

Index of notation

Roman symbols			
<p>ad 20 ad* 20 Ad 20 Ad* 20 A_{op} 101 A^{op} 103 A^0 113 $A^{\mathcal{F}}$ 130 $\mathcal{A}(s)$ 158 A 288 A^0 478 $A(L)$ 498 a_{ij} 562 B_+ 39 bun$_G$ 141 braid 143 \mathcal{B}_m 143 b_U 154 \mathcal{B}_g 256, 282, 565 $\mathcal{BMW}_\ell(r, \epsilon)$ 334 B_V 479 $B(\lambda)$ 479 B_0 481 \mathcal{B}_0 481 B_σ 486 \mathcal{B} 487 \mathcal{B}_∞ 488 $C^\infty(M)$ 16 corep$_A$ 141 $C_{rs}^{(m)}$ 226 ch 315, 353 \mathcal{C}_ℓ 362 c 376 \mathcal{C} 392 C_w 406 $C_{\nu, s; \mu, r}^\lambda$ 432 $\mathcal{C}_\epsilon(G)$ 451 $\mathcal{C}_\epsilon^{1,u}(G)$ 452 $\mathcal{D}(\mathfrak{g})$ 34 $\mathcal{D}(G)$ 34</p>	<p>$\mathcal{D}(A)$ 128 dtangle 145 dribbon 146 dribbon 147 dribbon(X) 147 $\det_{q,\eta}(\lambda)$ 321 $\det_{\epsilon,\eta}(\lambda)$ 326 \mathcal{D} 341, 414 $d\mathcal{L}$ 501 D_L 503 $d\lambda$ 518 \mathcal{D}_m 538 d_i 562 ev_U 139 exp_q 273 ev_a 386, 400 ev^a 387, 400 $\tilde{\text{ev}}_a$ 407 $\tilde{\text{ev}}^a$ 407 $\mathcal{F}(G)$ 106 $\mathcal{F}_R(M_m)$ 228 $\mathcal{F}_{R,q}(GL_m)$ 232 $\mathcal{F}_{R,q}(SL_m)$ 233 $\mathcal{F}_h(G)$ 234–5 Fun_R 237 Fun_h 237 Fr 305 Fr_ϵ 305 \mathcal{F} 410 $\mathcal{F}_\epsilon(G)$ 430 \mathcal{F}_ϵ^+ 433 \mathcal{F}_ϵ^- 433 $\mathcal{F}_\epsilon^{++}$ 433 $\mathcal{F}_\epsilon^{--}$ 433 $\mathcal{F}_{\epsilon,w}$ 435 $\mathcal{F}_\epsilon^{1,u}(G)$ 440 $\mathcal{F}_\epsilon^{0,u}(G)$ 442 $\tilde{\mathcal{F}}_A$ 443 $\tilde{\mathcal{F}}_\epsilon$ 443 $\mathcal{F}_\epsilon(F_\lambda)$ 447 $\mathcal{F}_\epsilon(S^{2n+1})$ 449</p>		

