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Banach spaces for analysts

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Banach Spaces For Analysts

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Preface

Banach space theory became a recognised part of the mathematical scene with the appearance of Banach [1932]. From its birth it maintained close ties with the rest of analysis. It turned out that Banach space theory offered powerful tools to other branches of analysis. The most useful ones are duality theory for spaces and operators, results about infinite dimensional convexity and results connected with the Baire category theorem, most notably the closed graph theorem. These powerful general concepts are now well known among analysts and appear in almost every textbook on functional analysis or real variable theory. They were already well understood in the forties and fifties, and at that time it seemed to many that Banach space theory was dead or at least relegated to an obscure corner of science. However, this was not the case. The sixties, and especially the seventies and eighties, saw an enormous eruption of activity in the field. Old problems were solved and, more importantly, new problems and new ties with the rest of mathematics emerged. Also, new and powerful methods and directions of research appeared.

For those in the field progress seemed to be constantly accelerating, but to those outside it may have looked as if the theory was dying again. Probably one of the reasons for this perception was that this research activity (maybe because it was so dynamic) unfortunately did not produce many books on the subject, and those that did appear were usually devoted to a special topic. There are two notable exceptions to this statement, Beauzamy [1982] and Lindenstrauss-Tzafriri [1977, 1979]. There are old favourites, still beautiful and in good shape like Banach [1932], Dunford-Schwartz [1958] and Day [1958 and newer versions], but naturally they do not present the more recent results. So the newcomer to the field, after learning what was generally known thirty years ago, had a choice of either starting on more specialized books or turning to Beauzamy or Lindenstrauss-Tzafriri. Now I like both these books and the present one is not intended to replace either of them. Beauzamy is a nice, easily readable introduction and Lindenstrauss-Tzafriri, although difficult in places for the novice, has every mark of a classic, especially if the long-promised volumes III and IV are added to the first two. But both these books have one feature in common: they study Banach spaces for their own sake and their own beauty. In this way they are great for the future specialist who is already under the spell of Banach spaces. However, there are some mathematicians (even a majority) who are not convinced that Banach space theory is the most enchanting branch of mathematics, and although I am not one of them, I understand them.

In fact, Banach spaces are not only beautiful and interesting, but also useful. This point is not, in my opinion, made clear in Lindenstrauss-Tzafriri or Beauzamy. The methods created in Banach space theory since the late 60's can be applied in other areas of analysis. Hence somebody interested primarily in harmonic analysis, functions of a complex variable, orthonormal series, approximation theory or probability theory can find it useful to know some Banach space theory. This book is directed towards such a person. Ideally I think of it as a textbook for a graduate course for students who have already learned some functional analysis and are interested in analysis or in some area of it. I also hope that a mature analyst may read it, or some of it, as part of the ongoing education process which is an important part of the life of any active mathematician. I hope too that the Banach space specialist will find some portions of the book interesting because they present some applications of the subject he was not aware of.

Let me digress a bit and comment on the possible profit a classical analyst might derive from Banach space theory. I do not claim that Banach space theory can solve all his important problems. But it may help him to see the problem in a new light which makes it easier to isolate essential features. He can also use general theorems and techniques in the solution. The general framework can also suggest interesting new problems. To mention one example, the power of duality methods makes it a standard procedure to try to find the dual of any Banach space considered, a problem that could not even arise without this more general framework. Another example is the corona theorem. It is now a major part of the theory of analytic functions, but its origin lies in the attempt to get some understanding of the maximal ideal space H_∞, a question which is unthinkable without the general theory of Banach algebras.

Mathematics and each of its parts lives and grows on the exchange of ideas, between mathematicians, between various branches of mathematics and between mathematics and other areas of human activity. Banach space theory is no exception to this rule. It utilises ideas and techniques from other fields and in turn provides other branches of mathematics with some of its own. In this respect this book is one-sided. It concentrates heavily on ideas and results that have a clear potential to be useful in other branches of analysis.

Formally the book is divided into three parts, numbered by roman numerals. Each of these parts is divided into chapters, distinguished by capital letters. Each chapter is divided into sections with arabic numbers. Each such section contains at most one Theorem, Proposition or Lemma. The Theorem appearing in section II.B.7 is later referred to as Theorem II.B.7, or within the same chapter just as Theorem 7 (or Lemma or Proposition as the case may be). Part I is of introductory character. It contains well known and some less well known results (without proofs) that will be used later. Chapter I.A contains basic results from general functional analysis and Chapter I.B discusses the examples of Banach spaces that are considered later and quotes some analytical results about these spaces. The main function of Part I is to establish notation and conventions and to serve as a refresher and reference for the background material needed later on. Part II is essentially an introduction to the language and basic techniques of Banach space theory. The real heart of the matter is Part III, where a selection of topics is studied in depth. The reader can get an idea of the contents of each chapter from the Table of Contents, so I will not attempt a more detailed description here. Also, each chapter of Parts II and III starts with a short description of its contents, so the reader can find additional details there.

