

THEORY OF ALGEBRAIC INTEGERS

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# Theory of Algebraic Integers

Richard Dedekind

Translated and introduced by John Stillwell



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## Contents

<b>Part one: Translator's introduction</b>	<i>page</i> 1
<i>Translator's introduction</i>	3
0.1 General remarks	3
0.2 Squares	6
0.2.1 Pythagorean triples	6
0.2.2 Divisors and prime factorisation	7
0.2.3 Irrational numbers	8
0.2.4 Diophantus	8
0.3 Quadratic forms	10
0.3.1 Fermat	10
0.3.2 The grit in the oyster	12
0.3.3 Reduction of forms	13
0.3.4 Lagrange's proof of the two squares theorem	15
0.3.5 Primitive roots and quadratic residues	16
0.3.6 Composition of forms	17
0.3.7 The class group	19
0.4 Quadratic integers	21
0.4.1 The need for generalised "integers"	21
0.4.2 Gaussian integers	22
0.4.3 Gaussian primes	24
0.4.4 Imaginary quadratic integers	25
0.4.5 The failure of unique prime factorisation	27
0.5 Roots of unity	29
0.5.1 Fermat's last theorem	29
0.5.2 The cyclotomic integers	30
0.5.3 Cyclotomic integers and quadratic integers	32
0.5.4 Quadratic reciprocity	36
0.5.5 Other reciprocity laws	38

vi	<i>Contents</i>	
0.6	Algebraic integers	39
0.6.1	Definition	39
0.6.2	Basic properties	40
0.6.3	Class numbers	41
0.6.4	Ideal numbers and ideals	42
0.7	The reception of ideal theory	44
0.7.1	How the memoir came to be written	44
0.7.2	Later development of ideal theory	45
	Acknowledgements	47
	<i>Bibliography</i>	48
	<b>Part two: Theory of algebraic integers</b>	51
	<i>Introduction</i>	53
1	Auxiliary theorems from the theory of modules	62
§1.	Modules and their divisibility	62
§2.	Congruences and classes of numbers	64
§3.	Finitely generated modules	67
§4.	Irreducible systems	71
2	Germ of the theory of ideals	83
§5.	The rational integers	83
§6.	The complex integers of Gauss	84
§7.	The domain $\mathfrak{o}$ of numbers $x + y\sqrt{-5}$	86
§8.	Role of the number 2 in the domain $\mathfrak{o}$	89
§9.	Role of the numbers 3 and 7 in the domain $\mathfrak{o}$	91
§10.	Laws of divisibility in the domain $\mathfrak{o}$	93
§11.	Ideals in the domain $\mathfrak{o}$	95
§12.	Divisibility and multiplication of ideals in $\mathfrak{o}$	98
3	General properties of algebraic integers	103
§13.	The domain of all algebraic integers	103
§14.	Divisibility of integers	105
§15.	Fields of finite degree	106
§16.	Conjugate fields	108
§17.	Norms and discriminants	111
§18.	The integers in a field $\Omega$ of finite degree	113
4	Elements of the theory of ideals	119
§19.	Ideals and their divisibility	119
§20.	Norms	121
§21.	Prime ideals	123
§22.	Multiplication of ideals	125
§23.	The difficulty in the theory	126
§24.	Auxiliary propositions	128

<i>Contents</i>		vii
§25. Laws of divisibility		129
§26. Congruences		134
§27. Examples borrowed from circle division		138
§28. Classes of ideals		146
§29. The number of classes of ideals		147
§30. Conclusion		149
<i>Index</i>		153