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Overview: Stars and Stellar Systems

1.1 Introduction

Stellar physics provides a natural starting point for the study of astrophysics for several reasons. To begin with, this is probably the best understood area of astrophysics. Second, there is a vast amount of reliable data dealing with stellar physics. This observational input motivates accurate and sophisticated theoretical modelling as well as provides the opportunity for a detailed comparison between models and observations. Finally, stellar physics also forms the basis for the study of several other related areas, even in the domain of extragalactic astronomy and cosmology. For example, measurements of cosmic distances and the ages of different structures – which are very important in cosmology – cannot be done without accurate modelling of the stellar phenomena that are used as tools; the study of formation and evolution of galactic systems requires an understanding of star formation and stellar evolution, etc. This volume deals with different aspects of the astrophysics of stellar systems.

The evolution of stars differs significantly, depending on whether the star is isolated or is a member of a binary system. The bulk of the chapters in the book (from Chap. 3 to Chap. 6) deal with stellar evolution and stellar remnants in isolated contexts. Chapter 7 is devoted to the study of evolution of binary stars, and Chap. 10 covers the dynamics of systems like globular clusters that have a very large number of stars. Because stars reside in – and are strongly coupled to – the interstellar medium (ISM), one full chapter (Chap. 9) is devoted to discussing the physics of the ISM.

Because of the interdependence of many of the concepts involved in this study, it is difficult to provide a completely logical and structured approach to stellar physics. It is necessary to take observational and phenomenological inputs within different contexts in order to guide the theoretical concepts appropriately. The purpose of this first chapter is to provide the necessary background so that the latter chapters can be developed in a streamlined fashion. In this chapter, most of the astronomical jargon, units, and a certain amount of observational and phenomenological inputs are introduced that will be required throughout the book. The observational input is kept at a general level, and aspects that are very specific to a particular topic will be discussed *in situ* in the relevant chapters.

It is also important to appreciate certain issues of observational astronomy that are very special to this subject and do not exist in other branches of applied physics, which draw inputs from laboratory experiments. Section 1.8 of this chapter highlights some of these issues and their relevance.[1–6] (The superscript numbers refer to the notes and the references given at the end of the book.)

1.2 Stars

There is sufficient evidence to believe that stars form when the local condensations of gas in a galaxy, contracting under self-gravity, reach sufficiently high central temperatures to ignite self-sustained nuclear reactions in the core. Although a fundamental theory that takes it into account all the relevant factors (rotation of the gas, magnetic and other nongravitational forces) and predicts the properties of the initial distribution of stars does not exist, the basic idea – that stars originate when sustained nuclear reactions take place in the core region of a self-gravitating cloud – seems to be well borne out, both theoretically and observationally. This idea will form the basis for the description of stellar structure and evolution in Chaps. 2 and 3.

In fact, this idea has sufficient predictive power to allow us to estimate several key properties of stars in an approximate manner. We shall briefly recall these results from Vol. I, Subsection 1.5.3. Consider a spherically symmetric cloud of, say, hydrogen gas, with mass M, radius R, and containing N protons, which is contracting under self-gravity. As the cloud contracts, the gravitational pressure $P_g \propto (GM^2/R^4)$ increases and needs to be balanced either by the thermal pressure of the gas or by the electron degeneracy pressure. This requires that the thermal energy or the Fermi energy of electrons (per particle) be comparable with the gravitational potential energy of the system (per particle); the latter is given by

$$\epsilon_g \equiv \frac{E_{\rm grav}}{N} = \left(\frac{Gm_p^2}{R}\right) N = \left(\frac{4\pi}{3}\right)^{1/3} Gm_p^2 N^{2/3} n^{1/3}, \tag{1.1}$$

where m_p is the mass of the proton and $n = (3N/4\pi R^3)$ is the number density of particles. To take into account both thermal and quantum degeneracy contributions to the energy, we take the total energy per particle to be $(k_B T + \epsilon_F)$, where $\epsilon_F = (\hbar^2/2m_e)(3\pi^2 n)^{2/3}$ is the Fermi energy of the electrons in the nonrelativistic limit. This energy will be comparable with gravitational energy if $(k_B T + \epsilon_F) \simeq Gm_p^2 N^{2/3} n^{1/3}$. Using the expression for ϵ_F for the nonrelativistic electrons, we get

$$k_B T \simeq G m_p^2 N^{2/3} n^{1/3} - \frac{(3\pi^2)^{2/3}}{2} \frac{\hbar^2}{m_e} n^{2/3}. \tag{1.2}$$

