

## Author Index

- 
- Abel 91  
 Aczél 54  
 Alekhine 80  
 Alladi 304  
 Artin 34  
 Atkinson 147
- Bachmann 305  
 Bateman 154  
 Bergh 13  
 Bombieri xiv, xvii, 46, 194, 273, 274  
 Borel 128  
 Brahms 320  
 Bruggeman 13  
 Brun 39  
 Buhler 318
- Carathéodory 128  
 Cauchy 9  
 Chebyshev xi, 306  
 Cherwell 147  
 Chudakov 130  
 Conway 13  
 Cramer 313
- Daboussi 110, 114, 148, 149, 305, 306  
 Davenport xi, xiv, 37, 38, 44, 46, 195  
 Delange xvi, 67, 84, 85, 110, 148, 305  
 de la Vallée Poussin 127  
 Deligne 152, 154, 156  
 Dirichlet xiv, 5, 9, 10  
 Duke 9
- Elliott xiv, xvii, 17, 50, 64, 67, 136, 137, 151, 154, 155, 157, 191, 234, 269, 273, 274, 284, 301, 305, 306, 313, 314, 316, 319  
 Eratosthenes 39
- Erdős xvi, 50, 84, 115, 129, 133, 137, 190, 191, 312, 313  
 Euclid xv  
 Euler xiv, 10
- Forti 203  
 Friedlander 274  
 Friedrichs 7
- Gallagher xiv, 46, 256  
 Granville 274
- Hadamard xiii, 127  
 Halász xvi, 84, 85, 92, 100, 110, 116, 121, 129, 134, 139, 144, 146, 150, 160, 269  
 Halberstam xiv, xvii, 37, 38, 39, 42, 224, 227, 273, 274  
 Hardy x, xi, 12, 64, 93, 151, 191  
 Heath-Brown 5  
 Hecke 196, 203  
 Heine xiii  
 Hewitt 305  
 Hildebrand 76, 78, 83, 189, 262, 274, 284, 301, 309, 310, 316  
 Hyers 54, 78, 306
- Ingham xiv  
 Iwaniec 9, 274
- Kátai xviii, 67, 191, 304  
 Kubilius 319  
 Kubilius 50, 189, 319  
 Kuznetsov 13
- Lakatos xiii, xv, 127, 128  
 Lee 189  
 Lehmer 157, 158  
 Lenstra 318  
 Levin 314  
 Lévy 313

- Linnik xiv, 7, 195, 204  
 Littlewood 93  
 Löfström 13
- Maass 13  
 Maier 274  
 Manstavičius 234  
 Mauclaire 147  
 Mertens 226  
 Möbius 46  
 Montgomery 38, 305, 306  
 Mordell 12, 151, 156  
 Moreno 155  
 Murty 154, 157
- Nachbin 13  
 Narkiewicz 64
- Pappus of Alexandria 83  
 Parseval 45  
 Pomerance 318  
 Pontryagin 5  
 Prachar xi, 14, 44, 53, 195, 197
- Rainville 183  
 Ramanujan 12, 64, 151, 152, 156, 191  
 Rankin 151, 152, 155, 157  
 Rassias 306  
 Rényi xiv, 37, 44, 110, 220  
 Richert 39, 42, 130, 224, 227  
 Riemann 10, 68, 71, 127, 306  
 Rosenthal xvii, 214, 232, 234  
 Roth xiv  
 Rudin 13  
 Ruzsa xvi, xvii, 53, 54, 160, 189, 190, 304
- Sarnak 13  
 Sato 154, 157
- Schmidt 154  
 Schwarz, W. 110, 305, 306  
 Selberg 13, 39, 42, 46, 129, 224, 260, 274  
 Serre 154, 158  
 Shahidi 155  
 Shiu 308  
 Siebert 194  
 Spilker 83, 110, 305, 306  
 Steinhaus 134, 136
- Tate 154, 157  
 Tenenbaum 311  
 Timofeev 314  
 Titchmarsh 13, 14, 71  
 Turán 319
- Ulam 54
- van Kampen 5  
 Vaughan 38, 203, 305  
 Venkov 13  
 Vinogradov xvii  
 Viola 203
- Wiener 7, 128, 205, 206, 262, 305  
 Williamson 305  
 Wintner 115, 137, 312  
 Wintner 84  
 Wirsing xvi, xvii, xviii, 67, 84, 85, 110, 115, 120, 121, 122, 125, 128, 129, 139, 143, 190, 191, 310  
 Wolke 194  
 Wright x, xi
- Yosida 13, 170

