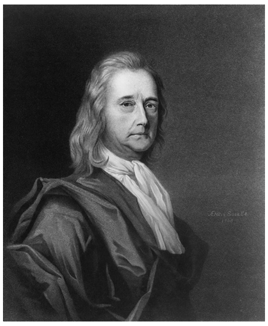
1 Waves versus particles

. . . I think I can safely say that nobody understands quantum mechanics.
Richard Feynman

Science and experiment

Science is a special kind of explanation of the things we see around us. It starts with a problem and curiosity. Something strikes the scientist as odd. It doesn't fit in with the usual explanation. Maybe harder thinking or more careful observation will resolve the problem. If it remains a puzzle, it stimulates the scientist's imagination. Perhaps a completely new way of looking at things is needed? Scientists are perpetually trying to find better explanations – better in the sense that any new explanation must not only explain the new puzzle, but also be consistent with all of the previous explanations that still work well. The hallmark of any scientific explanation or 'theory' is that it must be able to make successful predictions. In other words, any decent theory must be able to say what will happen in any given set of circumstances. Thus, any new theory will only become generally accepted by the scientific community if it is able not only to explain the observations that scientists have already made, but also to foretell the results of new, as yet unperformed, experiments. This rigorous testing of new scientific ideas is the key feature that distinguishes science from other fields of intellectual endeavour – such as history or even economics – or from a pseudoscience such as astrology.



Isaac Newton (1642–1727) published his book *Optics* in 1704 that explained the rainbow and put forward the 'corpuscular' theory of light. In his 1687 book *Mathematical Principles of Natural Philosophy* Newton set down the principles of mechanics and gravity that guided science until the mid nineteenth century.

In the seventeenth century Isaac Newton and several other great scientists developed a wonderfully successful explanation of the way things move. This whole theoretical framework is called 'classical mechanics', and its scope encompasses the motion of everything from billiard balls to planets. Newton's explanation of motion in terms of forces, momentum and acceleration is encapsulated in his 'laws of motion'. These principles are incorporated into so many of our machines and toys that classical mechanics is familiar from our everyday experience. We all know what to expect in

