

## Notation Index

### GENERAL NOTATION

( $F$  and  $G$ , generic species of structures)

$\text{aut}(\mathbf{n})$	number of automorphisms, 20
$[x^n]f(x)$	coefficient of formal power series, 13
$\text{coeff}_{\mathbf{n}}$	coefficient of index series, 20
$=_n$	contact of order $n$ , 23
$\text{cyc}(\psi)$	connected components of $\psi$ , 86
$\text{des}$	number of descents, 346
$\text{Des}$	descent set, 346
$\text{fix } \sigma$	number of fixed points of $\sigma$ , 16
$\text{Fix } \sigma$	fixed points, 16
$\text{inv}$	number of inversions, 346
$\text{Inv}$	inversion set, 346
$\mathcal{O}(x)$	orbit of $x$ , 395
$\mathcal{G}_x$	stabilizer of $x$ , 396
$\mathcal{L}_n^{(\alpha)}(x)$	Laguerre polynomials, 95
$p_n$	power sum, 278
$r_n(\lambda)$	binomial type sequence, 169
$\text{rec}(\psi)$	recurrent points of $\psi$ , 86
$\lambda^{(n)}$	rising factorial, 170
$\lambda_{(n)}$	falling factorial, 170
$\mathfrak{A}$	set of atomic species, 146
$\mathfrak{M}$	set of molecular species, 142
$F[n]$	$F$ -structures on $[n]$ , 13
$F[\sigma]$	transport function, 5
$\underline{F}(x)$	generating series, 12
$\overline{F}(x)$	type generating series, 15
$P_{G;Y}$	cycle index polynomial, 397
$Z_F$	cycle index series, 17
$\Gamma_F$	asymmetry index series, 324
$F'$	derivative, 47
$F^{(k)}$	$k^{\text{th}}$ derivative, 51
$F^\bullet$	pointing, 60
$F^{\bullet n}$	pointing $n$ times, 69
$\int F$	integral, 348
$F_w$	weighted species, 83
$F_{w(\alpha)}$	" , with connected components counter, 97
$F_\alpha$	$F$ -structures with constant weight

$\alpha$ , 86	
$F^c$	connected $F$ -structures, 46
$\kappa_n(F)$	average number of connected $F$ -structures, 63
$F_n$	restriction to cardinality $n$ , 30
$F_{\text{odd}}$	odd part, 38
$F_{\text{even}}$	even part, 38
$\Phi^-$	negative part, 123
$\Phi^+$	positive part, 125
$\overline{F}$	flat part, 322
$\mathbf{T}(F)$	types of $F$ -structures, 14
$F^{\text{inj}}(X_t)$	$F$ -injective colorings, 325
$F \simeq G$	isomorphism (of species), 21
$F = G$	combinatorial equality (isomorphism), 21
$F \equiv G$	equipotence, 20
$F \circ G$	substitution, 41
$F^{(n)}$	iterates for substitution, 46
$F \times G$	Cartesian product, 64
$F \square G$	functorial composition, 70
$F * G$	convolution product, 348
$F \cdot \mathcal{O} G$	ordinal product, 348
$s(n, k)$	Stirling numbers of the first kind, 53
$c(n, k)$	signless " , 53
$S(n, k)$	Stirling numbers of the second kind, 53
$\ell_1 + \mathcal{O} \ell_2$	ordinal sum, 343

### PARTICULAR SPECIES

$\mathcal{A}_-$	rooted trees, 7
$\mathcal{A}_R$	$R$ -enriched rooted trees, 165
$\mathcal{A}_L$	ordered rooted trees, 167
$\mathbf{a}$	trees, 7
$\mathbf{a}_R$	$R$ -enriched trees, 178
$\text{Alt}$	alternating permutations, 345
$\mathcal{B}$	binary rooted trees, 8
$\mathcal{B}^+$	increasing binary trees, 345
$\mathcal{B}_c^+$	complete increasing binary trees, 345
$\text{Bal}$	ballots, 10

$\text{Bal}^{[k]}$	$k$ level ballots, 36	$\Lambda^{(a)}$	connected components weighting species, 98
$\text{Bij}$	bijections, 112	$L$	linear orders, 7
$\mathcal{C}$	cyclic permutations, 7	$\text{Lag}$	Laguerre configurations, 95
$\mathcal{C}_{\text{alt}}$	alternating cyclic permutations, 108	$n$	$n \in \mathbb{N}$ as a species, 31
$\text{Cha}$	chains, 11	$\text{Oct}$	octopuses, 12
$\mathcal{C}^{[m]}$	$m$ -complexes, 78	$\text{Oct}_{\text{alt}}$	alternating octopuses, 113
$\text{Cov}$	coverings, 78	$\text{Oct}_{\text{reg}}$	regular octopuses, 56
$\text{Cov}^{[m]}$	$m$ parts coverings, 78	$\text{Ord}$	order relations, 47
$\mathcal{D}$	directed graphs, 7	$\wp$	subsets, 7
$\text{Der}$	derangements, 10	$\wp^{[k]}$	$k$ -element subsets, 34
$E$	sets, 8	$\mathcal{P}$	commutative parenthesizations, 10
$E^{\pm}$	oriented sets, 144	$\mathfrak{p}$	plane trees, 178
$\text{End}$	endofunctions, 7	$\mathcal{P}$	plane rooted trees, 168
$\text{End}_R$	$R$ -enriched endofunctions, 172	$\text{Par}$	set partitions, 7
$\text{End}^{\wp}$	partial endofunctions, 172	$\text{Par}^{[k]}$	partitions with $k$ parts, 31
$\text{End}_R^{\wp}$	$R$ -enriched partial endofunctions, 172	$\text{Par}_P$	partial set partitions, 49
$\Phi$	functions, 112	$P$	polygons, 10
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$\mathcal{G}$	simple graphs, 4	$\text{Rel}^{[m]}$	$m$ -ary relations, 74
$K_n$	complete graphs, 297	$\text{Red}$	reduced posets, 57
$\mathcal{G}^c$	connected simple graphs, 27	$\mathcal{S}$	permutations, 7
$\mathcal{G}^d$	disconnected simple graphs, 27	$\mathcal{S}^{[k]}$	permutations with $k$ cycles, 31
$\Gamma(X, Y)$	graphs on vertices and edges, 111	$\mathcal{S}^{<k>}$	permutations with all cycles of size $k$ , 40
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$\mathfrak{h}$	homeomorphically irreducible trees, 283	$\mathcal{S}^{\text{mix}}$	mixed permutations, 114
$\mathcal{H}$	hedges of rooted trees, 10	$\text{Sur}$	surjections, 112
$\text{Inv}$	involutions, 7	$\mathcal{V}$	vertebrates, 61
$\text{Inj}$	injections, 112	$X$	singletons, 8
$\text{Jac}$	Jacobi endofunctions, 203	$1$	empty set species, 8
		$0$	empty species, 8
		$\varepsilon$	elements, 8

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