## Subject Index

- Abel's theorem, analogue of 91, 98  
 action of a function, notation 1  
 additive function ix, 16, 49, 64, 67  
 additive function, difference 3, 47, 50, 66, 151, 190–192, 198, 204, Chapter 31, Chapter 32, 303–304  
 additive function, discrete derivative 3, 47, 50, 64, 190, 192, 198, 204, Chapter 31, Chapter 32  
 additive function, Kac' simulation of 50  
 additive function, normal order 64  
 additive functions, finitely distributed xvi, Chapter 16, 140, 150, 190  
 adjoint map/operator 2, 199  
 aesthetic 147, 203  
 algorithm, euclidean 288  
 almost periodic function 112, 148–149, 304–305  
 almost periodic function, Daboussi Theorem 114, 149  
 almost prime xiv  
 American Math. Soc. annual general meeting 154–155  
*Analysis*, method of ancient Greece xv  
 analytic continuation 7, 9, 10, 13–14, 70–75, 91, 152, 155, 306  
 analytic number theory, nature of xiii  
 approximation, Newton 295  
 arithmetic functions on short intervals 306–315  
 arithmetical propositions xiv  
 automorphic functions 11
- background philosophy Chapter 1, 27, 44, 79–80  
 Banach space 3  
 basis 2
- Bessel's inequality 7  
 bilinear form 4, 113, 122, 206, 305  
 Bohr–Besicovitch space 113–114  
 Bombieri–Vinogradov theorem xvii, 67, 193–196, 204, 263  
 boundary coordinates 66  
 bounded variation 14, 93  
 Brun sieve 39  
 Brun–Titchmarsh theorem 194, 227
- $\mathbb{C}$ -divisible 36  
 canons of stable duality 100  
 Cauchy, approximate functional equation 53–54, 62–63, 76  
 Cauchy functional equation 54, 134  
 Cauchy law 313  
 celestial mechanics 66  
 Cesàro summation 262, 269  
 characteristic function 8, 300, 315  
 characters 5–6, 111, 198, 203  
 characters, Dirichlet 9, 44–47, 112–114, 120, 148–149, 194–197, 200–203, 205–210, 245, 255–260, 262, 317  
 characters, Dirichlet, induced 44  
 characters, Dirichlet, primitive 44–46, 149, 196, 207, 245  
 characters, generalised 146  
 characters on  $Q^*$  9, 111–112, 118, 124, 205  
 characters on  $\mathbb{R}$  7  
 characters on  $\mathbb{R}^*$  10  
 characters on  $\mathbb{R}/\mathbb{Z}$  6  
 characters on  $\mathbb{Z}$  111–113  
 characters on an object 35–36  
 characters on finite abelian groups 34–35, 195–196, 198–200  
 Chebyshev inequality in number theory 20, 128–129, 226  
 Chebyshev inequality in probability 16

- chess 80  
 circle method of Hardy–Littlewood  
 7, 12, 203  
 Class H of Kubilius 157  
 class number of imaginary quadratic  
 field 5, 11  
 computation through the adjoint 2  
 computation through the dual 1  
 concentration function 160, 189  
 conics 1  
 conjecture(s) of Elliott 154, 157–158,  
 204, 272–274, 315  
 conjecture of Elliott and Halberstam  
 xvii, 273–274  
 conjecture of Erdős 190  
 conjecture(s) of Kátai xviii, 67, 191  
 conjecture of Lehmer 157–158  
 conjecture of Narkiewicz 64  
 conjectures of Ramanujan 151–152,  
 156  
 conjecture of Riemann 222  
 conjecture, Riemann–Piltz 197, 223  
 conjecture of Serre (Sato–Tate) 154,  
 157  
 conjecture of Wirsing 121  
 connected topological space 57  
 continuity, approximate/asymptotic  
 52, 62–64, 94, 116, 279, 310  
 converse and duality xv  
 convolution 14–15  
 convolution, Dirichlet 43, 95, 101,  
 103, 105–106, 108, 125, 142, 195,  
 197, 206, 209  
 convolution of distribution functions  
 308  
 correlation of multiplicative functions  
 xvi, 67, 214, 297–301  
 Cramer’s rule 178  
 Cramer’s theorem 313  
 cross ratio 73  
 cubic forms xiv  
 cusp forms 12  
 cyclotomic polynomials, coefficients  
 of 305  
  
