CAMBRIDGE TRACTS IN MATHEMATICS

125 *The Hardy–Littlewood method*

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The Hardy–Littlewood method

Second Edition

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Preface

There have been two earlier Cambridge Tracts that have touched upon the Hardy–Littlewood method, namely those of Landau, 1937, and Estermann, 1952. However there has been no general account of the method published in the United Kingdom despite the not inconsiderable contribution of English scholars in inventing and developing the method and the numerous monographs that have appeared abroad.

The purpose of this tract is to give an account of the classical forms of the method together with an outline of some of the more recent developments. It has been deemed more desirable to have this particular emphasis as many of the later applications make important use of the classical material.

It would have been useful to devote some space to the work of Davenport on cubic forms, to the joint work of Davenport and Lewis on simultaneous equations, to the work of Rademacher and Siegel that extends the method to algebraic numbers, and to the work of various authors, culminating in the recent work of Schmidt, on bounds for solutions of homogeneous equations and inequalities. However this would have made the tract unwieldy. The interested reader is referred to the Bibliography.

It is assumed that the reader has a familiarity with the elements of number theory, such as is contained in the treatise of Hardy and Wright. Also, in dealing with one or two subjects it is expected that the reader has a working acquaintance with more advanced topics in number theory. Where necessary, reference is given to a standard text on the subject.

The contents of Chapters 2, 3, 4, 5, 9, 10 and 11 have been made the basis of advanced courses offered at Imperial College over a number of years, and could be used as part of any normal post-graduate training in analytic number theory.

Preface to second edition

At the time that the first edition was written, there had been relatively little recent work on the central theory of the Hardy–Littlewood method, namely that surrounding Waring's problem and associated questions. Indeed, the work of Davenport and Vinogradov had taken on the aspect of being written on tablets of stone. This is in complete contrast to the current situation. In the last decade or so there has been a series of important developments in the area. The tract is, therefore, ripe for revision, and the opportunity has been taken to give an introduction to this new material, and especially to the important work of Wooley. Chapter 5 has been extensively rewritten to take account of our new understanding of Vinogradov's mean value theorem, and a completely new chapter has been added to describe the new work on Waring's problem. Fortunately the large bulk of the material has not been superseded and the underlying ideas still play an important rôle in many of the new developments.

Notation

The letter k denotes a natural number, usually with $k \geqslant 2$, and the statements in which ε appear are true for every positive real number ε. The letter p is reserved for prime numbers.

The Vinogradov symbols $\ll$, $\gg$ have their usual meaning, namely that for functions f and g with g taking non-negative real values $f \ll g$ means $|f| \leqslant Cg$ where C is a constant, and if moreover f is also non-negative, then $f \gg g$ means $g \ll f$.

Implicit constants in the O, $\ll$ and $\gg$ notations usually depend on k, s and ε. Additional dependence will be mentioned explicitly.

As usual in number theory, the functions $e(\alpha)$ and $\| \alpha \|$ denote $e^{2\pi i\alpha}$ and $\min_{h \in \mathbb{Z}} |\alpha - h|$ respectively. Occasionally the expression $\min(X, 1/0)$ occurs, and is taken to be X.

The notation $p^r \| n$ is used to mean that p^r is the highest power of p dividing n.