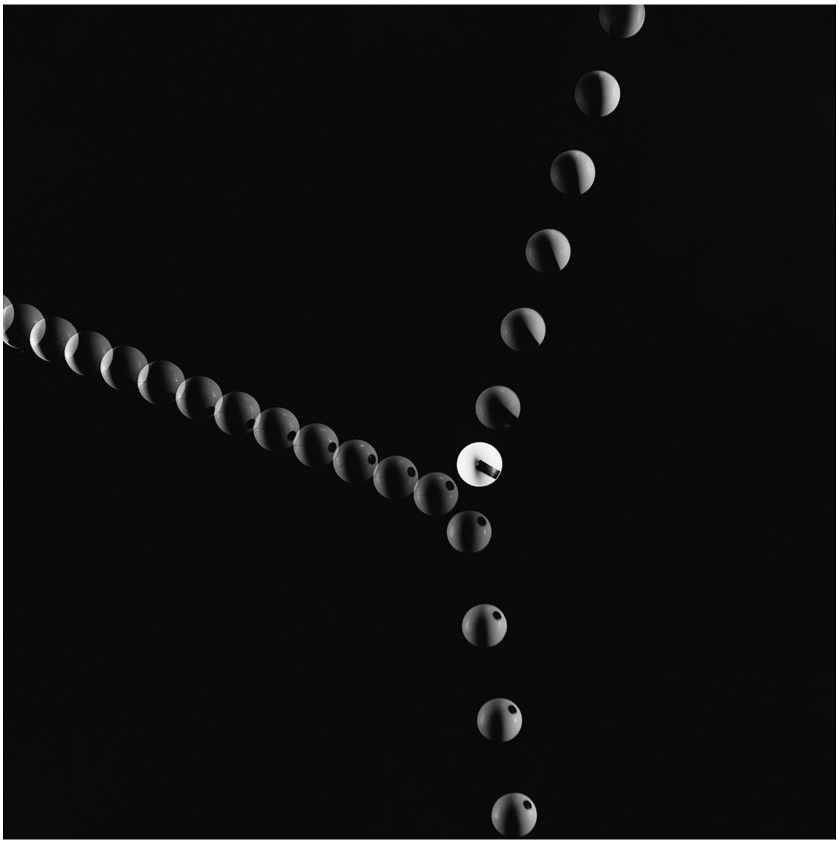


Fig. 1.1 A multi-flash photograph of a billiard ball collision. The motions of the balls can be calculated using Newton's laws but we have a good feel for what will happen from watching snooker on television or playing ourselves.

the collision of two billiard balls. Perhaps the most spectacular application of classical mechanics is in the exploration of space. Nowadays, it surprises no one that the astronaut and the space shuttle float side by side and neither falls dramatically to Earth. A hundred years ago it was not so 'obvious', and in Jules Verne's famous story *A Trip Around The Moon* the passengers of the spacecraft were amazed to find the body of a dog that died on takeoff, and which they had jettisoned outside the craft, floating side by side with them all the way to the Moon. Today, you may not know how Newton's theory works in detail but you can see that it works. It is part of our daily experience.

All this brings us to the problem most of us have in coming to terms with 'quantum mechanics'. It is just this. At the very small distances involved in the study of atoms and molecules, things do *not* behave in a familiar way. Classical mechanics is inadequate and an entirely new explanation is needed. Quantum mechanics is that new explanation, and it is cunningly constructed so that it not only works in the quantum realm of very-short length scales, but also so that, for larger distances, its predictions are identical with those of Newton. An atom is a typical quantum thing – it cannot be understood from the standpoint of classical physics. One popular

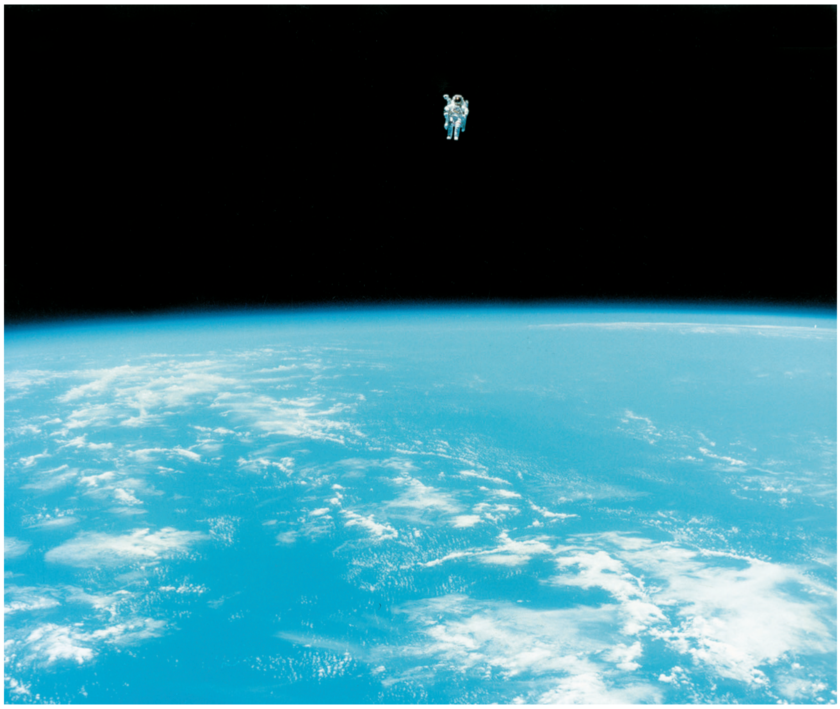


Fig. 1.2 Astronaut Bruce McCandless floats in space during the first untethered space walk on February 7th, 1984. The astronaut is essentially an independent spacecraft in orbit near the shuttle. McCandless commented 'Well that may have been one small step for Neil [Armstrong] but it's a heck of a big leap for me!'