 Daboussi theorem on almost periodic  
 functions 114, 149  
  
*Data*, of Euclid xv  
 Davenport dictum 195  
 decompositions, convolution 313  
 density, logarithmic 149  
 derivative, discrete xvi, 47, 50, 67,  
 190, 192, 198, 204, Chapter 31,  
 Chapter 32  
 determinant, van der Monde 178,  
 180, 210  
 Deuring–Heilbronn phenomenon 11  
 dichotomy 85, 120–121  
 dictum, Davenport 195  
 dictum, Hecke 196, 203  
 difference operator 289–293  
 differential equation, approximate  
 66, 69, 77  
 differential equation, Riccati 70–75  
 differential equation, stability of  
 solutions of 296–297  
 differential equations over the  
 complex plane 73  
 Dirac function 135  
 Dirichlet characters 9,44–47,  
 112–114, 120, 148–149, 1194–197,  
 200–203, 205–210, 245, 255–260,  
 262, 317  
 Dirichlet convolution 43, 95, 101,  
 103, 105–106, 108, 125, 142, 195,  
 197, 206, 209  
 Dirichlet series 10–11, 87, 91–92,  
 120, 122–124  
 Dirichlet’s function,  $\tau(n)$  151  
 disc, Gershgorin 36, 196  
 discoveries, methodological xiii  
 distribution, Normal 50, 157–158,  
 313–314  
 distribution function 8, 312–314, 317  
 distribution function, tail of 8  
 distribution functions, convolution of  
 308  
 divisibility and independence 16, 50  
 divisible objects 35–36  
 divisor function  $\tau(n)$  of Dirichlet 151  
 dollar, American 151  
 Doppeltgänger xiii  
 dual map/operator 1, 3–4, 27, 214,  
 223, *see also* dual spaces

- dual objects 35–36  
 dual space(s) 1, 3, 27–29, 200–203,  
   212, 219–220, 223, 230, 236–237,  
   251, 265–266, 271  
 duality xiv, Chapter 0  
 duality and converse xv  
 duality principle xviii, 3–5, 10, 30,  
   36–38, 44, 48, 209, Chapter 25,  
   261  
 duality theorem of Pontryagin 5–6,  
   35, 199  
  
 eigenspace 2  
 eigenvalue 1–2  
 eigenvalue, approximate 163, 179,  
   189  
 eigenvector 1–2  
 eigenvector, approximate 80, 163,  
   170, 178, 189, 295  
 eigenvectors, simultaneous 2  
 Eisenstein series 13  
 Elliott conjecture(s) 154, 157–158,  
   204, 272–274, 315  
 Elliott–Halberstam conjecture xvii,  
   273–274  
 embedding, isometric 27  
 Eratosthenes sieve 39  
 Erdős' conjecture 190  
 Erdős' generosity 191  
 Erdős–Kac theorem 50  
 Erdős–Wintner theorem 137, 301  
 Erdős–Wintner theorem, localised  
   312  
 euclidean algorithm 288  
 euclidean norms 3  
 euclidean triangle 30  
 Euler product 50, 87–95, 100,  
   103–104, 118, 120, 124, 141, 143,  
   153, 155, 305  
 exercise routine, physical 145  
 expectation/mean in probability 62,  
   215, 217, 228–229, Chapter 26,  
   249–250, 254  
 exponential function, characterisation  
   of 69–75  
 extra divisible group 36  
  
 factorisation, computer 317  
 Fellowship, John Simon Guggenheim  
   66, 151  
 fixed point 72  
 forms, cubic xiv  
 forms, cusp 12  
 forms, holomorphic modular 12  
 forms, Maass 13  
 formula of Selberg 128–129  
 Fourier analysis (mod 1) 5–7  
 Fourier analysis on  $\mathbb{R}^*$  10  
 Fourier transform 6, 199, 320  
 Fourier transform on  $\mathbb{R}$  4–8, 14  
 Friedrichs' theorem 7  
 function, almost periodic 112,  
   148–149, 304–305  
 function, almost periodic, Daboussi  
   theorem 114, 149  
 function of Dirac type 135  
 function, Liouville 209, 299–300  
 function, slowly-oscillating 84, 121  
 functional equation 54  
 functional equation, approximate 16,  
   48–49, Chapter 6, Chapter 7, 80,  
   115, 117, 122, 161–163, 306–307  
 functional equation of Cauchy 54,  
   134  
 functional equation of Cauchy,  
   approximate 53–54, 62–63, 76  
 fundamental domain 11  
  
