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Telescope optics

This is a book on advanced techniques in amateur astronomy, not a textbook on optics. However, the amateur astronomer should understand the basics of optical theory as applied to telescopes. How else might he/she make sensible choices as to the equipment to be purchased or constructed? The characteristics and performance of the telescope and its auxiliary equipment depend heavily on design. Aperture, focal length, focal ratio, image scale, resolving power, image brightness, image contrast, magnification and diffraction pattern structure are just some of the interrelated factors of crucial importance. The purpose of this chapter is to provide a summary of the optical matters relevant to the needs of the telescope user.

Focal length and image scale

Before considering specific types of telescopes let us take the imaginary case of a single, perfect, converging lens forming an image of a distant object. Further, imagine that the object is a point source, such as a star. With reference to Figure 1.1(a), the rays from the star will arrive at the lens virtually parallel. The line perpendicular to the plane of the lens and passing through its centre is known as the *optical axis*. If, as is shown in the diagram, the optical axis of the lens is aligned in the direction of the star, then all the arriving rays will be parallel to the optical axis.

If the lens is truly perfect then all the rays from the star will be brought together at a common point after passing through the lens. A screen placed at this point would show a focused image of the star. This position is known as the *principal focus* of the lens. The distance from the centre of the lens to the principal focus is known as the *focal length*. Since the optical axis of the lens is aligned to the direction of the star, the focused image is, itself, formed on the optical axis.