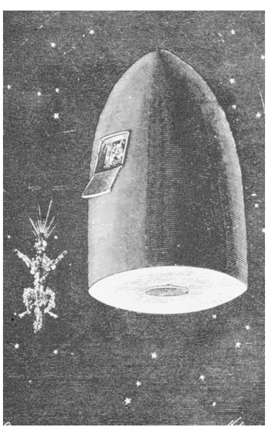


Fig. 1.3 In the story by Jules Verne *A Trip Around The Moon*, published in 1865, the dog 'Satellite' died on take-off and was jettisoned from the space-ship. Much to the surprise of the occupants the dog's body floated along with them all the way to the Moon!

visualization of an atom imagines electrons orbiting the nucleus of the atom much in the way planets orbit the Sun in the solar system. In fact, for negatively charged electrons in orbit round a positively charged nucleus, this simple model is unstable! According to classical physics the electrons would spiral into the centre and the atom would collapse. This nice and comforting model of the atom cannot account for even the existence of real atoms, let alone predict their expected behaviour. It is important to be aware at the outset that there is *no* simple picture that can accurately describe the behaviour of electrons in atoms. This is the first hurdle faced by the newcomer to the quantum domain: the inescapable and unpalatable fact that the behaviour of quantum objects is totally unlike anything you have ever seen.

How can we convince you that quantum mechanics is both necessary and useful? Well, a physicist, just like a good detective, sifts through the evidence and remembers the old maxim of Sherlock Holmes that 'when you have excluded the impossible, whatever remains, however improbable, must be the truth'. Nonetheless, it was only with much reluctance that twentieth-century physicists became convinced that the whole magnificent edifice of classical physics was not 'almost right' for describing the behaviour of atoms, but had, instead, to be radically rebuilt. Nowhere, was the confusion generated by this painful realization more evident than in their attempts to understand the nature of light.

Fig. 1.4 The interference pattern produced by two vibrating sources in water.

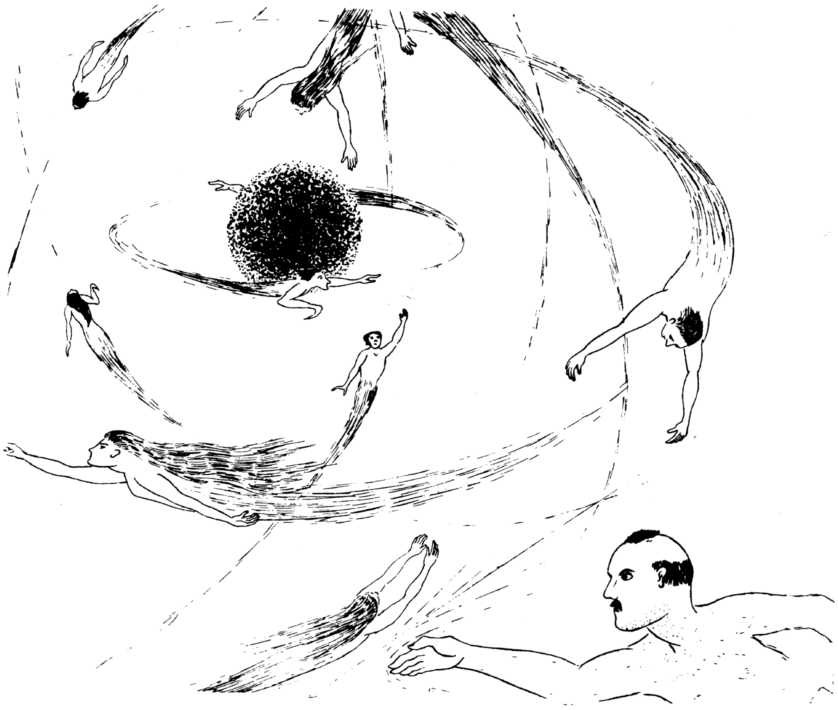
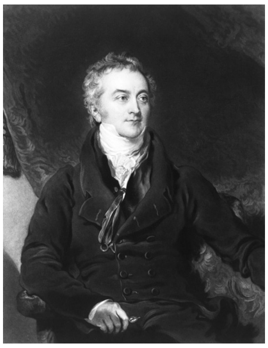