As the radius R of the system is reduced, the second term on the right-hand side ($\propto n^{2/3}$) grows faster than the first ($\propto n^{1/3}$); hence the temperature of the system will first increase, then reach a maximum, and finally decrease again. The maximum temperature $T_{\max}$ is reached when $n = n_c$, with

$$n_c^{1/3} \cong \frac{\alpha_G}{(3\pi^2)^{2/3}} \left(\frac{N^{2/3}}{\lambda_e} \right), \quad k_B T_{\max} \simeq \frac{\alpha_G^2}{2(3\pi^2)^{2/3}} (N^{4/3} m_e c^2), \tag{1.3}$$

where $\lambda_e \equiv (\hbar / m_e c) \approx 3.8 \times 10^{-11}$ cm is the Compton wavelength of the electron and $\alpha_G \equiv (G m_p^2 / \hbar c) \approx 6 \times 10^{-39}$ is the gravitational equivalent of the fine-structure constant.

At temperatures higher than $\sim 10^3$ K, hydrogen will be ionised and we will have a plasma of electrons and protons. If the maximum temperature $T_{\max}$ of the plasma is sufficiently high to trigger nuclear fusion in the system, then we obtain a gravitationally bound, self-sustained nuclear reactor. For two protons to fuse together and undergo nuclear reaction, it is necessary that their de Broglie wavelengths $\lambda_{\mathrm{deB}} \equiv (h / m_p v)$ overlap. Because this requires overcoming the Coulomb repulsion, such direct interaction can take place only if the kinetic energy of colliding particles is at least of the order of the electrostatic potential energy at the separation λ_{deB}. This requires kinetic energies of the order of $\epsilon \approx (e^2 / \lambda_{\mathrm{deB}}) \approx (\alpha^2 / 2\pi^2) m_p c^2 \approx 1$ keV, where $\alpha \equiv (e^2 / \hbar c)$ is the fine-structure constant. It is conventional to write this expression as $\epsilon_{\mathrm{nucl}} \approx \eta \alpha^2 m_p c^2$, with $\eta \simeq 0.1$. The quantity ϵ_{nucl} sets the scale for triggering nuclear reactions in astrophysical contexts. The energy corresponding to the maximum temperature $k_B T_{\max}$ [obtained in expression (1.3) above] will be larger than ϵ_{nucl} when

$$N > N_* \equiv (2\eta)^{3/4} (3\pi^2)^{1/2} \left(\frac{m_p}{m_e} \right)^{3/4} \left(\frac{\alpha}{\alpha_G} \right)^{3/2} \approx 4 \times 10^{56} \tag{1.4}$$

for $\eta \simeq 0.1$. The corresponding condition on mass is $M > M_*$, with

$$M_* \approx (2\eta)^{3/4} (3\pi^2)^{1/2} \left(\frac{m_p}{m_e} \right)^{3/4} \left(\frac{\alpha}{\alpha_G} \right)^{3/2} m_p \approx 4 \times 10^{32} \text{ gm}, \tag{1.5}$$

which is comparable with the mass of the smallest stars observed in our universe. The radius is $R_* \cong (G M_* m_p / k_B T_{\max}) \simeq 3 \times 10^{10}$ cm. The mass and the radius of the Sun, for example, are $M_\odot = 2 \times 10^{33}$ gm and $R_\odot \simeq 7 \times 10^{10}$ cm, respectively. Most of the stars in the universe powered by nuclear reactions have masses in the range (0.1–60) $M_\odot$.

According to the above description, stars form in overdense regions of primordial gas in the galaxy. To understand the relationship between the stars and the galaxy, it is necessary to model the origin of the galaxy itself. This is somewhat more uncertain and will be discussed in detail in Vol. III. It is, however, possible to understand the characteristic mass and size of a galaxy by analysing the cooling processes operating in a protogalactic cloud. Such an analysis in Vol. I, Subsection 1.5.1, showed that the size and the mass of a typical galaxy are

$$\begin{aligned} R_g &\simeq \alpha^3 \alpha_G^{-1} \lambda_e \left(\frac{m_p}{m_e}\right)^{1/2} \simeq 74\,\text{kpc}, \\ M_g &\simeq \alpha^5 \alpha_G^{-2} \left(\frac{m_p}{m_e}\right)^{1/2} m_p \simeq 3 \times 10^{44}\,\text{gm}, \end{aligned} \tag{1.6}$$