Index of notation

$\mathcal{F}_\epsilon(\tilde{E}_2)$	460-1	\mathcal{K}_i	392
$\mathcal{F}_\epsilon(E_2)$	461	$K(L)$	500
$\mathcal{F}_\epsilon^{(r)}$	464	KZ_m	543
(\mathfrak{g}, δ)	25	\bar{L}_+	497
$(\mathfrak{g}, \mathfrak{g}^*)$	25	\bar{L}_-	497
G^*	33	\bar{L}_0	497
$\text{Gr}(\mathbb{C})$	137	\bar{L}_∞	500
\mathfrak{g}	294	$\text{lk}(L, L')$	501
G^0	294	$\ell(w)$	565
G_{F_p}	304	mod_k	141
\mathcal{G}_V	484	$M_q(\omega)$	314
$\hat{\mathfrak{g}}^{(2)}$	553	$M_\epsilon(\omega)$	324
$\hat{\mathfrak{g}}^{(3)}$	554	$M_\epsilon(\lambda)$	324
$\widehat{\text{GT}}$	558	$M(\mathfrak{h})$	383
$\underline{\text{GT}}$	559	$M_{\mathfrak{a}}$	406
$\mathfrak{g}'(A)$	562	m_{ij}	564
$\mathfrak{g}(A)$	562	$M(\lambda)$	566
\mathfrak{g}_α	564	n	562
g	564	N	565
H_{Lie}^2	84	O_∞	414
$H^*(A, A)$	174	o_∞	414
$H_{\text{alg}}^*(A, A)$	175	\mathbf{O}_κ	551
$H_{\text{coalg}}^*(A, A)$	175	$\text{Par}(\eta)$	320
H_h	182	P_i	383
h_q	284	$P_{i,V}$	396
h_ϵ	290	$P_{i,V}^-$	399
$\mathcal{H}_\ell(\epsilon)$	332	prim	454
$H_{i,r}$	378	$p_n(z; \alpha, \beta; q)$	466
$\tilde{\mathcal{H}}_\ell(\epsilon)$	403	$P_n(z; \alpha, \beta; q)$	468
$\text{height}(\alpha)$	563	$P(L)$	497-8
h	565	QFT	159
I_π	406	$\text{qdet}(T)$	231
int_κ	556	Q_2^*	283
j_U	153	q_i	289
$J(x)$	376	Q	563
J_π	406	Q^+	563
\mathcal{J}	409	$\text{Rep}(G)$	106
$\mathcal{J}_\epsilon(w, \lambda)$	435	\mathcal{R}	119, 123
$J_n(z; q)$	470	rep_A	141
$J(L)$	498	ribbon	146
K_w	39	ribbon	147
k_U	154	\tilde{R}	228
K_β	282	$\tilde{\mathcal{R}}_h$	264
k_β	291	\mathcal{R}_h	201, 271
K_{ρ^*}	365	$\tilde{\mathcal{R}}_{h,i}$	266
		repr	307

$\text{Rep}(\Gamma)$	308	U_ϵ^-	289
$\mathcal{R}(\lambda)$	418	\tilde{U}_ϵ	293
$\tilde{\mathcal{R}}(\lambda)$	423	$U_A^{\text{res}}(\mathfrak{g})$	297
R_σ'	487	$U_A^{\text{res}+}$	297
\mathcal{R}_{KZ}	543	$U_A^{\text{res}0}$	297
S	103	$U_A^{\text{res}-}$	297
$\text{Sym}(V)$	107	$U_{\epsilon,Z}^{\text{res}}$	300
$\text{Sym}_{R,q}$	228	U_ϵ^{res}	300
$\text{Spec}(Z_0)$	291	$U_\epsilon^{\text{res}+}$	300
$\text{Spec}(Z_\epsilon)$	291	$U_\epsilon^{\text{res}0}$	300
$S(r)_\alpha$	389	$U_\epsilon^{\text{res}-}$	300
S_w	435	$U_{\epsilon,Z}^{\text{res}+}$	300
$s_i(\Gamma)$	488	$U_{\epsilon,Z}^{\text{res}0}$	300
S_L	503	$U_{\epsilon,Z}^{\text{res}-}$	300
$s^{\lambda\mu}$	518	U_ϵ^{fin}	302
s_i	564	$U_{\epsilon,Z}^{\text{fin}}$	302
TM	17	$U_{\epsilon,Z}^{\text{fin}+}$	302
T^*M	17	$U_{\epsilon,Z}^{\text{fin}0}$	302
t	55	$U_{\epsilon,Z}^{\text{fin}-}$	302
tangle	145	U_i^{res}	304
T_i	256, 282	\tilde{U}_1^{res}	304
$T\mathcal{L}_t(\lambda)$	333	U_{F_p}	304
$T_\epsilon(\lambda)$	362	$U_{F_p}^{\text{fin}}$	304
\bar{T}	368, 370	U_ϵ^X	340
tilt $_t$	370	$U_h(\mathfrak{g}(u))$	376
tilt' $_t$	370	$U_q(\mathfrak{g})$	392
$\overline{\text{tilt}}_t$	370	$U_\epsilon(\mathfrak{g})$	394
trace_j	506	$U_\epsilon(L(\mathfrak{g}))$	396
$U(\mathfrak{g})$	107	$U_\epsilon(\mathfrak{gl}_{n+1}(\mathbb{C}))$	399–400
$U_h(\mathfrak{g})$	208	$\hat{U}(o_\infty)$	416
U_h^+	260	\tilde{U}_A^{res}	443
U_h^0	260	$U_\epsilon(\mathfrak{e}_2)$	460
U_h^-	260	\mathcal{U}^+	486
u_h	275	\mathcal{U}_σ^+	486
$U_q(\mathfrak{g})$	280–1	$\tilde{U}_\epsilon^{\text{fin}}$	519
U_q^+	282	$V_h^{\mathfrak{h}}$	276
U_q^0	282	V_ω	314
U_q^-	282	$V_q(\omega)$	314
$U_q^{0\tilde{W}}$	285	$V_q(\lambda)$	315
U_A	289	V_A^{res}	316
U_ϵ	289	$V_\epsilon(\omega)$	324
$U_\epsilon^{0\tilde{W}}$	290	$V_\epsilon(\lambda)$	324
U_ϵ^+	289	$V_{\mathfrak{h}}$	332
U_ϵ^0	289		

