

Contents

	Preface	<i>page</i> xi
1	Preliminaries	1
1.1	Conventions	1
1.2	Differential forms	2
2	Spacetimes admitting Killing fields	6
2.1	Killing fields	7
2.2	Basic identities	9
2.3	The vacuum variational principle	12
2.4	Asymptotic flatness and stationarity	14
2.5	Static spacetimes	17
2.6	Foliations of static spacetimes	21
2.7	Stationary and axisymmetric spacetimes	24
3	Circular spacetimes	31
3.1	The metric	32
3.2	The orbit manifold	33
3.3	The orthogonal manifold	36
3.4	Weyl coordinates and the Papapetrou metric	39
4	The Kerr metric	42
4.1	The vacuum Ernst equations	43
4.2	Conjugate solutions	45
4.3	The Kerr solution	48
5	Electrovac spacetimes with Killing fields	56
5.1	The stress–energy tensor	57
5.2	Maxwell’s equations with symmetries	61
5.3	The electrovac variational principle	65
5.4	The electrovac circularity theorem	69
5.5	The circular Einstein–Maxwell equations	72
5.6	The Kerr–Newman solution	75

6	Stationary black holes	84
6.1	Basic definitions	86
6.2	The strong rigidity theorem	88
6.3	The weak rigidity theorem	92
6.4	Properties of Killing horizons	94
6.5	The topology of the horizon	97
7	The four laws of black hole physics	102
7.1	The zeroth law	103
7.2	The first law	109
7.3	The second law	117
7.4	The generalized entropy	119
8	Integrability and divergence identities	122
8.1	The circularity theorem	123
8.2	The staticity theorem	124
8.3	Divergence identities	128
8.4	Static identities and further applications	135
9	Uniqueness theorems for nonrotating holes	140
9.1	The Israel theorem	141
9.2	Uniqueness of the Schwarzschild metric	146
9.3	Uniqueness of the Reissner–Nordström metric	151
9.4	Uniqueness of the magnetically charged Reissner–Nordström metric	157
9.5	Multi black hole solutions	161
10	Uniqueness theorems for rotating holes	166
10.1	Outline of the reasoning	167
10.2	The Ernst system and the Kinnersley group	169
10.3	The uniqueness proof	172
10.4	The Robinson identity	175
10.5	Appendix: The sigma model Lagrangian	178
11	Scalar mappings	180
11.1	Mappings between manifolds	182
11.2	Harmonic mappings	185
11.3	Skyrme mappings	188
11.4	The SU(2) Skyrme model	193
11.5	Conformal scalar fields	199

Contents

ix

12	Self-gravitating harmonic mappings	205
12.1	Staticity and circularity	206
12.2	Nonexistence of soliton solutions	209
12.3	Uniqueness of spherically symmetric black holes	211
12.4	Divergence identities	215
12.5	The uniqueness theorem for nonrotating black holes	220
12.6	The uniqueness theorem for rotating black holes	222
	References	230
	Index	246