

Index

- action, *see* Lagrangian
- ADM mass 15
- angular momentum 25
 - horizon 110
 - Kerr metric 53
 - Kerr–Newman metric 80
- angular velocity
 - Killing horizon 91
 - uniformity 91
 - weak rigidity 94
- anti-derivative 3
- apparent horizon 88
- area theorem 119
- asymptotic expansion 16
 - electric potential 151
 - Kerr–Newman metric 83
 - stationary metric 11, 16
- asymptotic flatness 14
 - Kerr metric 54
 - static spacetime 146
- asymptotic predictability 87
- axial Killing field 90
- axisymmetry 25
 - electromagnetic field 62, 69
 - scalar mapping 207
- Bach tensor 20
 - electrovac spacetime 155
 - vacuum spacetime 149
- Bekenstein black hole solution 203
- Bianchi identity 11
- bifurcate Killing horizon 89, 97, 106
- bifurcation surface 89
- black hole 87
 - at time Σ 88
 - rotating, nonrotating 91
- Bochner identity 188
- Boyer–Lindquist coordinates 52
 - Kerr–Newman metric 52, 79
- Cartan, structure equations 19
- Cauchy horizon 55
- Cauchy surface 87
- causal past 87
- Christoffel symbols
 - circular spacetime 36
 - static spacetime 21
- circularity 26
 - Ricci-circularity 27
 - zeroth law 107
- circularity theorem 28
 - electrovac 70, 124
 - general 124
 - scalar mappings 208
- closed form 3
- Codazzi–Mainardi equations 22
- co-derivative 4
 - product 5
 - stationary 1-form 33
- conformal flatness 20, 149, 154
- conformal scalar field 200
 - field equations 200
 - stress-energy tensor 201
- conformal transformation 147
- conjugate electromagnetic potential 80
- conjugate Ernst 1-form 46
- conjugate solution 46, 51
- connection 1
 - along a mapping 182, 183, 184
 - coordinate representation 183
- connection-forms 19
- cosmic censor conjecture 87
- covariant derivative,
 - commutation rule 2
- curvature 2
 - along a mapping 184
 - singularity in Kerr spacetime 54
- curvature-forms 19

- degree, of Skyrme mapping 188
 differential, of mapping
 between manifolds 185
 differential form 2
 closed, exact 3
 harmonic 4
 dominant energy condition
 cosmic censor conjecture 87
 topology 98
 zeroth law 105

 Einstein equations 2
 Einstein–Hilbert principle 114
 Einstein–Maxwell equations
 circular spacetime 73
 static 152, 158
 stationary 136
 Einstein tensor 2
 static metric 21
 submanifold 22
 electric charge 80, 130
 electromagnetic action 57
 electromagnetic field 57, 58, 60
 algebraic relations 60
 invariance conditions 62
 Kerr–Newman spacetime 80
 Ricci–circularity 70
 stationarity and axisymmetry 69
 stationary 62
 electromagnetic potential
 asymptotic expansion 151
 complex circular 73
 and gravitational potential 137
 Kerr–Newman solution 80
 Reissner–Nordström solution 157
 uniformity on Killing horizon 129
 energy density
 harmonic 185
 Skyrme 188
 entropy 120
 ergoregion 15
 Kerr spacetime 55
 staticity 127
 Ernst equations
 electrovac 65, 74, 76
 vacuum 44, 49
 variational principle 13, 45, 50, 76
 Ernst potential, vacuum 12, 43
 conjugate 46
 electrovac 66, 73, 74

 Euler characteristic 98
 Euler–Lagrange equations
 electrovac Ernst potential 66, 73
 harmonic mappings 186
 sigma models 172
 event horizon 87
 future and past 90
 Kerr spacetime 55
 Killing horizon 89
 at time Σ 88
 exact differential form 3
 expansion, of null congruence 99, 118
 exterior algebra 2
 exterior derivative 3
 exterior product 5
 extrinsic curvature 21, 142, 145, 150

 first law, *see* mass, variation formula
 flux integrals 130

 gauge potential, symmetry
 condition 62
 Gauss–Bonnet theorem 98, 144
 Gauss curvature of horizon 98
 Gauss equation 22
 generalized entropy 120
 generating technique 202
 geodesic congruence 97
 geodesic equation 187
 global hyperbolicity 87
 gravitational collapse 87

 harmonic energy density 185
 harmonic form 4
 harmonic function 187
 harmonic mapping 185
 solitons 211
 symmetric space 172, 178
 uniqueness theorem 218, 222, 227
 variational equation 186
 Hawking radiation 104, 120
 Hawking temperature 214
 Higgs field 210, 217
 Hodge dual 3
 horizon
 bifurcate 89, 97
 event horizon 87
 Kerr–Newman 83
 Killing horizon 88, 90, 92
 Papapetrou–Majumdar 163

248

induced connection, along a
 mapping 182, 183
 integrability conditions
 circularity 26
 Killing 2-form 28
 staticity 18
 interior multiplication 3, 5
 invariance conditions 8
 electromagnetic field 62, 63
 gauge potential 62
 isometry 7