where 1 kpc $\simeq 3 \times 10^{21}$ cm. A comparison of M_g with expression (1.5) for M_* shows that the number of stars $N_{\text{star}} \simeq (M_g/M_*)$ in a typical galaxy will be given by a combination of fundamental constants $N_{\text{star}} = \alpha^{7/2}\alpha_G^{-1/2}(m_e/m_p)^{1/4} \simeq 10^{12}$. Typical galaxies indeed have approximately 10^{11}–10^{12} stars, although there is a fair amount of spread in this number. Most of the visible mass in the galaxy is contained in a region somewhat smaller than the size estimated above, at $R_{\text{gal}} \approx 20$ kpc.

Given the radius of the galaxy and the number of stars in it, we can estimate the mean distance between the stars to be $d_{\text{star}} \approx (R_{\text{gal}}/N_*^{1/3}) \approx 3 \times 10^{18}\,\text{cm} \equiv 1$ pc. Thus we expect to see stars in our galaxy to be distributed at distances varying from a few parsecs to a few tens of kiloparsecs. A star like the Sun, with a radius of $\sim 10^{11}$ cm and located at a distance of 10 pc from us, will subtend an angle of ~ 1 milliarcsecond; it is clear that most stars will look like point objects.

One of the important questions in observational astronomy is the determination of spatial distribution of stars in the galaxy, which – to a large extent – is independent of the structure and the dynamics of the stars. Thus the simplest observation we can make regarding a star is to measure its position in the sky, which requires the specification of two suitable angular coordinates. However, because most of the astronomical data are either gathered from Earth or from satellites, which have systematic motion at short time scales with respect to a fixed cosmic reference frame, it is necessary to define accurately the coordinate system used in any given astronomical observation. Although several coordinate systems are possible, each having its own domain of applicability and utility, there is one coordinate system called *the equatorial system* that appears to be natural to the observations based on Earth. We shall now briefly describe how such a coordinate system is defined, as the measurement of the position of an object in the sky is of central importance in any branch of astronomy.

The rotation of the Earth about its axis defines two unique directions in the sky called the north celestial pole (NCP) and the south celestial pole (SCP), which are obtained by the extension of the Earth's axis of rotation to the celestial sphere (see Fig. 1.1). The great circle in the celestial sphere, formed by the plane perpendicular to this axis, defines the celestial equator. Treating the Earth

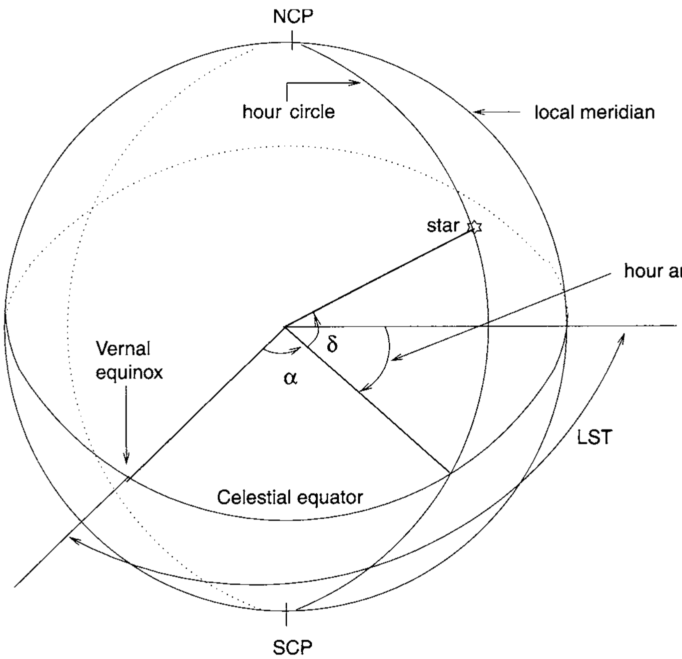

Fig. 1.1. The equatorial coordinate system.

as a sphere of radius R, we can define a local normal at any given location; the intersection of this normal with the celestial sphere defines the direction of the zenith at any given location on Earth. The great circle on the celestial sphere passing through the celestial poles and the zenith is called the local meridian.