Each chapter concludes with Notes and Remarks containing bibliographical data and comments about generalizations, extensions or applications of the results presented in the chapter. I have tried really hard to find the correct reference and credits. On the other hand, I have not conducted a full scale historical investigation into the origin and development of each idea and result. *I would like to offer sincere apologies for any omissions and inaccuracies in this respect.* In the main text theorems are only given names if it is common usage. The absence of a name in the text or of a credit in the Notes and Remarks does not mean that the result is due to the author. It means either that the result is so well known that I judged it to be folklore or simply an omission on my part.

Each chapter of Parts I and II contains exercises. These exercises range from routine to very hard and I have not given any indication of their difficulty. There is a hint for each exercise located in the special chapter at the end of the book. These hints range from almost complete solutions to the reference only. I have tried to point out, if possible, where the solution of an exercise can be found in the literature and to give proper credit. There are also some repetitions in the exercises. I have simply put the same or similar problems into different chapters if they fit well into the material. This should be useful for those readers (the majority?) who read only selected parts of the book. It also indicates different approaches to the same question.

The bibliography contains only the works actually referred to somewhere in the book. I have made no attempt to make it complete. Also, it does not include any data about translations (this is particularly important with respect to publications in Russian) or reprints and republishing (it happens sometimes that an East European book is published originally in English and later republished without any changes by a West European or American company). If the work appeared in Russian, this is indicated in the references and the author's English translation of the title is given. This translation should be close to the one used in Mathematical Reviews, but need not be identical.

As with most mathematics books, it will be unusual for a reader to read this book from beginning to end. It is also unnecessary. The reader interested in a particular theorem or chapter should start right there.

The choice of material in the book reflects my philosophy, taste and, last but not least, knowledge and ability. Here I would like to mention some subjects which really should have been included but which were not because of limitations of space and time and (probably most important) my poor understanding of them. The first is the deep connections between Banach spaces, descriptive set theory and the classical theory of sets of uniqueness for trigonometric series (see Kechris-Louveau [1987] and Lyons [1989]). The second is the applications of the study of finite dimensional spaces to problems on convex bodies. This in turn has applications to harmonic analysis, number theory and other subjects. This whole area is currently one of frantic activity and is undergoing constant and fascinating changes. Probably anything I could write about it now would be outdated by the time this book reaches the reader. As an introduction to this area I suggest Pisier [P], Milman-Schechtman [1986] or Milman [1986]. The next subject which I regrettably had to omit is the connection between Banach space theory and probability theory. Actually probabilistic methods underly much of the current research in Banach spaces. This shows even in this book despite my unfortunate lack of knowledge of probability theory. The study of probability in Banach spaces is developing too: see Linde [1983]. The last subject I would like to have included is the general area of vector valued functions and operators acting on such functions. There is considerable activity in this area as well. As an introduction to it I suggest Burkholder [1986] and Pisier [P1].

While writing the book I received helpful advice from many mathematicians. I am grateful to all of them for their time, advice and help. First of all I would like to express my profound gratitude to my teacher and colleague, Prof. Aleksander Pełczyński. He convinced me that I should write the book in the first place and offered plenty of advice on what it should contain. Much of the time I did not follow his advice, but the effort needed to refute his arguments helped greatly to clarify my own ideas. I would like to thank Prof. Keith Ball and Prof. Ben Garling for reading large parts of the manuscript and providing numerous and invaluable pieces of advice on language and presentation. The following other mathematicians helped me greatly by generously offering their advice, knowledge and insight: Dan Amir, Sheldon Axler, Don Burkholder, Joe Cima, John Fournier, Ben Garling, Nassif Ghossoub, Yehoram Gordon, Paweł Hitczenko, Bill Johnson, Serguey Kislyakov, Boris Kashin, Stanisław Kwapień, Elton Lacey, Joram Lindenstrauss, Niels Nielsen, Gilles Pisier, Richard Rochberg and Walter Schachermayer.

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Finally I would like to express my deep gratitude to my wife Anna and my children Ola and Kuba for all the love, support and distractions they generously provided over the years. Without their presence (and at suitable times their absence!) writing this book would have been much more difficult.