte, Serge Ferleger, Eugene Gutkin, Moisey Guysinsky, Miaohua Jiang, Tasso Kaper, Alexej Kononenko, Viorel Nițică, Ralf Spatzier, Garrett Stuck, Andrew Török, and Chengbo Yue.

In particular Howie Weiss, Tasso Kaper, Garrett Stuck, Ralf Spatzier, and Misha Brin taught from parts of the book and were very helpful in polishing it.

We had fruitful discussions with Michael Jakobson, Welington de Melo, Mikhael Lyubich, and Zbigniew Nitecki concerning one-dimensional maps and with Eduard Zehnder on variational methods. These were useful in crystallizing the content and presentation of those respective chapters. Gene Wayne helped by providing references concerning infinite-dimensional dynamical systems and Mike Boyle gave some useful guidance for sources in symbolic dynamics.

A number of corrections were made between printings. We would like to thank Luis Barreira, Marlies Gerber, Karl Friedrich Siburg, Garrett Stuck and Andrew Török who pointed out many small errors. Peter Walters found inaccuracies in Lemma 4.5.2 and Lemma 20.2.3. Robert McKay pointed out that some results of Section 14.2 needed recurrence hypotheses and Jonathan Robbins noted problems with the first version of Step 5 in the proof of the Hadamard-Perron Theorem 6.2.8. Tim Hunt corrected the DA construction. Corrections are listed at <http://www.tufts.edu/~bhasselb/thebook.html>.

A serious omission survived three printings: Section 20.6 is entirely due to Charles Toll, an attribution we inadvertently failed to make. Our sincere apologies.

Last, and most importantly, we wish to thank Svetlana Katok and Kathleen Hasselblatt for constant support and inspiration.