 Gauss sums 9, 44  
 Gelfand representation 305  
 generosity of Erdős 191  
 geometric procedure 100  
 geometry, Greek xv, 83  
 geometry, projective xiv–xv, 1, 51,  
   146  
 Gershgorin disc 36, 196  
 global to local 68–69  
 graduate student xiv  
 grammar school student 51  
 ground field 2  
 group characters 5  
 group, dual 5, 195, 210  
 group, extra divisible 36

- group, locally compact abelian 5–6,  
 13, 195, 210
- Haar measure 6, 13  
 Haar measure on  $\mathbb{R}^*$  62, 77–78  
 Hahn–Banach theorem 35  
 Halász’ method 91–92  
 Hamilton–Jacobi equation 66  
 Hardy–Littlewood circle method 7,  
 12, 203  
 Hardy–Ramanujan problem 64  
 Hecke dictum 196, 203  
 Hecke operator 12–13  
 high school student 51  
 Hilbert space xvi, 3, 26, 43–44, 160,  
 198, 295, 308  
 Hildebrand’s theorem 309  
 homomorphism, approximate 295,  
 317
- imaginary quadratic field, class  
 number of 5, 11
- independence and divisibility 16, 50
- inequality, maximal 204, 209,  
 Chapter 29, 306
- inequality, maximal gap Chapter 29
- inequality of Chebyshev in number  
 theory 20, 128–129, 226
- inequality of Chebyshev in  
 probability 16
- inequality of Gallagher 256
- inequality of Manstavičius 234
- inequality of Rosenthal xvii,  
 214–215, 229, Chapter 26, 249
- inequality, Young–Hausdorff 198–200
- inner product 2
- inner product of Petersson 12–13
- integral functions, Hadamard’s  
 theory of 127
- interpolation theorem of  
 Riesz–Thorin xvii, 199, 217–218,  
 247, 253, 268
- inverse, Dirichlet convolution 103,  
 105
- inversion 43, 64
- invertibility theorem of Hewitt and  
 Williamson 305
- invertibility theorem of Wiener 305
- isometric embedding 27
- isometry 199
- isometry, approximate xvi, 308
- iteration 49, 65
- Jacobi polynomials 183–184
- Kátai conjecture(s) xvii, 67, 191
- Kloosterman sums 284, 304
- Kronecker 206–207
- L-series of Dirichlet 10, 120
- L-series for  $\mathbb{Q}^*$  11, 124
- Laplace transform 8, 14
- Laplacian, hyperbolic 12–13
- large numbers, law of 62
- Large Sieve xiv, 3, 7, 30, Chapter 4,  
 194, 197, 204, 207, 212–213, 220,  
 Chapter 29
- Large Sieve, fractional power xvii,  
 47, 203, Chapter 25, 237, 243–244,  
 251, 265–266, 268
- Large Sieve, lower bound 37–38
- Large Sieve maximal gap 262
- law, improper 314–315
- law, infinitely divisible 313
- law, Cauchy 313
- law, Normal 50, 157–158, 313–314
- law, Poisson 313
- laws, probability, decomposition of  
 313
- lecture, Bordeaux 313
- lecture, Cambridge 313
- Lehmer conjecture 157–158
- line conic 1
- line envelope 1
- linear function, characterisation of  
 68–69
- Liouville function 209, 299–300
- Lipschitz condition 187, 268
- loop argument xiv, 27–29, 100
- loop diagrams 29, 219–220, 230
- Maass forms 13
- map, collapse 235–236, 251, 266
- map, inflation 236

- matrix, complex conjugate transpose 2–4  
 matrix, transpose 1, 3–4  
 mean, local 48–49, 86–87, 100, 115–117, 122–124, 129–131, 140–146, 150, 161, 309–312  
 mean value, asymptotic 84, 120–121, 305  
 mean value, non-zero 85, 305  
 mean value, zero 115–116, 120–121, 146–147  
 measure, atomic 173  
 measures, projection valued 3  
 measures, weak convergence of 136–137, 173, 175, 184–189, 301, 312–317  
 mechanics, celestial 66  
 mechanics, stability in 63, 66  
 Mellin transform 10, 14, 91, 204, 206, 239, 257–260, 262, 266, 268–269  
 methodological discoveries xiii  
 metric on  $\mathbb{R}/\mathbb{Z}$  56, 66, 111  
 metrics on  $(Q^*)^\wedge$  111–112, 118  
 modular forms, cusp 12  
 modular forms, holomorphic 12  
 modular function coefficient  $\tau(n)$  of Ramanujan xvi, 12–13, Chapter 19, 304, 309  
 monodromy theorem 13–14  
 multiplicative function, correlation of xvi, 67, 214, 297–301, 315–317  
 multiplicative function, upper bound 19–20, 115, 117, 141–142, 308, 311–312  
  