Fig. 1.5 George Gamow's rather whimsical view of the planetary model of the atom in *Mr Tompkins Explores The Atom.*



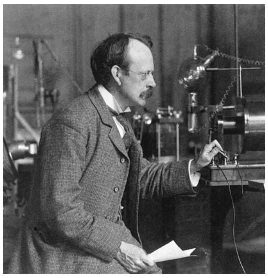
Thomas Young (1773–1829) was an infant prodigy who could read at the age of two. During his youth he learnt to speak a dozen languages. He is best remembered for his work on vision and for establishing the wave theory of light. However, he was also the first to make progress on deciphering the hieroglyphic script of the ancient Egyptians.

Light and quantum mechanics

In the seventeenth century, Isaac Newton suggested that light should be regarded as a stream of particles, rather like bullets from a machine gun. Such was Newton's reputation that this view persisted, apart from some isolated pockets of opposition, until the nineteenth century. It was then that Thomas Young and others conclusively showed that the particle picture of light must be wrong. Instead, they favoured the idea that light was a kind of wave motion. One property of waves that is familiar to us is that of 'interference', to use the physicists' term for what happens when two waves collide. For example, in Fig. 1.4 we show the 'interference' patterns produced by two sources of water waves on the surface of the water. Using his famous 'double-slit' apparatus to make two sources of light, Thomas Young had observed similar interference patterns using light.

Alas, physicists were not able to congratulate themselves for long. Experiments at the end of the nineteenth century revealed effects that were inexplicable by a wave theory of light. The most famous of such experiments concerns the so-called 'photo-electric' effect. Ultraviolet light shone onto a negatively charged metal caused it to lose its charge, while shining visible light on the metal had no effect. This puzzle was first explained by Albert Einstein in the same year that he invented the 'theory of relativity' for which he later became famous. His explanation of the photo-electric effect resurrected the particle view of light. The discharging of the metal was caused by electrons being knocked out of the metal by light energy concentrated into individual little 'bundles' of energy, which we now call 'photons'. According to Einstein's theory, ultraviolet photons have more energy than visible-light ones, and so no matter how much visible light you shine on the metal, none of the photons has enough energy to kick out an electron.

After several decades of confusion in physics, a way out of this dilemma was found in the 1920s with the emergence of quantum mechanics, pioneered by physicists such as Heisenberg, Schrödinger and Dirac. This theory is able to provide a successful explanation of the paradoxical nature of light, atoms and much else besides. But there is a price to pay for this success. We must abandon all hope of being able to describe the motion of things at atomic scales in terms of everyday concepts like waves or particles. A 'photon' does not behave like anything anyone has ever seen. This does not, however, mean that quantum mechanics is full of vague ideas and lacks predictive power. On the contrary, quantum mechanics is the only theory capable of making definite and successful predictions for systems of atomic sizes or smaller, in much the same way that classical mechanics makes predictions for the behaviour of billiard balls, rockets and planets. The difficulty with quantum things such as the photon is that, unlike billiard balls, their motion cannot be visualized in any accurate pictorial way. All we can do is summarize our lack of a picture by saying that a photon behaves in an essentially quantum mechanical way.



J. J. Thomson (1856–1940) measured the charge-to-mass ratio of the electron thus establishing it as a new elementary particle. He was awarded the Nobel Prize in 1906.