Index of notation 641

$V_\epsilon^{\mathfrak{h}}$	334	$Y(\mathfrak{gl}_\infty(\mathbb{C}))$	416
$V_\epsilon^{\text{res}}(\lambda)$	355	Z_q	284
$V_{F_p}(\lambda)$	358	Z_ϵ	290
$V_{\mathfrak{h}}$	383	Z_0^+	291
$V(\mathfrak{h})$	383	Z_0^0	291
$V(\lambda_i, a)$	384	Z_0^-	291
V_a	387, 401, 407	\hat{Z}_0	293
V^a	387, 401, 407	\hat{Z}_ϵ	293
$V(\varphi)$	396	$Z(\bar{L})$	512
V^{nat}	415		
$V_q^{\mathfrak{h}}$	477	Greek symbols	
V^0	479	$\alpha_{U,V,W}$	138
$V^\kappa(\lambda)$	551	$\tilde{\alpha}$	353
$V(\infty)$	552	α	506
V_λ	566	α_A	529
$V(\lambda)$	566	α_i	563
w_M	18	γ_q	284
w_h	262	γ_ϵ	290
\tilde{w}_h	263	$\Gamma(2)$	553
$w_{h,i}$	265	$\Gamma(3)$	554
$\tilde{w}_{h,i}$	265	δ	25, 178
$\tilde{w}_{h,0}$	274	δ_R	42
$W_\epsilon^{\text{res}}(\lambda)$	354	$\delta_{a,u}$	42
\tilde{W}_ℓ	360	Δ	102, 564
$w(\bar{L})$	499	Δ^{op}	103
$W^\kappa(L)$	551	δ_R	231
W	564	Δ^+	564
w_0	565	Δ^-	564
X_f	17	ϵ	102
X^L	60	ϵ_i	293
X^R	60	θ	564
$(X_i^\pm)^{(r)}$	257	ι	101
X_β^\pm	259	$\Lambda(V)$	107
x_α^\pm	291	λ_U	138
\underline{x}_i^\pm	293	$\Lambda_{R,q}$	228
$X_{i,r}^\pm$	378	Λ_ℓ	405
$\mathcal{X}_{i,r}^\pm$	392	λ_i	564
$X_{\sigma,t}^+$	486	μ	101
X_t^+	487	μ_{op}	101
X_m	558	Ξ	340
$Y(\mathfrak{g})$	377–8	Π	80
Y^+	382	π_U	138
Y^0	382	π_t	227, 437, 462
Y^-	382	π_{s_i}	438
		$\hat{\pi}_1$	557