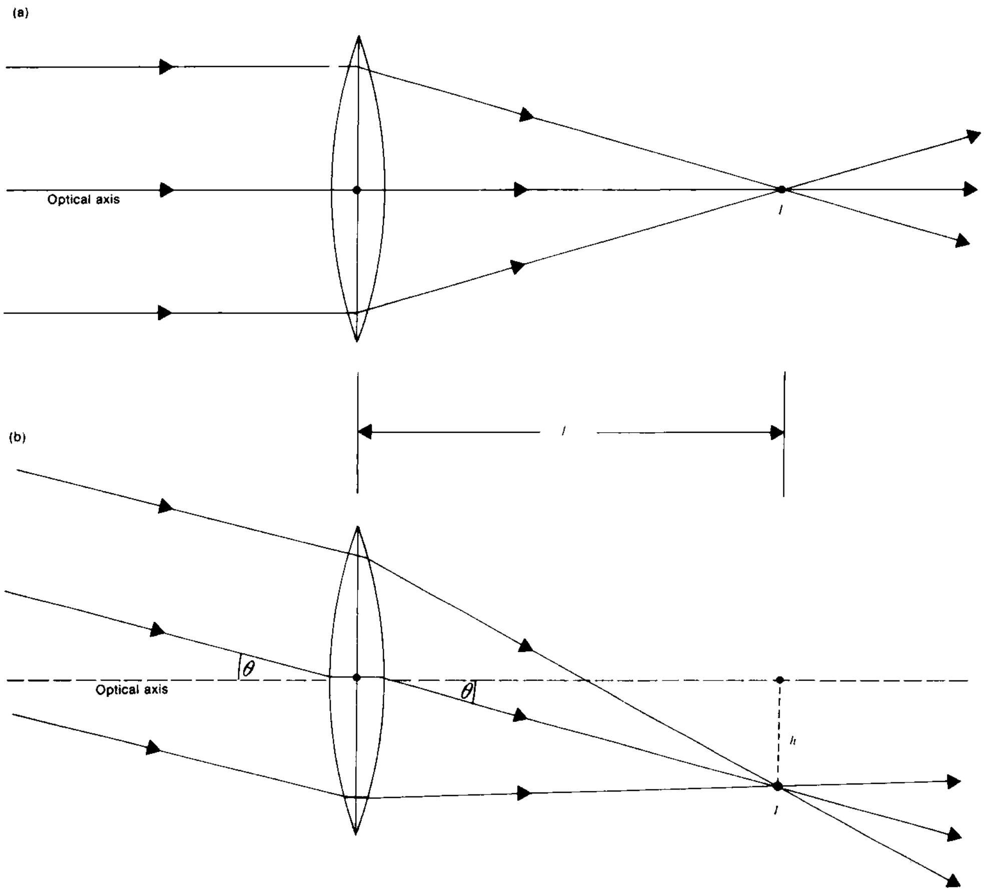


Figure 1.1. Light from a star being focused by an optically perfect lens, of focal length *f*. In (a) the optical axis of the lens is aligned to the direction of the star. After refraction through the lens the light achieves a focus at a point *I*. In (a) this point is on the optical axis. In (b) the arriving starlight makes an angle θ to the optical axis. In this case the point of focus is on the focal plane of the lens but is a distance *h* from the optical axis.

Figure 1.1(b) illustrates the case of the starlight entering the lens at an angle to the optical axis. Notice that the ray passing through the centre of the lens makes the same angle, θ, with the optical axis both before and after refraction by it. In this case the image is formed a distance *h* from the optical axis. In both (a) and (b) the focal length of the lens is *f*. The plane defined by the positions of sharp focus for distant point objects at varying angles to the optical axis is known as the *focal plane* of the lens.

For small values of θ we can make the approximation that:

$$\theta_{radians} = h/f,$$

where θ is measured in radians (see Figure 1.2 for an explanation of radian measure).

Expressing θ in arcseconds:

$$\theta_{arcsec} = 206\,265 \times h/f$$

From which:

$$\theta_{arcsec}/h = 206\,265/f$$

Now, θ_{arcsec}/h is the *image scale* – the linear distance in the focal plane that corresponds to a given angular distance in the sky/distant object.

$$\therefore \textbf{Image scale} = \mathbf{206\,265}/\boldsymbol{f}$$

If the focal length of the lens is measured in millimetres, then the image scale is in arcseconds per millimetre. For example, suppose that the diameter of the Moon subtends an angle of 2000 arcseconds. If the lens has a focal length of 1000 mm then the image scale in its focal plane is, to 3 significant figures, 206 arcseconds per millimetre. The focused image of the Moon will, in this case, be just under 10 mm across.

The image scale in the focal plane actually varies with the distance from the optical axis, though only by a negligible amount for small values of θ.

Aperture, focal ratio and light grasp

The *aperture* (diameter), focal length and *focal ratio* of our hypothetical lens are related by:

$$\text{Focal ratio} = \text{focal length/aperture},$$

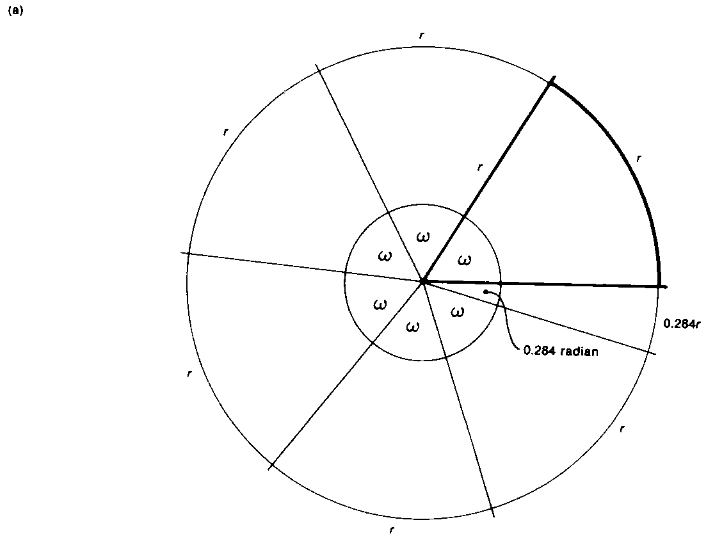
the aperture and the focal length being expressed in the same units.