There is one sense in which Nature has been kind to us. Viewed from the perspective of classical physics, photons and electrons are very different kinds of objects. Remarkably, in the quantum domain both photons and electrons, and indeed all quantum objects, behave in the same strange quantum mechanical way. This is at least some compensation for our inability to picture quantum things! There is a curious little irony in the history of our attempts to understand the nature of electrons. In 1897 J. J. Thomson measured the charge-to-mass ratio of the electron and established the electron as a new elementary particle of Nature. Thirty years later, his son, G. P. Thomson, and also Davisson and Germer in the USA, performed a beautiful series of experiments that conclusively revealed that electrons behave like waves. The historian Max Jammer wrote: ‘One may feel inclined to say that Thomson, the father, was awarded the Nobel Prize for having shown that the electron is a particle, and Thomson, the son, for having shown that the electron is a wave’.

Our intention in this book is to impress even the most skeptical reader with the enormous range and diversity of the successful predictions of quantum mechanics. The apparently absurd ideas of de Broglie, Schrödinger and Heisenberg have now led to whole new technologies whose very existence depends on the discoveries of these pioneers of quantum mechanics. The modern electronics industry, with its silicon chip technology, is all based on the quantum theory of materials called semiconductors. Likewise, all the multitude of applications of lasers are possible only because of our understanding, at the fundamental quantum level, of a mechanism for radiation of light from atoms first identified by Einstein in 1916. Moreover, understanding how large numbers of quantum objects behave when packed tightly together leads to an understanding of all the different types of matter ranging from ‘superconductors’ to ‘neutron stars’. In addition, although originally invented to solve fundamental problems concerned with the existence of atoms, quantum mechanics was found to apply with equal success to the tiny nucleus at the heart of the atom, and this has led to an understanding of radioactivity and nuclear reactions. As everyone knows, this has been a mixed blessing. Not only do we now know what makes the stars shine, but we also know how to destroy all of civilization with the awesome power of nuclear weapons.

Before we can explain how quantum mechanics made all these things possible, we must first attempt to describe the strange quantum mechanical behaviour of objects at atomic distance scales. This task is clearly difficult given the absence of any accurate analogy for the mathematical description of quantum behaviour. However, we can make progress if we use a mixture of analogy and contrast. Young’s original ‘double-slit’ experiment used a screen with two slits in it to make two sources of light which could interfere and produce his famous ‘interference fringes’ – alternating light and dark lines (Fig. 1.6). We shall describe the results of similar ‘double-slit’ experiments carried out using bullets, water waves and electrons. By