 Narkiewicz' conjecture 64  
 Newton approximation 295  
 Normal distribution 50, 157–158, 313–314  
 norms, euclidean 3  
 norms, operator xv, Chapter 23  
  
 operator, adjoint 2, 199  
 operator, compact 170  
 operator, difference 289–293  
 operator, dual 1, 3–4, 27, 214, 223, *see also* dual spaces  
 operator, dual, support of 197  
 operator, self-adjoint 3, 6, 13, 163–164, 169–170, 191, 196, 220, 230–231, 295  
 operator, shift 191–193, 239, 268, Chapter 30, 277–278, 284, 286–294, 298, 300, 317  
 operator, spectral radius of xiv, 4, 172, 189, 320  
 operator and tangent xv  
 operator norms xv, Chapter 23  
 operators, perturbation of 63  
 operators, ring of 286–287, 317  
 orthogonal basis 2  
 oscillation, slow 94–95, 99–100, 116–117, 140–141, 143–146, 149, 279  
  
 Pappus of Alexandria 83  
 Parseval's relation 7, 35  
 partition function 12  
 period, finite 5  
 Petersson inner product 12–13  
 phenomenon, Deuring–Heilbronn 11  
 phenomenon, Wiener 7, 206, 209–210, 262, 268  
 Ph.D. thesis of Elliott 1  
 Ph.D. thesis of Richert 129  
 philosophy, background Chapter 1, 24, 27, 44, 51, 79–80, 100  
 philosophy of Alekhine 80  
 physical exercise routine 145  
 physics, partial differential equations in 5  
 Plancherel's relation 7, 35, 124, Chapter 15  
 planetary orbit, perturbation of 66  
 point conic 1  
 point locus 1  
 Poisson law 313  
 Poisson summation 8–9, 36–38, 42  
 Pólya–Vinogradov inequality 46, 201  
 polynomials, Jacobi 183–184  
 Pontryagin duality theorem 5–6, 35, 199

- pound, English 151  
 power residue symbol 112  
 prime ideal theorem 128  
 prime number theorem 49–50, 53,  
 85, 122, Chapter 15, 299, 306  
 prime number theorem, elementary  
 proof 128–129, 131, 306  
 probabilistic number theory,  
 axiomatic treatment 50  
 probability laws, decomposition of  
 313  
 procedure, geometric 100  
 process, acceleration 316  
 product representation by integer/  
 rationals 9, 36, 64, 67, 304, 317  
 product representation by  
 polynomials 317  
 projection valued measures 3  
 projective geometry xiv–xv, 1, 51,  
 146  
  
 $Q$ , freedom of 16  
 $Q^*$ , characters on 9–10, 111–112,  
 118, 124, 205  
 $Q^*$ , dual of 9, 305  
 $Q^*$ , harmonic analysis on 9–10, 209  
 $Q^*$ , L-series for 11, 124  
 $(Q^*)^\wedge$ , metrics on 111–112, 118  
 quadratic reciprocity law 9  
  
 $\mathbb{R}^*$ , characters on 10  
 $\mathbb{R}^*$ , Fourier analysis on 10  
 Ramanujan conjectures 151–152, 156  
 Ramanujan sum 45–47, 305  
 Ramanujan's function,  $\tau(n)$  xvi,  
 12–13, Chapter 19, 304, 309  
 random variable, expectation/mean  
 62, 215, 217, 228–229, Chapter 26,  
 249–250, 254  
 random variable, variance Chapter  
 26  
 random variables 8, 50, 62  
 random variables, independent 8, 50,  
 62, 216, 220, 229, Chapter 26, 249,  
 314, 316–317  
 random variables, uncorrelated 220  
 Rationalists xiii  
  
 rationals, product of 9, 36, 64, 67,  
 304, 317  
 reciprocating action of dualisation  
 214  
 reciprocity law, analytic 68  
 reciprocity theorem 5, 9, 112  
 representation, Gelfand 305  
 rescaling of vectors 28  
 Riccati differential equation 70–75  
 Riemann hypothesis 222  
 Riemann–Piltz conjecture 197, 223  
 Riesz representation 2  
 Riesz–Thorin interpolation theorem  
 xvii, 199, 217–218, 247, 253, 268  
 ring of operators 286  
 root, weak 296  
 Rosenthal's inequality xvii, 214–215,  
 217, 229, Chapter 26, 249  
  