As the Earth revolves around the Sun annually, the Sun appears to move from west to east on the celestial sphere in a path called the ecliptic, inclined at approximately $23°27'$ to the celestial equator. The two great circles – the ecliptic and the celestial equator – intersect at two different points called the vernal and the autumn equinoxes. The Sun passes through the vernal equinox (VE) on approximately 21 March, moving from south to north of the celestial equator.

Given the celestial equator and the VE, it is possible to define the position of any celestial object – say, a star – by the following procedure. We first draw the great circle (called the hour circle) passing through the celestial poles and the star. We can now specify the coordinates of the star by giving two angles: (1) the *declination* δ, which is the angular distance measured from the celestial equator to the star along the star's hour circle and (2) the *right ascension* (RA) α, which is the angular distance along the celestial equator from the VE to the star's hour circle. (In terms of the standard spherical polar coordinate system, $\alpha \equiv \phi$ and $\delta \equiv 90° - \theta$.) By convention, the RA is usually expressed in terms of hours, minutes, and seconds rather than in degrees, with the convention of 24 h corresponding to 360°. RA increases from west to east so that the stars with larger RA rise later than those with smaller ones. The declination is taken to be positive northward of the celestial equator and negative southward.

The motivation for using time units rather than angular units to measure RA comes from the fact that a star's RA is very nearly equal to the time between the meridian transit of the star and the meridian transit of the VE. The *local sidereal time* (LST) is the RA of the meridian expressed in units of time. It is also usual to define a quantity called the *hour angle*, which is the angle along the celestial equator between the meridian and the hour circle that, by convention, is measured in the sense opposite to α. It follows that a star's hour angle (1) is the difference between the LST and the star's RA and (2) will be equal to the time since the star crossed the meridian. Obviously, a star will be at the meridian when the LST is equal to the star's RA.

The coordinate system based on the NCP and the VE suffers from the difficulty that neither of these directions remains static in the celestial sphere because of the complicated dynamical process acting in the solar system (see Chap. 8.) Careful corrections have to be applied in order to define the coordinate system with respect to a hypothetical mean NCP and mean VE. Further, it is also necessary to define a particular instant in time with respect to which α and δ are measured. Several such epochs are used, with the most popular ones being those based on the Julian year 1950 or 2000.

Exercise 1.1

Practice with coordinate systems I: An astronomer wants to observe a star with RA and dec$(X, Y) = (23^h 20^m 39^s, +18°08'33'')$, say. It would be best if the source is as high as possible in the sky at the time of observation. Which part of the year is best suited for this? [Answer: If the declination is positive, it is best to do the observation from the northern hemisphere whereas for $Y < 0$ it is better to use a telescope in the southern hemisphere. What we really need to determine is the time of the year at which the observation can best be carried out. To have the star as high as possible in the sky, an observer would like to choose the time of the year such that the star crosses the observer's meridian near the middle of the night. We begin by noting the two relations: (1) The LST when a star crosses the meridian is equal to the RA of the star and (2) the LST at midnight on any particular day is the RA of the Sun on that particular day ($\mathrm{RA}_\odot$) plus the hour angle of the Sun at midnight. Because the latter is 12^h, it follows that LST (midnight) = $\mathrm{RA}_\odot + 12^h$. Because we want the LST at the time of the star's crossing the meridian to be the same as the LST at midnight, we can equate the two expressions for the LST. This gives the $\mathrm{RA}_\odot = \mathrm{RA}_* - 12^h = (X - 12)^h$. This is the expression we need. Because the RA of the Sun is zero on the spring equinox (roughly March 21) and varies by 2 h/month, the RA of the Sun will have the value $(X - 12)^h$ after approximately $(X - 12)/2$ months since the spring equinox. In this specific case mentioned in the question, $(X - 12)/2 \approx 6$ months, and hence the second or third week of September will be the best time for the indicated observation.]

Exercise 1.2

Practice with coordinate systems II: A star has RA $(+6^h)$ and dec $+30°$. What is its altitude above the horizon for an observer at 30° north when the LST is 9^h? At what LST will it be overhead? At what time of the year will the star be directly overhead at local midnight?

