

## Theoretical Astrophysics

### Volume II: Stars and Stellar Systems

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After reviewing the key observational results and nomenclature used in stellar astronomy, the book develops a solid understanding of central concepts including stellar structure and evolution, the physics of stellar remnants (such as white dwarfs, neutron stars and black holes), pulsars, binary stars, the Sun and the planetary system, the interstellar medium, and globular clusters. Throughout, the reader's grasp of all of the topics is developed and tested with more than seventy-five exercises.

This indispensable volume provides graduate students with a self-contained introduction to stellar physics and will allow them to master the material sufficiently to read and engage in research with heightened understanding.

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T. Padmanabhan

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# THEORETICAL ASTROPHYSICS

## Volume II: Stars and Stellar Systems

T. PADMANABHAN

Inter-University Centre for Astronomy and Astrophysics  
Pune, India



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Dedicated to the memory of my father, Shri. N. Thanu Iyer,  
who insisted on Excellence

## **THEORETICAL ASTROPHYSICS**

*– in three volumes –*

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### **VOLUME II: STARS AND STELLAR SYSTEMS**

1: Overview: Stars and Stellar Systems; 2: Stellar Structure; 3: Stellar Evolution; 4: Supernova (Type II); 5: White Dwarfs, Neutron Stars, and Black Holes; 6: Pulsars; 7: Binary Stars and Accretion; 8: The Sun and the Solar System; 9: The Interstellar Medium; 10: Globular Clusters.

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## Preface

“Yadhyadh vibuthimatsthvam srimadhurjithameva va  
thatthhadevava gachchatwam mama tejoamsa sambhavam”  
(“. . . Whatever that is glorious, prosperous or powerful anywhere,  
know that to be a manifestation of a part of My splendour. . .”)

— Bhagawad-Gita, Chapter 10, verse 41.

During the past decade or so, theoretical astrophysics has emerged as one of the most active research areas in physics. This advance has also reflected the greater interdisciplinary nature of the research that has been carried out in this area in recent years. As a result, those who are learning theoretical astrophysics with the aim of making a research career in this subject need to assimilate a considerable amount of concepts and techniques, in different areas of astrophysics, in a short period of time. Every area of theoretical astrophysics, of course, has excellent textbooks that allow the reader to master that *particular* area in a well-defined way. Most of these textbooks, however, are written in a traditional style that focusses on one area of astrophysics (say stellar evolution, galactic dynamics, radiative processes, cosmology, etc.) Because different authors have different perspectives regarding their subject matter, it is not very easy for a student to understand the key unifying principles behind several different astrophysical phenomena by studying a plethora of separate textbooks, as they do not link up together as a series of core books in theoretical astrophysics covering everything that a student would need. A few books, which *do* cover the whole of astrophysics, deal with the subject at a rather elementary (first-course) level.

What we require is clearly something analogous to the famous Landau-Lifshitz course in theoretical physics, but focussed to the subject of theoretical astrophysics at a fairly advanced level. In such a course, one could present all the key physical concepts (e.g., radiative processes, fluid mechanics, plasma physics, etc.)

from a unified perspective and then apply them to different astrophysical situations.

This book is the second of a set of three volumes intended to do exactly that. The three volumes form one single coherent unit of study, using which a student can acquire mastery over all the traditional astrophysical topics. What is more, these volumes will emphasise the unity of concepts and techniques in different branches of astrophysics. The interrelationship among different areas and common features in the analysis of different theoretical problems will be stressed throughout. Because many of the basic techniques need to be developed only once, it is possible to achieve significant economy of presentation and crispness of style in these volumes.

Needless to say, there are some basic “boundary conditions” one has to respect in such an attempt to cover the whole of theoretical astrophysics in approximately  $3 \times 560$  pages. Not much space is available to describe the nuances in greater length or to fill in details of algebra. For example, I have made conscious choices as to which parts of the algebra can be left to the reader and which parts need to be worked out explicitly in the text, and I have omitted detailed discussions of elementary concepts and derivations. However, I do *not* expect the reader to know anything about astrophysics. All astrophysical concepts are developed *ab initio* in these volumes. The approach used in these three volumes is similar to that used by Genghis Khan, namely (i) cover as much area as possible, (ii) capture the important points, and (iii) be utterly ruthless!

To cut out as much repetition as possible, the bulk of the physical principles are presented at one go in Vol. I and are applied in the other two volumes to different situations. These three volumes also concentrate on *theoretical* aspects. Observation and phenomenology are, of course, discussed in Vols. II and III to the extent necessary to make the motivation clear. However, I do not have the space to discuss how these observations are made, the errors, reliability, etc., of the observations or the astronomical techniques. (Maybe there should be a fourth volume describing observational astrophysics!)

The target audience for this three-volume work will be fairly large and comprises (1) students in the first year of their Ph.D. program in theoretical physics, astronomy, astrophysics, and cosmology; (2) research workers in various fields of theoretical astrophysics, cosmology etc.; and (3) teachers of graduate courses in theoretical astrophysics, cosmology, and related subjects. In fact, anyone working or interested in some area of astronomy or astrophysics will find something useful in these volumes. They are also designed in such a way that parts of the material can be used in modular form to suit the requirements of different people and different courses.

