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Contents

Preface

This book deals with the symbiotic relationship between

(i) function spaces on $\mathbf{R}^n$ and in domains,
(ii) entropy numbers in quasi-Banach spaces, and
(iii) distributions of eigenvalues of degenerate elliptic differential and pseudodifferential operators,

as it has evolved in recent years.

We are mainly interested in the two scales of function spaces B^s_{pq} and F^s_{pq} with $s \in \mathbf{R}, 0 < p \leq \infty, 0 < q \leq \infty$, which cover many well-known classical spaces such as (fractional) Sobolev spaces, Hölder–Zygmund spaces, Besov spaces and (inhomogeneous) Hardy spaces. The theory of these spaces has been developed in its full extent in [Triα], [Triβ] and [Triγ]. Here we also deal with some recent modifications and refinements connected with spaces of Orlicz type and logarithmic Sobolev spaces.

Let $B^s_{pq}(\Omega)$ be the corresponding spaces on an (arbitrary) bounded domain Ω in $\mathbf{R}^n$. Then the embedding

$$\mathrm{id} : B^{s_1}_{p_1 q_1}(\Omega) \to B^{s_2}_{p_2 q_2}(\Omega) \tag{1}$$

is compact if

$$s_1 - s_2 > n\left(\frac{1}{p_1} - \frac{1}{p_2}\right)_+, \quad 0 < q_1 \leq \infty, 0 < q_2 \leq \infty. \tag{2}$$

Let $e_k(\mathrm{id})$ be the corresponding entropy numbers. Then there exist two positive numbers c_1 and c_2 such that

$$c_1 k^{-(s_1-s_2)/n} \leq e_k(\mathrm{id}) \leq c_2 k^{-(s_1-s_2)/n}, \quad k \in \mathbf{N}. \tag{3}$$

The history of assertions of this type begins in 1967 when M.S.Birman and M.Z.Solomyak [BiS1] proved (3) for the embedding of the Sobolev

(–Besov) spaces $W_p^s(\Omega)$ in $L_q(\Omega)$, their proof being based on the method of piecewise-polynomial approximations. Our method for proving (3) in its full extent relies on Fourier-analytical techniques and has been developed in the last few years in [ET1], [ET2], [Tri3] and [ET4].

The connection between (ii) and (iii) comes from Carl's observation (1980) that

$$|\mu_k| \le \sqrt{2} e_k, \; k \in \mathbf{N}, \tag{4}$$

where μ_k and e_k are respectively the eigenvalues (counted according to their algebraic multiplicities and ordered by decreasing modulus) and the entropy numbers of a compact operator acting in a given (quasi-) Banach space; see [Carl1], [CaT]. It is the main aim of this book to combine observations of type (3) and (4), and to apply them in order to study eigenvalue distributions of degenerate elliptic differential and pseudodifferential operators and their inverses on the basis of some recent progress made in the theory of spaces of B^s_{pq} and F^s_{pq} type.

This book may be considered as a research report mostly based on results of the authors and their co-workers obtained in the last few years. On the other hand, we review the basic material which is needed and give proofs of new results and of assertions not available in relevant books. In this sense we have tried to present a self-contained treatment, accessible to non-specialists.

There are five chapters. Chapter 1 contains elements of a spectral theory in quasi-Banach spaces. We also introduce entropy and approximation numbers and establish some of their basic properties in the context of quasi-Banach spaces, including certain results about the behaviour under interpolation of entropy numbers. Although we focus mainly on entropy numbers in this book, it is helpful to have simultaneously a close look at approximation numbers of abstract and concrete compact operators. Chapter 2 deals with function spaces of type B^s_{pq} and F^s_{pq}. In addition to providing a description of the basic notation and facts, we prove some specific assertions needed in the following chapters. Thus the chapter may be considered as a complement to [Triα], [Triβ] and [Triγ]. In Chapter 3 we calculate the entropy and approximation numbers of compact embeddings between the spaces B^s_{pq} and F^s_{pq} on bounded domains. Chapter 4 concentrates on corresponding problems for weighted spaces of type B^s_{pq} and F^s_{pq} on $\mathbf{R}^n$. Finally, Chapter 5 is devoted to applications of all these results to the distribution of eigenvalues of degenerate elliptic differential and pseudodifferential operators (and their inverses) with non-smooth coefficients.

The book is organised by the decimal system: "n.k.m" refers to subsection n.k.m, "Theorem n.k.m/1" means Theorem 1 in n.k.m, etc. All unimportant positive numbers will be denoted by c (with additional indices if there are several cs in the same formula).

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