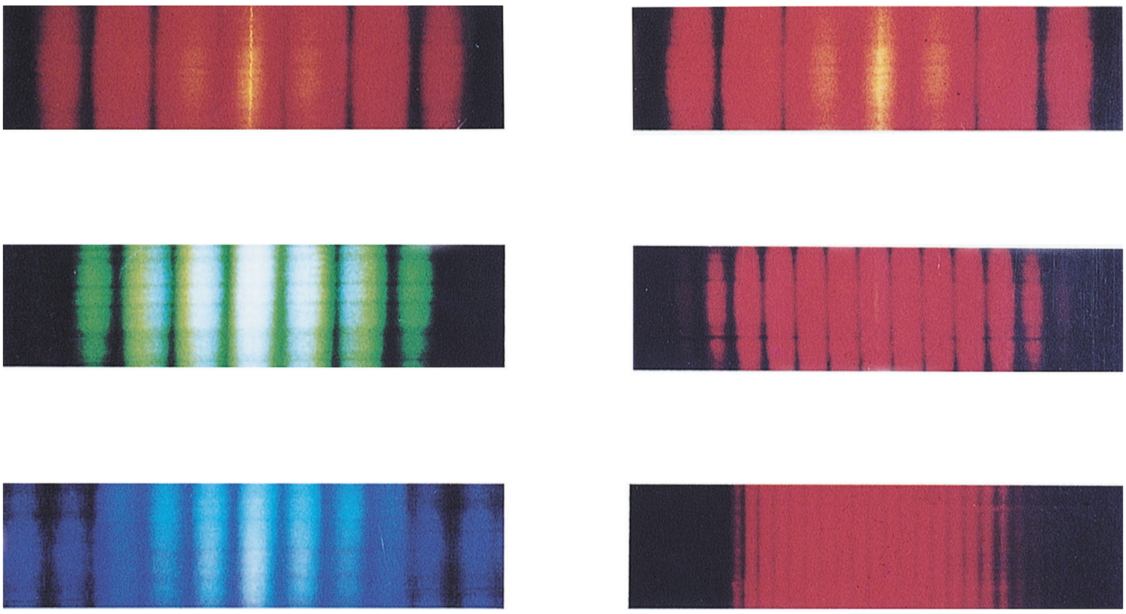


Fig. 1.6 Double-slit interference patterns for light, usually taken as demonstrating that light is a wave motion. In the left-hand pictures, as the wavelength of the light is decreased and the colour changes from red to blue, the interference fringes become closer together. On the right, for red light, the decrease in the fringe separation is caused by increasing the separation of the slits.

comparing and contrasting the results obtained with the three different materials we shall be able to give you some idea of the essential features of quantum mechanical behaviour. Quantum mechanics textbooks contain detailed discussion of many types of experiment, but this double-slit experiment is sufficient to reveal all the mystery of quantum mechanics. All of the problems and paradoxes of quantum physics can be demonstrated in this single experiment.

A word of warning before we begin. To avoid running into a frustrating psychological cul-de-sac, try to be content with mere acceptance of the observed experimental facts. Try not to ask the question 'but how can it be like that?' As Richard Feynman says 'nobody understands quantum mechanics'. All we can give you is an account of the way Nature appears to work. Nobody knows more than that. Only **after** we have convinced you that quantum mechanics really works will we examine what quantum mechanics has to say about the very nature of reality, with a discussion about Schrödinger's cat, Einstein and dice.

The double-slit experiment

This section may be rather hard going first time through. If so, just glance at the pictures and pass on quickly to the next chapter!

With bullets

Source: a wobbly machine gun that, as it fires, spreads the bullets out into a cone, all with the same speed but with random directions.
Screen: armour plate with two parallel slits in it.
Detector: small boxes of sand to collect the bullets.
Results: the gun fires at a fixed rate and we can count the number of bullets that arrive in any given box in a given period of time. The bullets that go through the slits can either go straight through or else bounce off one of the edges, but must always end up in one of the boxes. The bullets we are using are made of a tough enough metal so that they never break up – we can never have half a bullet in a box. Moreover, no two bullets ever arrive at the same time – we have only one gun, and each bullet is a single identifiable 'lump'.

If we let the experiment run for an hour and then count the bullets in each of the boxes, we can see how the 'probability of arrival' of a bullet varies with the position of the detector box. The total number of bullets arriving at any given position is clearly the sum of the number of bullets going through slit 1 plus the number going through slit 2. How this 'probability of arrival' varies with position of the sand boxes is shown in Fig. 1.7. We shall label this result P_{12} – the probability of arrival of bullets when both slits are open. We also show in Fig. 1.7 the results obtained with slit 2 closed, which we call P_1, and those obtained with slit 1 closed, which we call P_2. Looking at the figures, it is evident that the curve labelled P_{12} is obtained by adding curves P_1 and P_2. We can write this mathematically as the equation:

$$P_{12} = P_1 + P_2$$

For reasons that will become apparent in a moment, we call this result the case of *no interference.*

With water waves

Source: a stone dropped into a large pool of water.
Screen: a jetty with two gaps in it.
Detector: a line of small floating buoys whose jiggling up and down gives a measure of the amount of energy of the wave at that position.
Results: Ripples spread out from the source and reach the jetty. On the far side of the jetty ripples spread out from each of the gaps. At the detector, the resulting disturbance of the water is given by the sum of the disturbances of the ripples coming from both gaps. As we look along the line of buoys, there will be some places where the crest of a wave from slit 1 coincides with the arrival of a crest from slit 2, resulting in