ρ_U	138
ρ^*	275
ρ_h^h	276
ρ_e^h	334
$\rho^{\lambda, \mu}$	462
ρ_m	504
ρ	564
Σ_w	39
$\sigma_{U, V}$	149
Σ_ℓ	332
$\Sigma(\tau)_a$	402
σ^ν	462
$\sigma_{\leq 0}(L)$	522
τ_ℓ	227, 437, 462
τ	291
τ_a	380
$\tau_a^*(V)$	384
ϕ	258
$\Phi_{i, r}^\pm$	392-3
$r\varphi_s \left(\begin{smallmatrix} a_1, \dots, a_r \\ b_1, \dots, b_s \end{smallmatrix}; q, z \right)$	465
Φ	529
Φ_{K2}	544
$\chi_{q, \lambda}$	285
$\chi_{e, \lambda}$	290
$\Omega^*(\text{Sym}_{R, q})$	240
$\Omega^*(\Lambda_{R, q})$	240-1
$\Omega^*(\mathcal{F}_R(M_m))$	242
$\Omega^*(\mathcal{F}_{R, q}(GL_m))$	244
$\Omega_{\text{inv}}^*(\mathcal{F}_{R, q}(GL_m))$	245-6
ω	258, 320, 552
Ω	196
$\omega_{\sigma, \lambda}$	314

Miscellaneous symbols

$\{ , \}$	16, 177
\langle , \rangle	17
$[[r, r]]$	51
$\{T \otimes T\}$	61
$\{ , \}^L$	61
$\{ , \}^R$	61
$f \bullet g$	104
$\mathbf{1}$	138, 315
$*_h$	184
$[n]_q$	200
$[n]_q!$	200
$\begin{bmatrix} m \\ n \end{bmatrix}_q$	200
\odot	222
\square	222
$[K_i; m]_{q_i}$	289
$\begin{bmatrix} K_i; c \\ r \end{bmatrix}_{q_i}$	297
$\omega' \leq \omega$	314
$\{u\}$	345
$\overline{\otimes}$	368
$[u, v]_{e^{1/2}}$	399
$\tilde{\boxtimes}$	407
\boxtimes	408
$*L$	446
$*R$	446
$(a; q)_k$	456
$\int_0^1 d_q z$	457
$-$	481
\bullet	554
\otimes	554
$\mu \leq \lambda$	564

General index

- absolute Galois group 557–60
- admissible 80, 92, 482
- affiliated element 462
- affine Hecke algebra 403
 - relation with p -adic groups 404
 - representations 405–8, 410–3
- affine Lie algebra 58, 541–3, 550–6, 565–6
- Alexander polynomial 498, 500
- algebra 101–2
 - deformation 43
 - opposite 101
- ambient isotopy 496, 514
- angle 1-form 501
- antipode 103, 529
 - square of 190–2
- arrow 482
- associativity constraint 139

- Baker–Campbell–Hausdorff formula 46, 182, 185–6
- big cell 294
- Birkhoff factorization theorem 96
- Birman–Murakami–Wenzl algebra 334, 526
- Boltzmann weight 247, 351, 510
- bond 246
- braid 142–4
 - closure 501
- braid group 143–4, 153, 256
 - action 257, 282, 336
 - and configuration spaces 538
- Brauer–Frobenius–Schur duality (see Frobenius–Schur duality)
- Bruhat decomposition 39, 41
- Bruhat ordering 360, 405, 565
- Burau representation 505

- Capelli identity 426
- Cartan matrix 562
 - extended 565–6
- Casimir element 196, 275–6
- category
 - abelian 136–7, 307
 - additive 136–7
 - balanced 154
 - braided monoidal 153
 - essentially small 148
 - k -linear 137
 - monoidal 138–47, 530
 - quasitensor 152–3, 329, 359, 370, 531, 555, 559–60
 - rigid 139–40, 530, 555
 - semisimple 137, 391, 556
 - small 148
 - strict 139
 - tensor 149–52, 531
- C^* -completion 451–2
 - type of 454
- central character 290, 315, 340
- central elements 122, 124, 168–9, 284, 321–3
- character 317, 325–6, 353–4, 361, 363
- characteristic p 296, 304–6, 358–61, 555–6
- classical limit
 - of a QF algebra 179
 - of a quasi-Hopf QUE algebra 536–7
 - of a QUE algebra 180–2
- classical r -matrix 54, 181–2, 183–4
 - and 2-cocycles 62–3
 - and integrable systems 71–7
 - non-degenerate 62, 87
- classical R -matrix 67–8
- classical Yang–Baxter equation (CYBE) 54, 125, 182, 187
 - and Lie bialgebras 51
 - class of solution of 96
 - equivalent solutions of 80, 84, 88
 - modified 55