The light collected by the lens is proportional to its area, and is thus proportional to the square of its aperture:

$$\text{Light grasp} \propto (\text{aperture})^2$$

Diffraction and resolution

It is an oversimplification to say that a lens, even a perfect one, will form a truly point image of a point object. In practice the image will consist of a



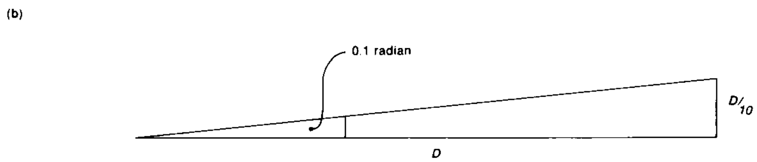


Figure 1.2. Radian measure. Consider a circle of radius r. If an arc of length equal to r is drawn along the circumference, the angle that the arc subtends from the centre of the circle is defined to be 1 radian. This is illustrated in (a) by the boldly outlined sector. In this diagram the angle ω is equal to 1 radian. Since the circumference of a circle is equal to 2π (6.284) times its radius, it follows that there are 2π radians in 360°. Hence:

$$1\ \textit{radian} = 360°/2\pi = 57°.29578$$
$$= 206\ 265\ \textit{arcseconds}.$$

In general, if the arc length is l and the radius of the arc is R, then the angle, θ, is given by:

$$\theta\ (\text{in radians}) = l/R,$$
$$\text{or}\ \theta\ (\text{in arcseconds}) = 206\ 265\ l/R$$

Diagram (b) shows the concept of the radian applied to smaller angles. In this case the angle subtended by a length $D/10$ (notice that the distiction between a straight line and an arc can be ignored for small angles) from a distance D is equal to 0.1 radian.

diffraction pattern, an effect caused by the wave nature of the light passing through the restricted aperture of the lens.

The diffraction pattern that our hypothetical lens might form of a star is represented, at a very large scale, in Figure 1.3. It consists of a central bright disc, the *Airy disc*, surrounded by concentric rings of decreasing brightness. The diffraction pattern of a star produced by a perfect lens has 84 per cent of the light energy contained in the Airy disc. The remainder of the light energy is shared amongst the more diffuse rings.

The diameter of the diffraction pattern is inversely proportional to the diameter of the lens producing it. Hence **larger** apertures produce **smaller** diffraction patterns. This effect is of crucial importance when the lens is used to image fine details. For instance, consider the case of two close stars being imaged by a lens of given aperture. The size of the aperture sets the sizes of the diffraction pattern images of each star. If the stars are too close together their diffraction patterns will merge and the stars will appear as one. In order to resolve the two stars as separate a lens of larger aperture must be used. Figure 1.4 (a), (b) and (c) illustrates the cases of two stars which are not resolved as separate, just resolved, and easily resolved, respectively.

The relationship between the limiting resolution of a lens and its aperture is expressed mathematically by the equation:

$$R = 206\,265 \times \frac{1.22\lambda}{d}$$

where R is the minimum angular separation of a pair of stars in order to be just resolved, measured in arcseconds; λ is the wavelength of the light being imaged and d is the diameter of the lens. Both λ and d are measured in the same units, for instance both in metres, or both in centimetres, etc.

You should notice that the resolution of the lens is wavelength dependent. The wavelength of light to which the eye is most sensitive is taken as 5.5×10^{-7}m (This corresponds to the yellow-green part of the spectrum). A perfect lens of 0.1m aperture should then be able to resolve two stars separated by 1.4 arcseconds when used visually.

Another formula for resolving power at visual wavelengths is that derived empirically by Dawes:

$$R = 4.56/D,$$

where R is the minimum resolvable separation of two stars in arcseconds, as before, and D is the aperture expressed in inches (1 inch = 25.4 mm). Dawes' formula predicts a minimum separation for resolution about 20 per