Exercise 1.3

Galactic coordinate system: Another coordinate system frequently used in extragalactic astronomy is based on our galaxy. In this scheme the galactic equator is chosen to be that great circle on the celestial sphere that closely approximates the plane of the Milky Way – which, in turn, is inclined at an angle of 62.87° to the celestial equator. The north galactic pole is located at $(\alpha_{\rm GP}, \delta_{\rm GP}) = (192.85948°, 27.12825°) \simeq (12^h 51^m, +27°7.7')$. The galactic latitude b of an object is the angle from the galactic equator to the star along the great circle through the star and galactic poles. The galactic longitude l is measured along the galactic equator from the direction of the galactic centre. This direction corresponding to $l = 0$, $b = 0$ has the equatorial coordinates $(\alpha, \delta) = (266.405°, -28.936°) \simeq (17^h 45.6^m, -28°56.2')$. Show that the galactic coordinates (l, b) are related to the equatorial coordinates (α, δ) by

$$\begin{aligned} \sin b &= \sin\delta_{\rm GP}\sin\delta + \cos\delta_{\rm GP}\cos\delta\cos(\alpha - \alpha_{\rm GP}), \\ \cos b\sin(l_{\rm CP} - l) &= \cos\delta\sin(\alpha - \alpha_{\rm GP}), \\ \cos b\cos(l_{\rm CP} - l) &= \cos\delta_{\rm GP}\sin\delta - \sin\delta_{\rm GP}\cos\delta\cos(\alpha - \alpha_{\rm GP}), \end{aligned} \tag{1.7}$$

where $l_{\rm CP} = 123.932°$ is the longitude of the NCP.

1.3 Stellar Magnitudes and Colours

Once the nuclear reactions occur in the hot central region of the gas cloud, its structure changes significantly. If the transport of this energy to the outer regions is through photon diffusion, then the opacity of matter will play a vital role in determining the stellar structure. In particular, the opacities determine the relation between the luminosity and the mass of the star.

A photon with mean free path $l = (n\sigma)^{-1}$, randomly walking through the hot plasma in the interior of the star, will have $N_{\rm coll} \simeq (R/l)^2$ collisions in traversing the radius R. This will take the time $t_{\rm esc} \simeq (lN_{\rm coll}/c) \simeq (R/c)(R/l)$ for the photon to escape. The luminosity of a star L will be proportional to the ratio between the radiant energy content of the star, E_γ, and $t_{\rm esc}$. Because $E_\gamma \simeq (aT^4)R^3 \propto T^4R^3$, we find that

$$L \propto \frac{R^3T^4l}{R^2} \propto RT^4l \propto \frac{RT^4}{n\sigma}. \tag{1.8}$$

For a wide class of stars, we may assume that the central temperature $k_BT \simeq (GMm_p/R)$ is reasonably constant because nuclear reactions – which depend strongly on T – act as a thermostat. If Thomson scattering dominates, then

$\sigma = \sigma_T \equiv [(8\pi/3)(e^2/m_e c^2)^2]$ and we get

$$L \propto \frac{RT^4}{\sigma_T n} \propto \frac{T^4 R^4}{\sigma_T N} \propto \frac{M^4}{M} \propto M^3. \qquad (1.9)$$

The situation is different if interaction of photons with partially ionised atoms provide the opacity. The cross section for bound–free and bound–bound opacity in thermal equilibrium has been obtained in Vol. I, Chap. 1, Subsection 1.4.4, where it was shown that $l \propto T^{7/2} n^{-2} \propto T^{7/2} R^6 M^{-2}$. [The bound–free opacity can be understood as follows: In equilibrium, the photoionisation rate, which removes energy from the radiation field, should match the recombination rate. The amount of energy removed by photoionisation is proportional to $dE_{\text{ion}} \propto n_{\text{atom}} \sigma_{\text{bf}} T^4$, where σ_{bf} is the photoionisation cross section. The energy supplied by recombination scales as $dE_{\text{rec}} \propto n_e n_i v \propto n_e n_i T^{1/2}$. Equating dE_{ion} to dE_{rec}, we get $n_{\text{atom}} \sigma_{\text{bf}} T^4 \propto n_e n_i T^{1/2}$. Introducing the bound–free opacity κ_{bf} by the definition $\kappa_{\text{bf}} = (n_{\text{atom}} \sigma_{\text{bf}}/\rho)$ and taking $n_e \propto \rho$, $n_i \propto \rho$, we find that $\kappa_{\text{bf}} \propto \rho T^{-3.5}$.] In this case, we have

$$L \propto RT^4 l \propto R^7 T^{15/2} M^{-2} \propto M^{11/2} R^{-1/2}. \qquad (1.10)$$

Taking $(GM/R) \approx$ constant so that $R \propto M$ now gives $L \propto M^5$. Taken along with expression (1.9), we expect the luminosity of a star to be related to its mass by $L \propto M^\alpha$, with $\alpha \simeq 3$–5.