- solutions of 80–98
 - with spectral parameters 87
- Clebsch–Gordan decomposition
 - 204, 326
- coadjoint orbits 20, 40–1
- coalgebra 102–3
- coassociativity 102
- coassociator 529
- coboundary 51, 84, 174
- cochain 173
- cocommutator 25, 535
- cocycle 25, 62, 174, 533
- cohomology of Hopf algebras 173–7
 - rigidity theorems for 176–7, 212
- co-Jacobi identity 178
- cokernel 137
- co-Leibniz identity 178
- comodule 108
- comodule algebra 109
- comodule coalgebra 109
- compact matrix pseudogroup
 - (see compact matrix quantum group)
- compact matrix quantum group
 - 453–4
 - Haar integral 454–8
- complete integrability 68–9
- complete reducibility 318–9, 324–5
- multiplication 102
- configuration spaces 537–8, 558
- conformal field theory 159, 371–2
 - orbifold 534
- conformal weight 542
- conserved quantities 69
 - in involution 69
- continued fraction 331
- continuous series 331
- contravariant form 320
- convolution product 104
- co-Poisson algebra 178
- co-Poisson Hopf algebra 178
- core 146
- corepresentation 110, 141
 - regular 110
 - unitary 118
- Coxeter automorphism 91
- Coxeter element 489, 565
- Coxeter number 92, 489, 565
- crystal basis 478–86
 - global 480–6
- crystal graph 484
- deformation
 - (mod \hbar^n) 172
 - of algebras 43
 - of bialgebras 171
 - of Hopf algebras 171
 - of Poisson algebras 44
 - quantization 44, 46–7
- degenerate affine Hecke algebra 405
- de Rham complex
 - homology of 243–4
 - of quantum GL_m 244–5
 - of quantum $m \times m$ matrices 242–3
 - of quantum plane 240–1
- derivation 190–2
- determinant formula 321, 327
- diagonal module 342
- diagram automorphism 309
- differential graded algebra 242
- directed tangle 145
- direct sum 136
- discrete series 330
- discriminant 340–1
- Dressing action 37–8
 - and symplectic leaves 38–41
- Dressing transformation 37–8
- dual Coxeter number 564
- dual basis lemma 141
- dual object 139
- dual object functor 140
- Dynkin diagram 92, 562
- Dynkin graph 307
- eight vertex model 253
- elliptic solutions
 - of the CYBE 90–1
 - of the QYBE 427
- enhanced quantum Yang–Baxter
 - operator 506
- epimorphism 137
- equivalence
 - of deformations 171
 - of solutions of the CYBE 80, 84, 88

