

Index of Notation

- $\lambda, (\lambda_1, \dots, \lambda_m), (d_1^{a_1} \dots d_s^{a_s}), \lambda \vdash |\lambda|$
partition, Young diagram, 1
- $\tilde{\lambda}$ conjugate partition, diagram, 2
- T^t transpose, 2, 189
- $s_\lambda(x_1, \dots, x_m)$ Schur polynomial, 3, 24, 26, 51, 75, 178
- x^T monomial of tableau, 3
- $h_n(x_1, \dots, x_m)$ complete symmetric polynomial, 3, 72
- $e_n(x_1, \dots, x_m)$ elementary symmetric polynomial, 4, 72
- $\mu \subset \lambda$ containment order, 4, 26
- λ/μ skew diagram, 4, 12–3
- $[m] = \{1, \dots, m\}$, 4
- $T \leftarrow x$ row-insertion, 7
- $T \cdot U$ product of tableaux, 11, 15, 23
- $\text{Rect}(S)$ rectification, 15, 58, 207
- $T * U$ skew tableau from T, U , 15
- $w(T) = w_{\text{row}}(T)$ row word, 17
- $u \leq v$ word order, 18
- $(K'), (K'')$ elementary Knuth transformations, 19
- $w \equiv w'$ Knuth equivalence, 19
- $P(w)$ tableau with word $\equiv w$, 22, 36
- $R_{[m]}$ tableau ring, 23–4, 63
- $S_\lambda = S_\lambda[m]$ in $R_{[m]}$, 24, 63
- $K_{\lambda\mu}$ Kostka number, 25, 53, 71, 75, 78, 92, 121, 204
- $\mu \leq \lambda$ lexicographic order, 26
- $\mu \trianglelefteq \lambda$ dominance order, 26
- $w_{\text{col}}(T)$ column word of T , 27
- $\equiv', \equiv'', K', K''$ -equivalence, 27–9
- $L(w, 1), L(w, k)$ lengths of sequences from word, 30
- $Q(w)$ recording tableau, 36, 57, 191, 193–5
- $\begin{pmatrix} u_1 u_2 \dots u_r \\ v_1 v_2 \dots v_r \end{pmatrix}$ two-rowed array, 38
- $\binom{u}{v} \leq \binom{u'}{v'}$ lexicographic order, 39
- $(P, Q) = (P(\omega), Q(\omega)) = (P(A), Q(A))$ tableau pair in R–S–K correspondence, 39, 41, 44
- West, west, northWest, etc., 42
- $A^{(1)}, A^b, A^{(2)}$ matrix-ball construction, 43–4, 199–200, 204–5
- f^λ number of standard tableaux on λ , 52–4, 56–7
- $d_\lambda(m)$ number of tableaux on λ , entries in $[m]$, 52, 55, 76
- $\lambda * \mu$ skew diagram from λ, μ , 60
- $\mathcal{S}(v/\lambda, U_\circ), \mathcal{T}(\lambda, \mu, V_\circ)$, 60–1, 189
- $(T_\circ)_S$ tableau from T_\circ, S , 61
- $c_{\lambda\mu}^v$ Littlewood–Richardson number, 62–71, 78, 92, 121–2, 146, 185
- $S_{v/\lambda} = S_{v/\lambda}[m]$, 63
- $U(\mu)$ tableau on μ whose i^{th} row consists of i , 65, 113, 141, 186, 194
- $s_{v/\lambda}(x_1, \dots, x_m)$ skew Schur polynomial, 67, 77
- $U(w)$ standard tableau of w , 68, 194
- $h_\lambda(x), e_\lambda(x), m_\lambda(x)$ symmetric polynomials, 72–3
- $p_r(x), p_\lambda(x)$ power sums, 73–4
- $z(\lambda)$, 74, 86
- $\Lambda = \bigoplus \Lambda_n$ ring of symmetric functions, 77, 103
- $s_\lambda, h_\lambda, e_\lambda, m_\lambda, p_\lambda$, 77
- \langle, \rangle inner product on Λ_n , 78

- χ_μ^λ , 78, 93
- ξ_μ^λ 78, 89
- ω involution on Λ , 78, 93
- S_n symmetric group, 79, 83
- $GL(E)$, $GL_m\mathbb{C}$ automorphism group of E , of \mathbb{C}^m , 79, 104
- S^λ Specht module, 79, 87–94, 99, 101–3
- E^λ Schur or Weyl module, 79, 104–26, 144
- \mathbb{I}_n trivial representation of S_n , 79
- $\text{Sym}^n E$ symmetric power, 79, 106
- $E^{\otimes n}$ tensor power, 79, 116