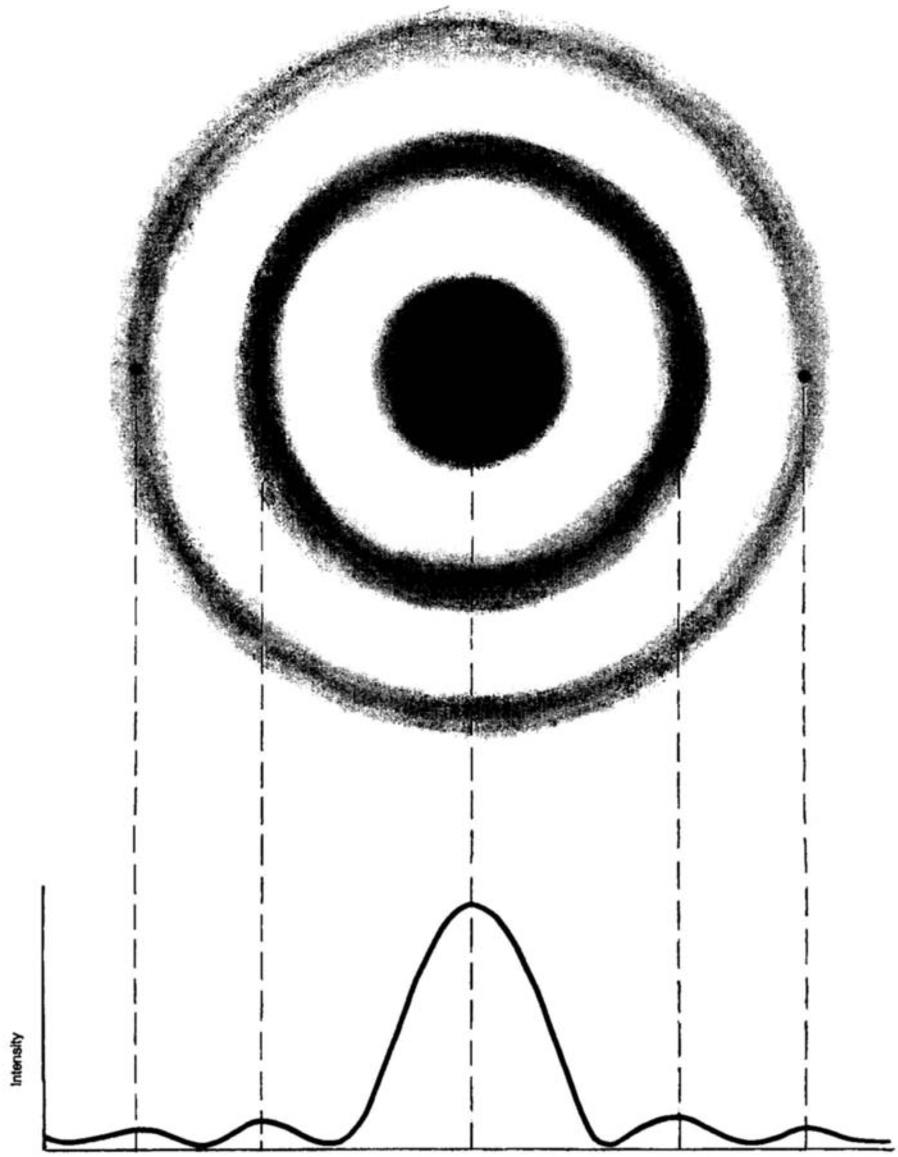


Figure 1.3. A representation of the diffraction pattern of a point source as seen through a limited aperture. The graph below illustrates the variation of the intensity of the light across the diameter of the diffraction pattern.