- euclidean group 459
- evaluation homomorphism
 - for affine Hecke algebras 407
 - for quantum loop algebras 399–401
 - for Yangians 386–7
- evaluation representations
 - of affine Hecke algebras 407
 - of quantum loop algebras 399–403
 - of Yangians 386–91
- exterior algebra 107–8
- Fateev–Zamolodchikov model 351
- Fibonacci sequence 331
- Frobenius Lie algebra 84
- Frobenius map 305–6, 357, 359
- Frobenius–Schur duality
 - classical 332
 - for quantum affine algebras 410–3
 - for $U_\epsilon(\mathfrak{g})$ 334–5, 408–10
 - for Yangians 413–4
- framing 502
- fuchsian differential equations 544
- functor
 - exact 148
 - faithful 148
 - monoidal 138–9
 - tensor 150
- fundamental reflection
 - (see simple reflection)
- fundamental representations
 - of quantum loop algebras 399
 - of Yangians 384, 387–8
- fundamental weight 564
- fusion matrices 155
- fusion ring 155, 159, 160, 371
- fusion rules 154–7, 160
- fusion tensor product 371, 552–5
- Gabriel's theorem 307–8, 489
- Gel'fand–Naimark–Segal state 439
- Gel'fand–Tsetlin basis 476–8
- generalized Cartan matrix 562
- graphs
 - admissible 482
 - coloured oriented 482
 - crystal 484
 - tensor products of 483–6
- Grothendieck group 137–8
- Grothendieck ring 139, 156
- Grothendieck–Teichmüller group
 - 558–9
 - and quasi-Hopf algebras 559–60
- group algebra 105, 112, 175
- group cohomology 533–4
- group-like 106, 193
- Haar integral
 - (see integral, invariant)
- h -adic topology 105
- hamiltonian 68–9
- hamiltonian vector field 17
- Harish Chandra homomorphism
 - 284, 290, 321–4
 - and quantum coadjoint action 296
- Harish Chandra's theorem 285, 290
- Hecke algebra 332–6, 404, 476
 - representations 405–6
- Heisenberg ferromagnet
 - (see XXX model)
- Heisenberg group 45, 218
- Heisenberg Lie algebra 44–5, 176, 186
- hermitian symmetric space 41
- highest weight 314, 354, 383, 434, 441
- highest weight module 314, 324, 354, 542
 - for QF algebras 434, 441
 - for Yangians 383
- highest weight vector 314, 383
- Hilbert space tensor product 415
- Hochschild cohomology 174, 254
 - HOMFLY polynomial 497–8, 509, 517
 - for trefoil knot 498–9
- *-homomorphism 117
- Hopf algebra 103–5
 - almost cocommutative 119
 - coboundary 123
 - cocommutative 102
 - cohomology 173–7

- deformations 171
- dual 111–5, 131–2, 223–6, 234–5
- graded 105
- integral on 115–6, 132
- quasitriangular 123, 129–30
- representations 108–12
- ribbon 125, 133
- *-structure 117–9, 309–11, 329, 431–2
- topological 105, 114, 123
- triangular 123, 150–1
- Hopf ideal 103–4
- Hopf link 165–6
- Hopf's theorem 108
- hyperalgebra 304, 358, 359–60
- identity object 138
- infinite-dimensional orthogonal group 414–7
- infinitesimal deformation 172
- integral 115–6
 - invariant 115
 - non-central 456
 - normalized 115
- integral form
 - non-restricted 289
 - restricted 297
- intersection cohomology 489
- intertwiner 151, 348–51, 542
- invariant bilinear form 563
 - orthogonality properties 564
- invariant differential form 245–6
- Ising model 371–2
- Jackson integral 457, 467
- Jacobi identity 17
- Jacobson topology 454
- jets 391
- Jones polynomial 168, 498, 500
- Kac–Moody algebra 562
- Kashiwara's tensor product
 - algorithm 485–6
- Kauffman polynomial 499, 500, 510, 517
- Kazhdan–Lusztig element 406
- Kazhdan–Lusztig polynomial 360
- Kazhdan–Lusztig tensor product 552–5
- kernel 137
- kink 501
- Kirby move 503, 522–5
 - special 503
- Kirillov–Kostant principle (*see* orbit principle)
- Knizhnik–Zamolodchikov equation 539–41
 - and hyperplane complements 561
 - and m -point functions 542–3
 - and quantization 543–9, 555
- knot 496
- Kohno–Drinfel'd monodromy theorem 549–50
- lattice model
 - exactly solvable 247
 - integrable 246–53
- Lax pair 69–71, 73, 76
- left dual 111, 122
- Leibniz identity 17
- length 565
- level 408–9
 - for affine Lie algebras 566
- Lie bialgebra 24–33
 - automorphism 32
 - coboundary 50–9
 - cocommutator of 25
 - derivation 32–3
 - double 34–5, 58–9
 - dual 33–4
 - factorizable 67
 - homomorphism 25
 - ideal 25
 - opposite 25
 - pseudotriangular 56
 - quasitriangular 54
 - quotient 25
 - standard 29–30, 34–6, 55–6, 59, 81, 93
 - tangent 26
 - triangular 54
- Lie–Poisson structure 19–20, 21
- linking number 500–1
- link invariants 167–8, 497