If we imagine the surface of the star to be at some effective temperature T_s, then the total blackbody luminosity from the star will be $L = (4\pi R^2)(\sigma T_s^4)$, where $\sigma = (\pi^2 k^4 / 60 \hbar^3 c^2)$ is the Stefan–Boltzmann constant. It is convenient to use this relation to define the surface temperature T_s of the star with a given luminosity L and radius R. If the radiation from the star is approximated as that of a blackbody, then the intensity f_ν (which is the energy per unit area per unit time per solid angle per frequency) of thermal radiation emitted by the star will be

$$f_\nu = \frac{dE}{dA\,dt\,d\Omega\,d\nu} = B_\nu \equiv \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/k_B T_s} - 1}. \qquad (1.11)$$

The quantity νB_ν (which gives the intensity per logarithmic band in frequency) reaches a maximum value near $h\nu \approx 4k_B T$, which translates to the fact that a blackbody at 6000 K will have the maximum for νB_ν at 6000 Å. The maximum intensity is $(\nu B_\nu)_{\max} \approx (T/100\text{ K})^4$ W m^{-2} sr^{-1}.

This description provides a few more useful observed characteristics of a star. In principle, we can fit the spectrum of a star to a blackbody spectrum (approximately) and obtain T_s. The total energy flux l received from the star (called the *apparent luminosity*) can also be measured directly. Because $l = L/(4\pi d^2)$, where d is the distance to the star, we can determine the *absolute luminosity* L of the star if the distance to the star is known. Assuming that the distance can be independently measured, we will be able to determine L and T_s and plot the location of the stars in a two-dimensional (L–T_s) plane.

From our definition, it follows that $T_s \propto L^{1/4}R^{-1/2} \propto L^{1/4}M^{-1/2}$. Combining with $M \propto L^{1/5}$, valid when the interior is only partially ionised, we get $T_s \propto L^{1/4}L^{-1/10} \propto L^{3/20}$. On the other hand, if Thomson scattering dominates with $L \propto M^3$, we get $T_s \propto L^{1/12}$. Thus, if the stars are plotted on a $\log T_s$–$\log L$ plane, (called the Hertzsprung–Russel diagram or the H-R diagram) we expect them to lie within the lines with slopes $3/20 = 0.15$ and $1/12 \simeq 0.08$. The observed slope is ~ 0.13, giving reasonable support to the basic ideas developed above. Observationally, it is found that stellar surface temperatures vary from approximately 3×10^3 K to 3×10^4 K as the mass varies from approximately $0.1\, M_\odot$ to $60\, M_\odot$. The corresponding variation in the radius is in the range of $(0.8\text{–}70) \times 10^{10}$ cm, and the luminosity ranges from $10^{-3}\, L_\odot$ to $10^{5.7} L_\odot$, where $L_\odot = 3 \times 10^{33}$ ergs s^{-1} is the luminosity of the Sun.

The intensity f_ν, defined above as the amount of energy received per second per unit area per unit frequency interval, is a fundamental quantity characterising the radiation from *any* celestial object. It is therefore important to stress some practical issues related to its measurement. Actual measurements in optical, IR, and UV bands do not measure the f_ν of a source directly. The observed intensity f can be usually expressed in the form

$$f \equiv \int_0^\infty f_\nu \mathcal{T}_\nu \mathcal{F}_\nu \mathcal{R}_\nu \, d\nu \equiv \int_0^\infty f_\lambda S_\lambda \, d\lambda, \tag{1.12}$$