- $v_1 \cdot \dots \cdot v_n$ product in $\text{Sym}^n E$, 79
- \mathbb{U}_n alternating representation of S_n , 80, 94
- $\wedge^n E$ exterior power, 80, 106
- $v_1 \wedge \dots \wedge v_n$ product in $\wedge^n E$, 80
- $R(T)$ row group of T , 84
- $C(T)$ column group of T , 84
- $T' > T$ ordering on numberings, 84–5
- $\{T\}$ (row) tabloid of T , 85
- $A = \mathbb{C}[S_n]$ group ring of S_n , 86
- a_T, b_T, c_T Young symmetrizers, 86
- $C(\lambda)$ conjugacy class of λ , 86
- M^λ representation based on row tabloids, 86–94, 96, 101
- v_T elements in M^λ and S^λ , 86–8, 102
- Ind induced representation, 90
- R_n Grothendieck group of S_n , 90
- $R = \bigoplus R_n$, 90
- $[V] \circ [W]$ product in R , 90
- (\cdot, \cdot) inner product on R_n , 90
- χ_V character of V , 91, 120
- ω involution of R , 91, 93–4
- $\varphi: \Lambda \rightarrow R$, 91
- $\psi: R \rightarrow \Lambda \otimes \mathbb{Q}$, 91–2
- $[T]$ column tabloid of T , 95
- \tilde{M}^λ representation based on column tabloids, 95–9
- $\tilde{S}^\lambda, \tilde{v}_T$, 95, 101
- $\alpha: \tilde{M}^\lambda \rightarrow S^\lambda, \beta: M^\lambda \rightarrow \tilde{S}^\lambda$, 96
- $\pi_{j,k}(T)$, 97–8
- Q^λ relations in \tilde{M}^λ , 98, 122
- $T' > T$ ordering on numberings, 98
- E^{x^λ} cartesian product indexed by boxes of λ , 105
- v element in E^{x^λ} , 105
- v^λ element in E^λ , 106
- $E^{\otimes \lambda}$ tensor product indexed by boxes of λ , 106
- \wedge^v element of $\bigotimes \wedge^{\mu_i}(E)$, 107
- $Q^\lambda(E)$ relations in $\bigotimes \wedge^{\mu_i}(E)$, 107, 122
- e_T basic element of E^λ , 107
- $R[Z] = R[Z_{1,1}, \dots, Z_{n,m}]$, 109
- D_{i_1, \dots, i_p}, D_T determinants, 109
- $D^\lambda \subset R[Z]$ Deruyts' space, 111
- $M_m R$ matrix algebra, 111
- $D = \wedge^m E$ determinant representation, 112
- $H \subset G = GL_m\mathbb{C}$ diagonal subgroup, 112
- V_α weight space, 113, 115
- $B \subset G$ Borel group of upper triangular matrices, 113, 155–6, 159
- $SL(E) = SL_m\mathbb{C}$ 114
- $\mathfrak{g} = \mathfrak{gl}_m\mathbb{C} = M_m\mathbb{C}$ Lie algebra, 114
- $E(M) = E^{\otimes n} \otimes_{\mathbb{C}[S_n]} M$ 116–9, 123–4
- $\underline{v}(U)$ element in $E^{\otimes n}$ 118
- $\tilde{Q}^\lambda(E)$ relations in $\bigotimes \text{Sym}^{\lambda_i}(E)$, 119
- $\text{Char}(V), \chi_V$ character, 120
- $\mathcal{R}(m)$ representation ring of $GL_m\mathbb{C}$, 122
- $\Lambda(m)$ ring of symmetric polynomials in x_1, \dots, x_m , 123
- $\text{Sym}^* V$ symmetric algebra, 124, 127
- $S^*(E; d_1, \dots, d_s) = S^*(m; d_1, \dots, d_s)$ 125–6, 136, 139
- $\mathbb{P}(E)$ projective space of E , 127
- $\mathbb{P}^*(E) = \mathbb{P}(E^*)$ projective space of hyperplanes in E , 127
- $[x_1 : \dots : x_m]$ point of projective space \mathbb{P}^{m-1} , 128
- $F\ell^{d_1, \dots, d_s}(E)$ partial flag variety, 128, 135–7
- $Gr^n E = Gr_{r-n} E$ Grassmannian of subspaces of codimension n , 128
- $I(X)$ ideal of X , 129
- $E = E_X$ trivial bundle, 130, 143, 161
- x_{i_1, \dots, i_d} Plücker coordinate, 132