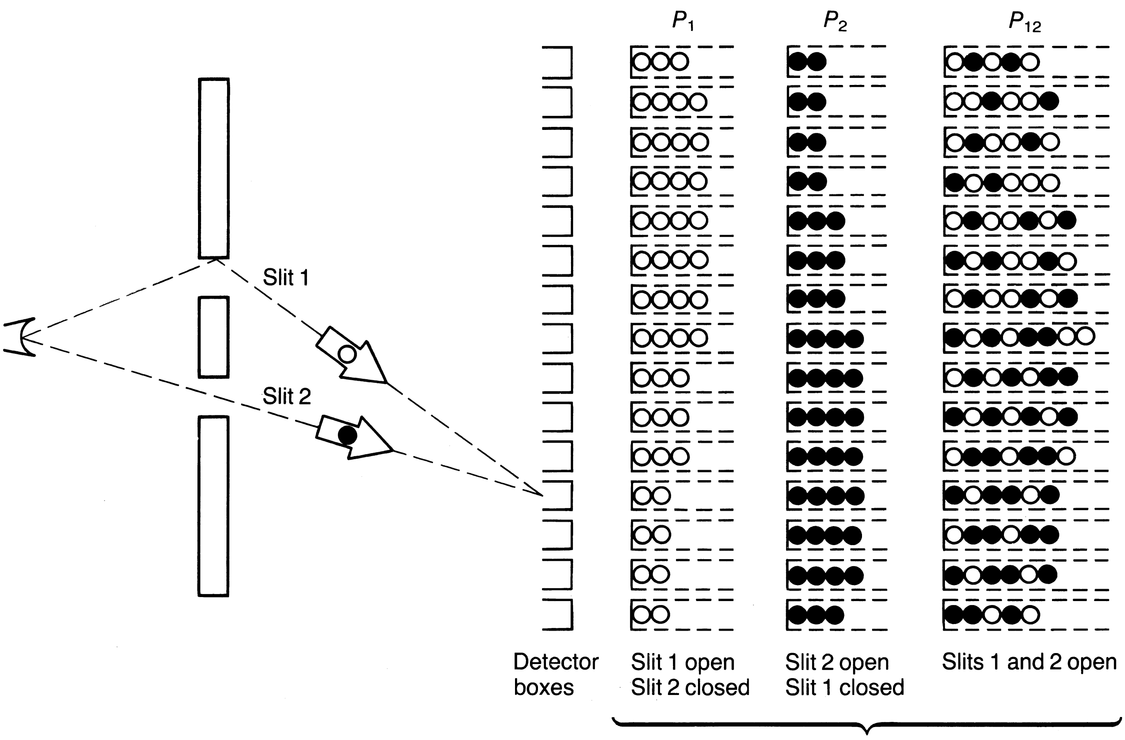
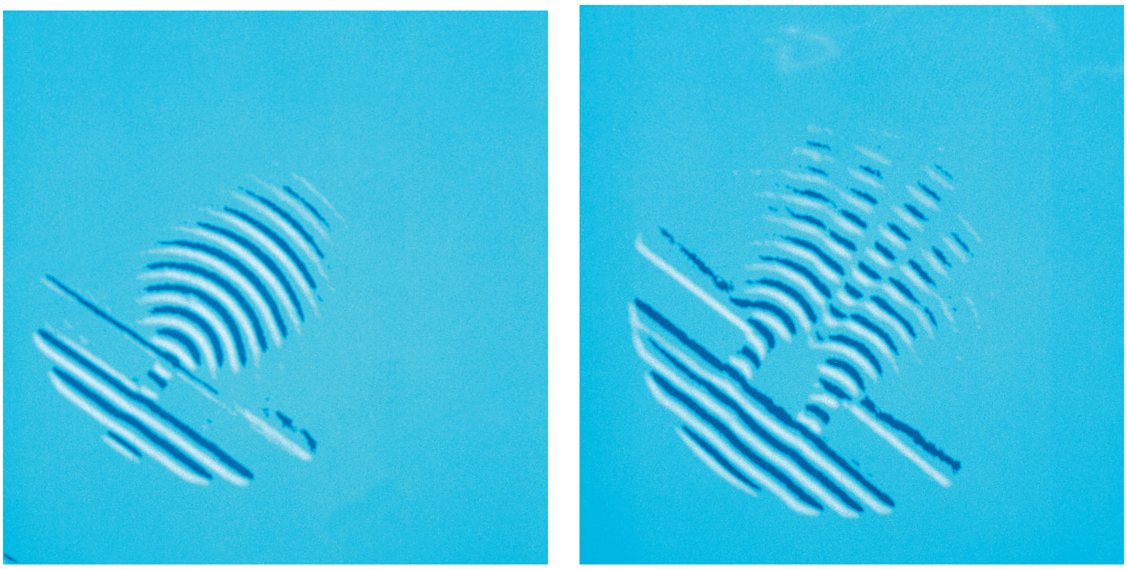