Kerr metric 52
 in Boyer–Lindquist
 coordinates 53
 Kerr–Newman metric 79, 175
 Killing equation 8
 Killing field 7
 axial 90
 horizon 90
 Killing horizon 88, 89, 90
 bifurcate 89, 97
 twist 92, 95
 Killing parameter 97
 Killing 2-form 28
 Kinnersley vector 68, 170
 Komar integral 130
 Kerr spacetime 54
 positivity 213
 Komar mass 16, 23, 54, 110, 130,
 210, 212

Lagrangian
 conformal scalar field 200
 electromagnetic 57
 electrovac Ernst equations 75, 169
 harmonic 179, 185
 sigma model 170, 179
 Skyrme 188, 190, 193, 198
 spherically symmetric gravity 198
 Ernst equations 13, 45, 50
 vacuum equations with
 Killing field 13
 Laplacian 4
 along mapping 185
 stationary and axisymmetric 33
 Lie derivative 7

mass, ADM 15
 electrovac spacetime 116, 134

Index

Kerr spacetime 53
 Kerr–Newman spacetime 80
 Komar 16, 23, 54, 110, 130, 210,
 212
 variation formula 109, 114, 116
 Maurer–Cartan form 193, 195
 Maxwell equations 63, 129, 136
 Mazur identity 168, 171, 172
 metric
 circular 33, 38, 43
 circular electrovac 73
 induced on null surface 98
 orbit manifold 33
 Papapetrou 39
 in prolate spheroidal
 coordinates 48
 multi black hole solutions 163

naked singularity 87
 null congruence 99, 118
 energy condition 118
 generator 91, 95
 infinity 87
 surface, *see* Killing horizon
 tetrad 118

Papapetrou metric 40
 Papapetrou–Majumdar solution 163
 Penrose conjecture 211, 214
 Poincaré lemma 3
 positive mass theorem 148
 potential space 68
 Poynting vector 58, 71, 127
 prolate spheroidal coordinates 48

Raychaudhuri equation 118
 Reissner–Nordström solution 157
 extreme 161
 uniqueness 157, 160
 Ricci–circularity 27
 circularity theorem 28, 124
 electrovac 70
 Ricci identity 8, 188
 Ricci 1-form 8, 10, 208
 Ricci scalar 2
 conformal transformation 147
 Ricci–staticity 18, 125
 Ricci tensor 2
 circular spacetime 27
 static spacetime 20, 146

- Riemann tensor 2
 - 3-dimensional 20
- rigidity theorem 89, 90, 94
- Robinson identity 176
- rotating black hole 91
- rotation-form, *see* twist

- scalar field, uniqueness theorem 213
- Schwarzschild metric 150
- second fundamental form 147
- second law 119
 - generalized version 120
- sigma model 178, 187
- singularity
 - Kerr metric 54
 - naked 87
 - Papapetrou–Majumdar metric 164
- Skyrme
 - energy bound 190, 194
 - energy density 188
 - Lagrangian, SU(2) 193
 - spherically symmetric action 190
- Smarr formula 133
- staticity 18, 107
- staticity theorem 125
 - electrovac 127, 138
 - scalar mappings 208
- stationarity
 - and staticity 125
 - strict 15
- stationary and axisymmetric spacetime 25
- stationary electromagnetic field 62, 69
- stationary mapping 207
- stationary spacetime 15
- Stokes' theorem
 - in presence of Killing field 8
 - staticity proof 126
- stress–energy tensor 2
 - circularity condition 124
 - conformal scalar field 201
 - electromagnetic 58
 - staticity condition 125
- strong energy condition 210
- strong rigidity theorem 89, 90
- structure equations, Cartan 19
- surface gravity 95, 96
 - geodesic congruence 97
 - Kerr solution 117
 - Reissner–Nordström metric 117
 - spherical symmetry 214
 - uniformity, *see* zeroth law
 - variation 112
- symmetric space 178
- symmetry conditions, *see* invariance conditions

- tangent space 1
- tangential field, along mapping 182
- third law 119
- topological censorship 100
- topological bound 190, 194
- topology of horizon 98
- torsion, along mapping 184
- trapped surface 88
- twist 9
 - circular spacetime 34
 - co-derivative 10
 - exterior derivative 10
 - Killing horizon 92, 95
 - staticity 18

- uniformity of surface gravity, *see* zeroth law
- uniqueness theorem
 - harmonic mappings 218, 222, 227
 - Kerr–Newman 175
 - Reissner–Nordström 157, 160
 - Schwarzschild 142, 150
 - sigma models 173
 - static Higgs fields 217

- variation formula for mass, *see* mass, variation formula
- variational principle, *see* Lagrangian
- vector field 1
 - along mapping 182
- volume-form 3
 - of SU(2) 196

- weak rigidity theorem 94

- zeroth law 104
 - bifurcate horizon 106
 - dominant energy condition 105
 - Ricci–staticity 106
 - static, circular spacetime 107