- $\langle v_1, \dots, v_r \rangle$ subspace spanned by v_1, \dots, v_r , 133, 135, 156
- P parabolic subgroup, 140, 142
- $\mathcal{O}_V(1), \mathcal{O}_V(n), \mathcal{O}_X(1), \mathcal{O}_X(a_1, \dots, a_s)$, 142

- $U_1 \subset \dots \subset U_m$ tautological filtration of vector bundles, 143, 161
- L^λ line bundle from λ , 143–5
- $L(\chi)$ line bundle from character, 143
- χ_λ character on P , 144
- $\Omega_\lambda = \Omega_\lambda(F_*)$ Schubert variety in Grassmannian, 146, 152, 179
- σ_λ class of Ω_λ , 146, 148
- σ_k class of $\Omega_k = \Omega_{(k)}$, 146
- $[Z]$ class of subvariety Z , 146, 212–3, 219–22
- $([Z_1], [Z_2])$ intersection number, 147, 214
- Ω_λ° Schubert cell in Grassmannian, 147
- F_k subspace spanned by first k vectors of a basis, 147
- \tilde{F}_k subspace spanned by last k vectors of a basis, 148
- $\tilde{\Omega}_\lambda = \Omega_\lambda(\tilde{F}_*)$, $\tilde{\Omega}_\lambda^\circ$ 148
- A_i, B_i, C_i subspaces, 148
- $F\ell(E) = F\ell(m)$ complete flag manifold, 154
- Z^T fixed points of T in Z , 154
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- X_w° Schubert cell in $F\ell(m)$, 157
- $\ell(w)$ length of w , 157, 158
- U_w neighborhood of $x(w)$, 157
- $n = m(m - 1)/2$ dimension of $F\ell(m)$, 157
- $D(w)$ diagram of w , 158
- Ω_w° dual Schubert cell, 158
- B' Borel group of lower triangular matrices, 159
- X_w, Ω_w Schubert varieties in $F\ell(m)$, 159, 176, 179
- $w_\circ = m m - 1 \dots 2 1$ in S_m , 160, 171
- $\sigma_w = [\Omega_w]$ Schubert class, 161
- $L_i = U_i/U_{i-1}$ line bundle, 161
- $c_1(L)$ first Chern class, 161, 214, 222–5
- $x_i = -c_1(L_i)$, 161, 182
- s_i transposition $(i, i + 1)$, 161, 165
- $R(m) \cong H^*(F\ell(m))$, 162
- $\mathbb{P}(V) \rightarrow Y$ projective bundle, 162
- $\iota : F\ell(m) \hookrightarrow F\ell(m + 1)$, 163
- $E' = E \oplus \mathbb{C}$, 163, 164
- ∂_i difference operator, 165
- \mathfrak{S}_w Schubert polynomial, 170–3
- $S_\infty = \cup S_m$, 172
- ∂_w difference operator, 173
- $r_w(p, q) = \#\{i \leq p : w(i) \leq q\}$, 173
- $u \leq v$ Bruhat order, 174
- $K_-(T)$ left key, 177, 210
- \mathcal{A}^*, w^* opposite alphabet, word, 183
- T^* dual tableau, 184
- $\Delta T, \Delta^2 T, \dots$ evacuation, 184
- $x \rightarrow T$ column-insertion, 186
- w^{rev} reversed word, 189, 207–8
- $w^\#, S^\#$ 193
- w^\natural, S^\natural , 194
- $I(i)$ index, 194
- $\sigma_{v/\lambda}$ permutation of skew diagram, 196
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- $w_-(T), w_+(T)$, 210
- $H_i X, H_* X = \bigoplus H_i X$ singular homology, 212
- $H^i X, H^* X = \bigoplus H^i X$ singular cohomology, 212
- $\alpha \cup \beta, \alpha \cap \beta$ cup, cap products, 212
- f^*, f_* pullback, pushforward, 212, 218
- $[V] \cdot [W]$ intersection class, 214
- $c_i(E)$ Chern class, 214, 222–5
- $H^i(X, Y)$ relative singular cohomology, 215
- T_M tangent bundle to M , 215
- γ_E Thom class, 215
- $\overline{H}_i X$ Borel–Moore homology, 216–9, 225
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