- from R-matrices 504–10
 - from vertex models 510–7
- links 167
 - equivalent 496
 - oriented 496
 - skein related 497
- local algebra 158
- loop representation 542
- Lusztig's canonical basis
 - algebraic construction 488–8
 - positivity property 487
 - topological construction 488–90
- Lusztig's conjectures 359–61
- 3-manifold invariants 522–5
- Manin triples 26–32
 - infinite-dimensional 28
- Markov equivalence 502
- Markov moves 502
- Markov trace 506
- matched pair 48
- matrix elements 106, 116, 430
 - and little q -Jacobi polynomials 466
 - and little q -Legendre polynomials 467
 - and q -Bessel functions 470–3
- modified classical Yang–Baxter equation (MCYBE) 55
- equivalent solutions 80
- modular Hopf algebra 517–21
- module 108
- module algebra 109
- module coalgebra 109
- monodromy 540
- monodromy theorem 549–50
- monoidal functor 138–9
- monomorphism 137
- non-linear Schrödinger equation 422
- non-restricted integral form 289
- non-restricted specialization 288–96
 - and conjugacy classes 295–6, 342–3, 347
 - and R-matrices 348–51
 - automorphisms 293–5
 - centre 290–2
 - defining relations 289
 - relation with $U(\mathfrak{g})$ 290
 - representations 339–48
- opposite
 - algebra 101
 - coalgebra 103
- orbit principle 180, 439, 442
- Ore condition 232, 244
- pairing of Hopf algebras 114–5, 224–6, 461
- partition function 247, 249, 510
- pentagon axiom 139
- perverse sheaves 490
- Peter–Weyl theorem 451
- Plücker relations 448
- Poincaré–Birkhoff–Witt 199–200, 260, 282–3, 291, 298, 302, 382, 394, 433
- Poisson action 36–7
- Poisson algebra 43, 177
 - deformation 44
- Poisson bivector 17
- Poisson bracket 16–7
- Poisson homogeneous space 22–4
 - Hopf fibration as 23–4
 - symplectic leaves of 38
- Poisson–Hopf algebra 177–8
- Poisson–Lie group 21–4, 292–3, 296
 - coboundary 59–68
 - compact 35–6
 - complex 26
 - double 34, 37–8, 43
 - dual 33–4, 37–8, 296
 - factorizable 67
 - homomorphism 21
 - Lie bialgebra of 26
 - subgroup 22
- Poisson manifold 16–21
 - complex 20–1
- Poisson map 18
- Poisson structure 16–7
 - compatible 41
 - product 18
 - rank of 19, 22

- second Adler–Gel'fand–Dickii 32
- symplectic 18
- trivial 18
- twisted 42–3
- Poisson submanifold 18
- Pontryagin product 108
- Pontryagin duality 135
- primitive element 107
- primitive ideal 454
- primitive vector 314, 354, 383
- principal alcove 362
- pro-finite fundamental group 557
- pseudo-differential operators 31–2
- pseudo-highest weight 396
- pseudo-universal R-matrix 419, 423
- pure braid group 144, 153, 538
- pure sphere braid group 538