where each of the factors signify the following features: (1) $\mathcal{T}_\nu$ measures the fractional transmission that is due to the Earth's atmosphere, thereby connecting the observed intensity and the intensity on top of the Earth's atmosphere. This is, of course, irrelevant for satellite-based observations. (2) No realistic apparatus can be equally sensitive at all frequencies or be sensitive at only a given frequency ν_0. The factor $\mathcal{F}_\nu$ is the fractional sensitivity of the filter used in the telescope or other measuring apparatus at frequency ν. For practical purposes, the apparatus can be characterised by a mean frequency ν_0 at which it is most sensitive and a full width at half maximum (FWHM) that specifies the band of frequencies over which significant sensitivity exists. (3) $\mathcal{R}_\nu$ represents the efficiency of the detector and is the ratio between energy detected and energy incident upon the detector. The second equality in Eq. (1.12) gives the corresponding equation in terms of wavelength, with S_λ combining the effects of all the three factors. Among these factors, the filter response $\mathcal{F}_\nu$ is probably most important and is often used to characterise the intensity in different frequency bands such as the ultraviolet (U) band, blue (B) band, visible (V) band, red (R) band, etc. For the sake of standardisation, each of these bands is specified in terms of an effective wavelength (λ_{eff}) at which the band is centred and a FWHM.

For historical reasons, astronomical measurements (especially those in the optical band) are quoted in terms of another quantity, called *magnitude*, which is related to f in a logarithmic manner. This unit is not of any intrinsic value

Table 1.1. Filter characteristics of broadband photometric systems

Band	$\frac{\lambda_{\rm eff}}{\rm nm}$	$\frac{W_\lambda}{\rm nm}$	$\frac{dE}{dt\,dA\,d\nu}$ (Jy)	$\frac{dE}{dt\,dA\,d\lambda}$*	$a^\dagger$	$b^\ddagger$	$c^\S$
U	365	66	1780	4.0×10^{-8}	22	150	9.9
B	445	94	4000	6.1×10^{-8}	23	100	9.4
V	551	88	3600	3.6×10^{-8}	22	170	15
R	658	138	3060	2.1×10^{-8}	21	250	35
I	806	149	2420	11.2×10^{-9}	18.5	1.5×10^3	223
J	1220	213	1570	3.07×10^{-9}	16	1.0×10^4	2.1×10^3
H	1630	307	1020	1.12×10^{-9}	13	5.6×10^4	1.7×10^4
K	2190	390	636	4.07×10^{-10}	12.5	4.4×10^4	1.8×10^3
L	3450	472	281	7.30×10^{-11}	5.5	8.0×10^6	3.8×10^6
M	4750	460	154	2.12×10^{-11}	2	1.0×10^8	4.6×10^7

*Flux density of a zero-magnitude star per unit wavelength [$f_\lambda(0)$/W m^{-2} μm^{-1}].
†Background intensity in magnitude arcsec^{-2}.
‡Background photon intensity per unit wave band [$I(\lambda)$/photons m^{-2} arcsec^{-2} s^{-1} μm^{-1}].
§Background photon intensity in standard wave band (I/ photons m^{-2} arcsec^{-2} s^{-1}), obtained as the product of the value given in b and W_λ.

and can be fairly confusing; however, as it is unlikely that optical astronomy will switch to a more rational and scientific unit of measurement in the near future, it is necessary for us to define and relate this archaic concept to the flux measured in physical units.

The total flux F_X in physical units integrated over the filter function with width W_X from an object with apparent magnitude m_X in the band X can be written as

$$F_X \equiv W_X f_X = (10^Q\, W_X)10^{-0.4 m_X}, \tag{1.13}$$

where Q is defined as log f_X for a reference star with $m_X = 0$ and W_X is the FWHM for the band X; that is,

$$Q = \log\left(\frac{f_\lambda}{\text{ergs cm}^{-2}\ \text{s}^{-1}\ \mu\text{m}^{-1}}\right), \tag{1.14}$$

where f_λ is the flux of a reference star with $m = 0$ and the width is conveniently measured in micrometers. For the U, B, V, R, I, J, and K bands, $Q = -4.37, -4.18, -4.42, -4.76, -5.08, -5.48$, and -6.40, respectively, by definition.

Table 1.1 gives the details for a commonly used photometric system. The fourth entry in the table (f_X) gives the flux density in the X band for an $m_X = 0$ reference star in units of 1 Jy $= 10^{-26}$ W m^{-2} Hz^{-1} $= 10^{-23}$ ergs cm^{-2} s^{-1} Hz^{-1} commonly used in radio astronomy. The fifth entry gives the flux density of a zero-magnitude star per unit wavelength [$f_\lambda(0)$/W m$^{-2}\mu$m^{-1}] $= 3 \times 10^{-6} f_X(\text{Jy})\lambda^{-2}$(nm). It