This timely review provides a self-contained introduction to the mathematical theory of stationary black holes and a self consistent exposition of the corresponding uniqueness theorems.

The opening chapters examine the general properties of space-times admitting Killing fields and contain a detailed derivation of the Kerr Newman metric. Strong emphasis is given to the geometrical concepts. The general features of stationary black holes and the laws of black hole mechanics are then reviewed. Subsequently, critical steps towards the proof of the 'no hair' theorem are discussed, including the methods used by Israel, the divergence formulae derived by Carter, Robinson and others, and finally the sigma model identities and the positive mass theorem. The book is rounded off with an extension of the electrovacuum uniqueness theorem to self-gravitating scalar fields and harmonic mappings.

This volume provides a rigorous textbook for graduate students in physics and mathematics. It also offers an invaluable, up-to-date reference for researchers in mathematical physics, general relativity and astrophysics.

CAMBRIDGE LECTURE NOTES IN PHYSICS 6

General Editors: P. Goddard, J. Yeomans

Black Hole Uniqueness Theorems

CAMBRIDGE LECTURE NOTES IN PHYSICS

1. Clarke: The Analysis of Space-Time Singularities
2. Dorey: Exact S-Matrices in Two Dimensional Quantum Field Theory
3. Sciama: Modern Cosmology and the Dark Matter Problem
4. Veltman: Diagrammatica - The Path to Feynman Rules
5. Cardy: Scaling and Renormalization in Statistical Physics
6. Heusler: Black Hole Uniqueness Theorems

Black Hole Uniqueness Theorems

MARKUS HEUSLER
University of Zurich, Switzerland

CAMBRIDGE UNIVERSITY PRESS

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011–4211, USA
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

First published 1996

A catalogue record for this book is available from the British Library

Library of Congress cataloguing in publication data

Heusler, Markus.
Black hole uniqueness theorems/Markus Heusler.
p. cm. – (Cambridge lecture notes in physics ; 6)
Includes bibliographical references and index.
ISBN 0 521 56735 1 (pbk.)
1. Black holes (Astronomy) – Mathematics. I. Title. II. Series.
QB843.B55H48 1996
523.8'875'0151 dc20 96–11787 CIP

ISBN 0 521 56735 1 paperback

Transferred to digital printing 2003

Contents

Preface

In a manuscript communicated to the Royal Society by Henry Cavendish in 1783, an English scientist, Reverend John Michell, presented the idea of celestial bodies whose gravitational attraction was strong enough to prevent even light from escaping their surfaces. Both Michell and Laplace, who came up with the same concept in 1796, based their arguments on Newton's universal law of gravity and his corpuscular theory of light.

During the nineteenth century, a time when the notion of "dark stars" had fallen into oblivion, geometry experienced its fundamental revolution: Gauss and Lobachevsky had already found examples of non–Euclidean geometry, when Riemann became aware of the full consequences which arise from releasing the parallel axiom. In a famous lecture given at Göttingen University in 1854, the former student of Gauss introduced both the notion of spatial curvature and the extension of geometry to more than three dimensions.

It is these features of Riemannian geometry which, more than fifty years later, enabled Einstein to reveal the connection between the gravitational field and the metric structure of spacetime. In February 1916 - only three months after having achieved the final breakthrough in general relativity - Einstein presented, on behalf of Schwarzschild, the first exact solution of the new equations to the Prussian Academy of Sciences.

It took, however, almost half a century until the geometry of the Schwarzschild spacetime was correctly interpreted and its physical significance was fully appreciated. The neutron had yet to be discovered and the theory of stellar evolution to be developed such that neutron stars could be understood; only then would it become clear that there existed no physical laws to prevent certain stars from undergoing total gravitational collapse. This ultimate fate of sufficiently massive stars had already been predicted in the early 1930s by Chandrasekhar, who also elaborated the critical limit for the masses of white dwarf stars. At the present time there is hardly any doubt concerning the existence of black holes in the Universe. In fact, the picture given by Oppenheimer and Snyder in 1939 has turned out to be in full agreement with current knowledge:

> When all the thermonuclear sources of energy are exhausted a sufficiently heavy star will collapse. Unless fission due to rotation, the radiation of mass, or the blowing off of mass by radiation, reduce the star's mass to the order of that of the sun, this contraction will continue indefinitely... *Oppenheimer and Snyder (1939)*

The mathematical theory of black holes has been steadily developing during the last thirty years. One of its most intriguing outcomes is the so–called "no–hair" theorem, which states that a black hole in a stationary electrovacuum spacetime is uniquely characterized by its mass, angular momentum and electric charge. This result bears a striking resemblance to the fact that a statistical system in thermal equilibrium is also described by a small set of state variables, whereas considerably more information is required to understand its dynamical behavior. This similarity is reinforced by the black hole mass variation formula and the area increase theorem, which are analogous to the corresponding laws of ordinary thermodynamics. These mathematical relationships are given physical significance from the observation that the temperature of the black–body spectrum of the Hawking radiation is equal to the surface gravity of the black hole.

The purpose of this text is to provide an introduction to stationary black holes and to present a self–consistent exposition of the corresponding uniqueness theorems. Although the emphasis is given to the new approach to these theorems, based on the positive energy theorem and sigma model identities, I have tried to take the traditional line of reasoning into account as well. In view of the recent developments in the field - notably the new black hole solutions which reflect the limited realm of the classical uniqueness theorems - some stress is laid upon the distinction between purely geometric results and conclusions which involve the matter fields. The book starts out with some general properties of spacetimes with Killing fields and with a systematic derivation of the Kerr–Newman metric. The body of the work is devoted to the properties of stationary black holes and their uniqueness theorems. The last chapters deal with self–gravitating mappings and include, in part, some recent results. I have also tried to provide a link with research in the field, by referring to problems which are currently under investigation.

This text is intended to be intelligible to the reader familiar with the basic notions of general relativity. Differential forms are used throughout, mainly for the sake of the improved efficiency of numerous derivations. It is therefore desirable that the reader is comfortable with both the calculus of tensor fields and differential forms. Most derivations are worked out in detail; however, I have given priority to a clear presentation of the geometrical concepts rather than to mathematical rigor.

I would like to acknowledge discussions with many colleagues. In particular, I wish to thank Robert Wald, Jürgen Ehlers and the Relativity Groups at the Enrico Fermi Institute in Chicago, the Max–Planck–Institute in Munich and the University of Zurich. I am very grateful to Piotr Chruściel for pointing out weak parts in the draft and providing me with the appropriate amendments. I owe especial thanks to Vivek Iyer for having read critically the manuscript and helping me to improve the style of this book with numerous valuable suggestions. Finally, I am particularly indebted to Norbert Straumann for many instructive discussions, his continuous support during the years and for drawing my attention to the uniqueness problem.

I gratefully acknowledge financial support from the Swiss National Science Foundation, the Max–Planck–Gesellschaft and the Tomalla Stiftung. It is a pleasure to thank Adam Black of the Cambridge University Press for his courtesy and cooperation.

I dedicate this book to my wife Regina, whose patience and understanding enabled me to write it.