 Sato–Tate conjecture 154, 157  
 Selberg sieve 39–44, 223–226, 263,  
 274, 284  
 Selberg's formula 128–129  
 self-adjoint extension 7  
 self-adjoint operator 3, 6, 13,  
 163–164, 169–170, 191, 196, 220,  
 230–231, 295  
 self-adjoint operators, commuting  
 family 2  
 semigroup 63  
 semigroup, freely generated  
 commutative 31, 317–319  
 Serre conjecture 154, 157  
 shift operator 191–193, 239, 268,  
 Chapter 30, 277–278, 284,  
 286–294, 298, 300, 317  
 Siegel–Walfisz theorem 194, 209, 235  
 Sieve, Large xiv, 3, 7, 30, Chapter 4,  
 194, 197, 204, 207, 212–213, 220,  
 Chapter 29  
 sieve of Brun 39  
 sieve of Eratosthenes 39  
 sieve of Selberg 39–44, 223–226, 263,  
 274, 284  
 slow oscillation 94–95, 99–100,  
 116–117, 140–141, 143–146, 149,  
 279



- slow variation (in sense of Karamata) 108–109, 146  
 smoothing a function 123  
 space, Bohr–Besicovitch 113–114  
 space, connected topological 157  
 space(s), dual 1, 3, 27–29, 200–203, 212, 219–220, 223, 230, 236–237, 251, 265–266, 271  
 spectral decomposition 2–3, 13, 163, 167, 170, 188, 295  
 spectral radius xiv, xvii, 172, 189, 320  
 spectrum, Fourier–Bohr 113–114, 304–305  
 stability xv, 63, 66, 68, 80, 295–297, 306, 320  
 stability in mechanics 63, 66  
 stable dual xiii, 304  
 stable duality, canons of 100  
 Steinhaus’ theorem 134  
 student, graduate xiv  
 student, undergraduate 66  
 student, grammar/high school 51  
 superposition of solutions 5
- $T$ , the operator 162, 199, 278, 280, 320  
 $T$ , the operator, spectrum of 165–167  
 tangent xiv–xv  
 tangent and operator xv  
 tauberian theorem of Hardy and Littlewood 93, 99–100, 102, 143, 155  
 tauberian theorem of Wiener 128  
 thesis of Elliott xiv  
 thesis of Richert 129  
 torsion groups 67  
 transpose of a matrix 1  
 transpose of a matrix, complex conjugate 2  
 Trinity College xiv  
 Turán–Kubilius inequality adjoint, underlying operator 3, 26, 160–161, 320  
 Turán–Kubilius inequality, dual 15, 17, 31, 48, 79–80, 86, 93, 115, 117, 122, 146, 304–305, 307  
 Turán–Kubilius inequality, dual, underlying operator 24, 277, 295  
 Turán–Kubilius inequality (generalised) xv–xvi, 16, 26–27, 31, 38, 48, 82–83, 102, 108–110, 138, 147, 160, 182, 189, 246, 277, 295, 302–304, 307–308  
 Turán–Kubilius inequality, underlying operator 26, 159–161, 246, 277, 295, 320
- Ulam question 54, 295–296  
 undergraduate student 66  
 upper half plane, complex 11
- van der Monde determinant 178, 180, 210  
 variance 159, 169, Chapter 26  
 variation, slow in the sense of Karamata 108–109, 146  
 variation of endpoints 78
- Wiener phenomenon 7, 206, 209–210, 262, 268  
 Wirsing’s conjecture 121  
 Wirsing’s theorem for the difference of additive functions 190
- Young–Hausdorff inequality 198–200
- $\mathbb{Z}$ , characters on 111–113  
 $\mathbb{Z}$ -divisible 35  
 $\mathbb{Z}$ -module 35  
 zeta function, local 152  
 zeta function of Dedekind 128  
 zeta function of Dedekind, analytic continuation of 128  
 zeta function of Riemann, analytic continuation of 72, 118, 127  
 zeta function of Riemann, bound for 125  
 zeta function of Riemann, as extremum 124