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R. B. Bapat and T. E. S. Raghavan
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This book presents an integrated treatment of the theory of nonnegative matrices, emphasizing connections with the themes of game theory, combinatorics, inequalities, optimization, and mathematical economics. Some related classes of positive matrices such as positive semidefinite matrices, M-matrices, P-matrices, and distance matrices are also discussed, but the main emphasis is on entrywise nonnegative matrices.

The book begins with the basics of the subject, such as the Perron-Frobenius Theorem. Only a minimal background in linear algebra is assumed, although familiarity with linear programming and statistics will be helpful in following some sections. Each of the later chapters is devoted to an area of applications, including doubly stochastic matrices (price fixing, scheduling, and the fair division problem), combinatorial matroids, and economics. These applications have been carefully chosen both for their elegant mathematical content and for their accessibility. The treatment is rigorous and almost all results are proved completely.

About half of the material in the book presents standard topics in a novel fashion, the remaining portion reports many new results in matrix theory for the first time in a book form.

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To the memory of my father,

Bhalachandra S. Bapat

- R. B. Bapat

To the memory of my father,

Eachambadi S. Narasimhachari

- T. E. S. Raghavan

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PREFACE

This book is aimed at first year graduate students as well as research workers with a background in linear algebra. The theory of nonnegative matrices is unfolded in the book using tools from optimization, inequalities and combinatorics. The topics and applications are carefully chosen to convey the excitement and variety that nonnegative matrices have to offer. Some of the applications also illustrate the depth and the mathematical elegance of the theory of nonnegative matrices. The treatment is rigorous and almost all the results are completely proved. While about half of the material in the book presents many topics in a novel fashion, the remaining portion reports many new results in matrix theory for the first time in a book form. Although the only prerequisite is a first course in linear algebra and advanced calculus, familiarity with linear programming and statistics will be helpful in appreciating some sections.

To give some examples, the Perron-Frobenius Theorem and many of its consequences are derived using the theory of matrix games where all rows and columns are essential for optimal play. The chapter on conditionally positive definite matrices and distance matrices has several new results appearing for the first time in a book. A transparent proof of the Alexandroff inequality for mixed discriminants is presented and a characterization of graphs giving rise to a finite Coxeter group is given in the chapter on combinatorial theory. The importance of P-matrices and M-matrices to several areas besides linear economic models is stressed and many of these results are seen via Game theory.

The application topics include, among other things, areas like game theory, Markov chains, probabilistic algorithms, numerical analysis, discrete distributions, categorical data, group theory, matrix scaling and economics. The chapter on doubly stochastic matrices contains applications to the pricing of houses in markets with known expectations, the problem of arriving at a fair division that respects peoples' individual preferences, scheduling jobs in a preemptive

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environment, and assigning people to jobs to minimize total job completion time. As described in the chapter on matrix scalings, while scaling is a powerful tool to speed up convergence, it is also a useful tool to estimate cell entries of an unknown matrix of the future parametric values from the known entries of the past. Such a procedure could be used to estimate the size of a population in a city or the growth of tumor after an operation and to reconstruct an image based on partial information.

The chapter on topics in economics describes the economic implications of properties of many special classes of matrices. The Perron-Frobenius Theorem can be used to explain both the Leontief and the Sraffa system. In the Leontief system, workers slave with fixed consumption and fixed wage to achieve targeted social output for the future generations. In the Sraffa system, all are entrepreneurs of identical skill who get equal rate of return. In international trade, the famous Hecksher-Ohlin Theorem shows that free trade could very well be a substitute for immigration. Another application for such matrices occurs in the price stability of an economy where any two goods are gross substitutes. In such economies, consumers switch from one brand to another if there is a steep price increase for a particular brand. An increase in price for a brand at a price equilibrium triggers a price increase for other substitute brands due to increased demand. However a dynamic price stability for the brand can be attained, allowing for varying speeds of price adjustments for other brands that are gross substitutes. It may be noted that these topics and results have been cited as major contributions to economic theory by P. Samuelson, Sir John Hicks and B. Ohlin (all Noble prize laureates) by the Noble prize committee.

This book was being written over a period of several years and it has benefitted from the comments, suggestions, and works of a large number of people. We only mention here some names that immediately come to mind. At the outset we wish to acknowledge the influence of the magnificent contributions of John von Neumann and Issai Schur on our approach to the subject at many places. We would also like to mention that the short note by Blackwell on minimax and irreducible matrices and Kaplansky's notion of completely mixed games have given the necessary game theoretic armory for many matrix problems.

We gratefully acknowledge comments and corrections, pertaining to various portions of the manuscript, due to Adi Ben-Israel, Charles Broyden, Gregory M. Constantine, S. Chandrasekaran, John Copas, M. V. Menon, S. R. Mohan, Dale Olesky, S. Panchapakesan, Ashok Ramu, Arunava Sen, Debapriya Sengupta, R. Sridhar, Pauline van den Driessche, and James Weber.

Evangelista Fe, Tamas Solymosi, Murali Srinivasan, Zamir Syed, and Julin Wu took a course based on the first three chapters from Raghavan and made helpful remarks. In particular, Fe made several corrections in spelling, grammer

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etc., Solymosi suggested improvements on the section on cooperative games and assignment problems and corrected some mathematical typos, while Murali carefully went through some topics on doubly stochastic matrices.

We owe a lot to S. Sankaran who made extensive corrections, particularly in the usage of syntax and grammar.

Dipankar Dasgupta convinced Raghavan of the Sraffa system as a much refined application of the Perron-Frobenius Theorem to Economics, whereas V. K. Chetty spent hours explaining to him the contributions and the depth of the works of Ricardo and Sraffa's formulation of the theory of production of commodities by means of commodities as a clear solution to the problem that Ricardo attempted to solve without success.

We warmly acknowledge the support given by our family members, Ragini, Sudeep, Usha, Deepa, Sampath, Santanu, Tara, and Manu, during the course of writing this book. The theory of Nonnegative matrices is indeed a fascinating and rewarding area. The Perron-Frobenius Theorem, the central result of the theory, was formulated at the beginning of the twentieth century and many significant developments have taken place during the past ninety years. In this book we have tried to touch upon some of these developments and the choice of topics clearly reflects our personal interests. We would consider our efforts amply rewarded even if a single curious and youthful mind is drawn to the subject and happens to share the same fascination after reading this book.

RBB
TESR

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