Fig. 1.7 A double-slit experiment with bullets. The experimental set-up is shown on the left of the figure and the results of three different experiments indicated on the right. We have shown bullets that pass through slit 1 as open circles and bullets through slit 2 as black circles. The column labelled P_1 shows the distribution of bullets arriving at the detector boxes when slit 2 is closed and only slit 1 is open. Column P_2 shows a similar distribution obtained with slit 1 closed and slit 2 open. As can be seen, the maximum number of bullets appears in the boxes directly in line with the slit that is left open. The result obtained with both slits open is shown in the column labelled P_{12}. It is now a matter of chance through which slit a bullet will come and this is shown by the scrambled mixture of black and white bullets collected in each box. The important point to notice is that the total obtained in each box when both slits are open is just the sum of the numbers obtained when only one or other of the slits is open. This is obvious in the case of bullets since we know that bullets must pass through one of the slits to reach the detector boxes.

a very large up-and-down motion for the buoy. At other places, a crest from one slit will coincide with a trough from the other so there will be no movement of the buoy at that position. At yet other places, the motion of the buoys will be somewhere between these two extremes. For water waves, it is certainly plausible that the energy of a wave at any given position is related to how big the waves are at that point. In fact, it can be shown that the energy of a wave depends on the square of the maximum height of the wave. Let us call the amount of energy arriving per second the 'intensity' and label this by the symbol I. If we label the



(*a*) (*b*)

Fig. 1.8 Wave patterns with water waves. (*a*) A wave spreading out from a single slit; (*b*) the interference obtained with two slits.

maximum height of the wave by h, we can write the relation between I and h as the following equation:

$$I = h^2$$

$$\text{intensity} = \text{height squared}$$

In contrast to our experiment with bullets, we see that the energy of the waves does not arrive at the detector in definite-sized lumps. There, bullets only arrived at one particular position at one particular time. Here, since the height of the resulting wave at the detector varies smoothly from zero up to some maximum value as we move along the detector, we see that the energy of the original wave is spread out. The curve showing how the intensity varies with position along the detector is shown in Fig. 1.9. Since this is the intensity obtained with both slits open, we shall call this curve I_{12}. This intensity pattern has a very simple mathematical explanation. The total disturbance of the water at any position along the detector is given by the sum of the disturbances caused by the waves from slit 1 and slit 2. If we label the height of the wave from slit 1 by h_1, the height from slit 2 by h_2, and the total height obtained when both slits are open as h_{12},we can write this result as the equation:

$$h_{12} = h_1 + h_2$$

Remember that each of these heights can be positive or negative depending on whether the corresponding wave disturbance raises or lowers the