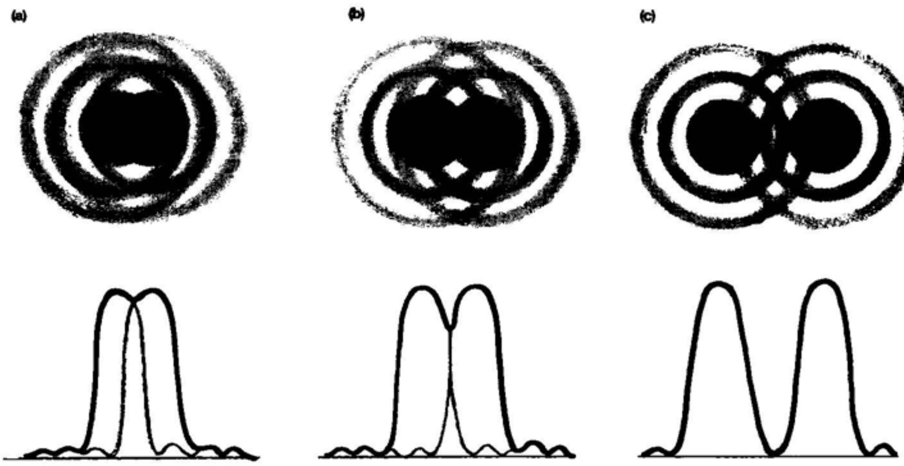


Figure 1.4. The overlapping diffraction patterns produced by two stars as seen through a given aperture. In (a) the stars are too close together to be resolved as separate. In (b) the stars are just resolvable and in (c) the stars are easily resolved.

cent closer than that of the mathematically derived formula but double star observers have found it to be more accurate in practice. Strictly, Dawes' formula applies to a pair of double stars of the sixth magnitude. Brighter stars, and those of unequal brightness, are less easily resolved.

The image of an extended object (anything which does not appear as a point), such as the surface of the Moon or a planet, can be thought of as a series of overlapping diffraction patterns. Here, too, the smaller diffraction patterns produced by larger apertures enable finer detail to be resolved. The situation has been compared to viewing a mosaic made of tiles. The finest details that can be seen in the mosaic are the size of the individual tiles. Making the mosaic from smaller tiles allows finer details to be represented.

The simple refractor

Figure 1.5(a) illustrates the optical principles of the simple refractor. Here the object glass and eyepiece are each represented as single lenses. The single-arrowed light rays originate from the top of some distant object and the double-arrowed light rays originate from the bottom of it. The object glass forms an inverted image of the object at position *I*. This is the object glass' focal plane.

When in normal adjustment, the eyepiece's focal plane is also made to coincide with position *I*. The rays then emerge from the eyepiece in parallel bundles. Notice that all the single-arrowed rays are parallel after passing through the eyepiece. The same is true for the other set of rays shown. An observer with normal vision can most comfortably view the image formed by the telescope when the rays emerge from the eyepiece in parallel bundles.

All the rays that pass through the object glass also pass through the plane at a position I'. This is variously known as the *Ramsden disc*, the *eye ring*, or, most popularly, the *exit pupil*. The exit pupil is an image of the object glass formed by the eyepiece (see Figure 1.6). It's significance is that all the rays collected by the object glass pass through this position and so it is the best location for the pupil of the observer's eye. If the observer were to place his/her eye far from the exit pupil then much of the light collected by the object glass would not pass through his/her eye pupil, and so would be wasted.

The magnification produced by the telescope can best be understood by referring to Figure 1.5(b), which shows only the relevant light rays. Notice that the object subtends an angle θ_1 from the telescope objective. It produces an image of height h at position *I*. h, θ_1 and the focal length of the object glass (f_o) are related by the equation:

$$\theta_1 = h/f_o.$$

(Strictly, this equation is only accurate if θ_1 is a small angle). The observer views this image through the eyepiece. Since the focal length of the eyepiece (f_e) is less than that of the object glass, the image subtends a larger angle, θ_2, from the eyepiece:

$$\theta_2 = h/f_e.$$

Dividing the two equations gives:

$$\theta_2/\theta_1 = f_o/f_e.$$

The telescope has increased the apparent angular size of the object from θ_1 to θ_2. θ_2/θ_1 is the *angular magnification, m*, produced by the telescope. Hence the well known equation for the magnification of the telescope in terms of the focal lengths of its object glass and eyepiece:

$$m = f_o/f_e.$$

Similar reasoning would show that the diameter of the exit pupil produced by the telescope is also linked to the magnification, as well as to the aperture of the telescope:

$$m = \text{aperture}/\text{diameter of exit pupil},$$

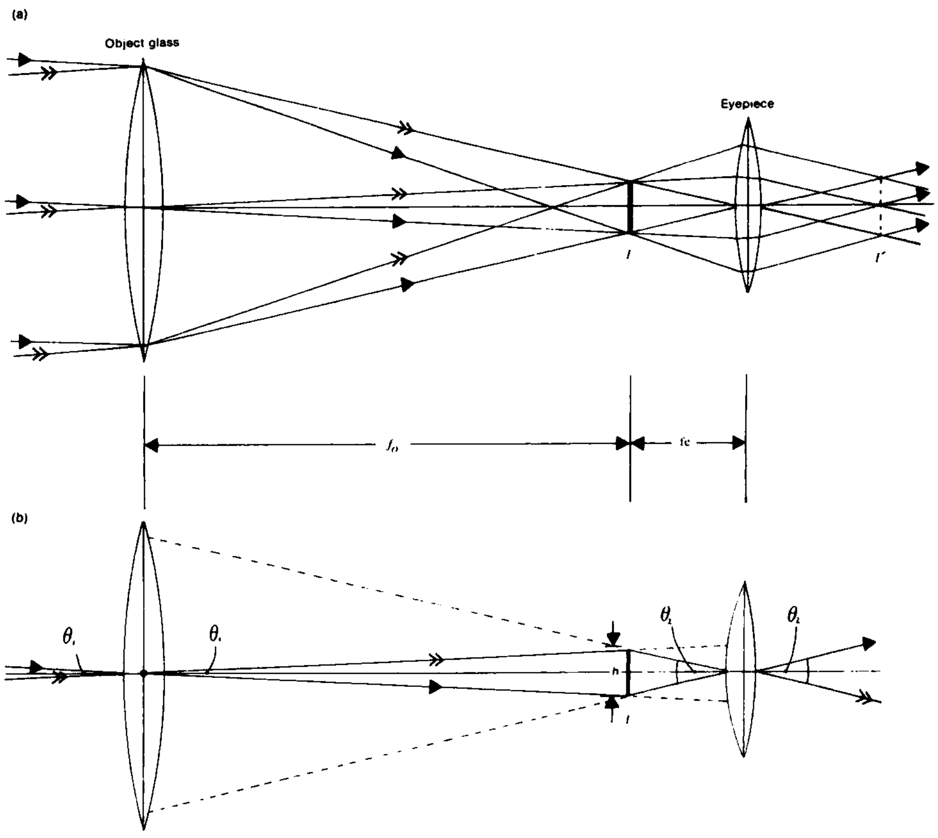


Figure 1.5. The simple refracting telescope. *I* denotes the positions of the coincident focal planes of the object glass and the eyepiece. In (a) *I′* marks the position of the exit pupil. (b) shows only the rays necessary for the explanation of the magnification the telescope produces (see text for details).

where the aperture and exit pupil diameter are measured in the same units. The exit pupil diameter is important – if it is larger than the observer's eye pupil then not all of the light collected by the object glass will enter his/her eye.

The iris of a youth's eye will open to about 8 millimetres diameter, when fully dark-adapted. In older people the maximum size of the opening decreases. Studies have shown that it averages 6.5 mm diameter for a thirty year old and further decreases with advancing years at the rate of about 0.5 mm per decade. The size of the eye pupil puts a lower limit on the magnification that can be used on a telescope without wasting any of the light gathered by its objective. If a particular

Figure 1.6. The bright image of the primary mirror of the author's telescope can be clearly seen in the eyelens of the eyepiece. This image is the exit pupil, the best position for the observer's eye in order to receive all the light collected by the mirror. The image is actually centred on the optical axis but is formed a short distance in front of the eyelens, which is why it appears displaced to the upper left in this oblique view.

observer's dark-adapted eye has a pupil diameter of 6 mm then any magnification less than the aperture in millimetres divided by 6 (this is roughly equal to ×4 per inch of aperture) will produce an exit pupil larger than 6 mm across. Some of the precious light gathered by the telescope's objective will then be wasted.

Optical aberrations

Any practical optical system will have its imperfections. The errors differ both in magnitude and type for different systems, depending upon their design and accuracy of manufacture.

The possible optical errors can be broadly grouped into *chromatic aberration* and five *seidal aberrations*. The following notes briefly detail these aberrations, using a single lens as an example.