Let me briefly highlight the features that are specific to Vol. II. The reader is assumed to be familiar with the material covered in Vol. I, having either studied that volume (which is the recommended procedure!) or through independent courses in basic physics. The spirit of the three coherent volumes is to avoid

repetition as much as possible, and hence I have merely referred to the relevant parts of Vol. I whenever some input is required. Given the familiarity with basic physical processes, it was fairly easy to order the topics of Vol. II in a logical sequence. The fundamentals of stellar structure, stellar evolution, and stellar remnants – treated as isolated systems – are covered in Chaps. 2–6. The behaviour of binary stellar systems is different enough to warrant a separate chapter, Chap. 7. Chapters 8–10 are in some sense special topics: Chap. 8 deals with the Sun and the solar system, which deserve a detailed discussion in any course of astrophysics; Chap. 9 covers the interstellar medium and the cross talk between stars and the rest of the contents of the galaxy; finally, the short Chap. 10 describes some aspects of globular clusters.

This volume provided a tough challenge as regards the discussion of phenomenological input, and a few words regarding my policy are in order. Stellar astronomy is probably one of those areas in which observations lead the theory and the availability of accurate data allows one to recognise the complexity of several phenomena. These volumes, however, are intended to be a course on *theoretical* astrophysics, and hence the emphasis naturally is on the theoretical aspects rather than on observational and phenomenological issues. Given this dichotomy, it is easy to fall into one of the two traps: (1) Drown the reader in an accurate but unclassified sea of astronomical data just because accurate data are available or (2) ignore the phenomenological input and treat the subject as a branch of applied mathematics. I have tried hard to avoid both these pitfalls by adopting the following approach. I describe the necessary observational issues (but not observational techniques) and provide a minimum of observational data whenever they are relevant. I have also tried to motivate theoretical developments based on specific observational inputs, especially when a more fundamental approach would be unwarranted or facetious. At the same time I have tried to bring some amount of method and order in the presentation of the topics so that the reader will be able to grasp how a theoretical astrophysicist goes about the task of developing the models. One major problem in this approach was the interdependency of concepts (and even jargon) that prevents a fully streamlined development of topics. I have attempted to solve this difficulty by providing an overview in Chap. 1 that develops the necessary astronomical jargon, introduces the *dramatis personae*, and summarises the observational data that are general enough to be presented right at the outset. Chapter 1 also discusses several key issues of observational astronomy that are generic and reasonably independent of the technology available at a given time. (I plead guilty of not having yet learnt how to write my first chapters; this is the worst chapter in the book.)

All this required the exercise of my judgement in deciding the choice of topics, their emphasis, and the proper blend of phenomenology, observations, and theoretical rigour. It is impossible to satisfy everyone as regards the “correctness” of such decisions and I have tried to do some optimisation so as to provide the maximum benefit to the reader.

Any one of these topics is fairly vast and often requires a full textbook to do justice to it, whereas I have devoted approximately 60 pages to discussing each of them! I would like to emphasise that such a crisp, condensed discussion is not only possible but also constitutes a basic matter of policy in these volumes. After all, the idea *is* to provide the student with the essence of several textbooks in one place. It should be clear to lecturers that these materials can be easily regrouped to serve different graduate courses at different levels, especially when complemented by other textbooks.

Because of the highly pedagogical nature of the material covered in this volume, I have not given detailed references to original literature except on rare occasions when a particular derivation is not available in standard textbooks. The annotated list of references given at the end of the book cites several other textbooks that I found very useful. Some of these books, of course, contain extensive bibliographies and references to original literature. The selection of core books cited here clearly reflects the personal bias of the author, and I apologise to anyone who feels their work or contribution has been overlooked.

Several people have contributed to the making of these volumes and especially to Vol. II. The idea for these volumes originated over a dinner with J.P. Ostriker in late 1994, while I was visiting Princeton. I was lamenting to Jerry about the lack of a comprehensive set of books covering all of theoretical astrophysics, and Jerry said, “Why don’t *you* write them?” He was very enthusiastic and supportive of the idea and gave extensive comments on and suggestions for the original outline I produced the next week. I am grateful to him for the comments and for the moral support that I needed to launch into such a project. I sincerely hope the volumes do not disappoint him.

Adam Black of Cambridge University Press took up the proposal with his characteristic enthusiasm and initiative. I should also thank him for choosing six excellent (anonymous) referees for this proposal whose support and comments helped to mould it into the proper framework. The processing of this volume was handled by Ellen Carlin of CUP and I thank her for the effort she has put in.

Many of my friends and colleagues carried out the job of reading the earlier drafts and providing comments. Of these, M. Vivekanand has gone through most of the book with meticulous care and has made extensive comments. H.M. Antia, D. Bhattacharya, S. Bhavsar, J. Chengalur, Nissim Kanekar, S. Konar, D. Narasimha, J.V. Narlikar, A.R. Ramprakash, S. Srianand, K. Subramanian, and F. Sutaria made detailed comments on selected chapters. Some of the figures and data were provided by H.M. Antia, Ranjan Gupta, Yashwant Gupta, Balchand Joshi, and A.R. Ramprakash. I thank all of them for their help.

I have been a regular visitor to the Astronomy department of Caltech during the past several years, and the work on the volumes has benefitted tremendously through my discussions and interactions with the students and staff of the Caltech Astronomy department. I would like to specially thank Roger Blandford, Peter Goldreich, Shri Kulkarni, Sterl Phinney, and Tony Readhead for several useful

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discussions and for sharing with me their insights and experience in the teaching of astrophysics.

This project would not have been possible but for the dedicated support from Vasanthi Padmanabhan, who not only did the entire TEXing and formatting but also produced most of the figures – often writing the necessary programs for the same. I thank her for the help and look forward to receiving the same for the last volume as well! I also thank Sunu Engineer, who was resourceful in solving several computer-related problems that cropped up periodically. It is a pleasure to acknowledge the library and other research facilities available at IUCAA, which were useful in this task.

T. Padmanabhan