- q -Bessel function 470
 - addition formula 471
 - Hansel–Lommel orthogonality relation 471
- q -binomial coefficient 200, 209, 301
- q -derivative 464
- q -exponential 273
- q -factorial 200
- QF algebra 176, 216–27, 234–40, 430
 - admits a faithful representation 438–9
 - and classical function algebra 306, 443–4
 - at roots of unity 301, 442–5
 - centre 219
 - differential calculus on 240–6
 - highest weight representations 227, 433–8, 441–2
 - multiparameter 238–40
 - PBW factorization 433
 - rational form 280–1
 - *-representations 433, 437–8, 441–2
 - *-structure 431–2
 - twisted 439–42
- QFSH algebra 190, 267–8
- q -Hahn polynomial 467
- q -hypergeometric series 430, 465, 561
- q -integral (*see* Jackson integral)
 - q -Jacobi polynomial
 - big 468–9
 - little 466–7
 - q -Legendre polynomial 467
 - q -number 200
 - q -spherical function 468–9
 - quantization
 - deformation 46–7
 - homogeneous 375–6
 - Moyal 46–7, 183–7
 - multiparameter 212–3, 238–40
 - non-standard 206–7
 - of co-Poisson–Hopf algebras 180
 - of Lie bialgebras 180–9
 - of Poisson–Hopf algebras 179
 - of Poisson–Lie groups 179
 - of quasi-Hopf algebras 537
 - R-matrix 187, 222–3, 228–34
 - standard 195, 208, 234–5
 - Weyl 44–5
 - quantized algebra of continuous functions 451–2
 - vanishing at infinity 461–2
 - quantized function algebra (*see* QF algebra)
 - quantum affine algebra 392–4
 - PBW basis 394
 - pseudotriangular structure 403, 423–5
 - quantum Casimir element 196, 285, 291–2, 323
 - quantum Clebsch–Gordan coefficient 205, 467
 - quantum coadjoint action 293–6
 - and conjugacy classes 295–6
 - quantum coordinate ring (*see* QF algebra)
 - quantum determinant 219, 220, 231–3, 236
 - quantum dimension 122–3, 126, 365–6, 371
 - quantum double 127–9, 132
 - quantum euclidean group 459–62, 469–70
 - automorphisms 463
 - Haar integral 463–5
 - representations 462–3

- quantum field theory 157–60
- quantum flag manifolds 447–8
- quantum G -space 445–7
- quantum Kac–Moody algebras 207–13
 - automorphisms 309
 - centre 212, 284–5, 290–3, 321–4
 - geometric realization 285–8, 308–9
 - real forms 309–11
 - representations 313–31
- quantum loop algebras 396
 - relation with Yangians 403
 - representations 394–403
- quantum orthogonal group 236–8
- quantum plane 217–8, 228, 240–1
- quantum projective space 451
- quantum R -matrix 133, 205, 276–7
 - elliptic 427
 - Frobenius 231
 - Hecke 232
 - rational 380, 419–23
 - singularities 425
 - strange 230
 - trigonometric 424
- quantum Schubert variety 448
- quantum Schur orthogonality relations 457–8
- quantum Serre relation 209
- quantum sl_2 192–206, 216–27
 - algebra structure 196
 - and little q -Jacobi polynomials 466–7
 - automorphisms 309
 - basis 199, 220–2, 226
 - centre 285, 291–2, 321–3
 - matrix elements 466
 - quantum Weyl group 262–3
 - quasitriangular structure 201–3
 - real forms 309–11
 - representations 203–6, 330–1, 343–4
 - tilting modules 364–6
 - universal R -matrix 201, 263–5
 - Weyl modules 355–6
- quantum special linear group 235–6
- quantum spheres 448–51, 467–9
- quantum symplectic group 236–8
- quantum torus 442
- quantum trace 122–3, 126–7, 365–6, 370
- quantum Weyl group 262–6, 439
 - and central elements 265
 - and universal R -matrix 263–5
 - for quantum affine algebras 424
- quantum Yang–Baxter equation (QYBE) 67, 124–5, 187, 203, 229, 250
 - and braid groups 504–5
 - and intertwiners 348–51
 - space of solutions of 233–4
 - with spectral parameters 250, 348–51, 380, 419–25, 511
- quasi-bialgebra 529
 - quasitriangular 531
 - triangular 531
- quasi-Frobenius Lie algebra 84
- quasi-Hopf algebra 130–1, 529
 - quasitriangular 531
 - triangular 531
- quasi-Hopf QUE algebra 534–7
 - quasitriangular 536
- quasi-Lie bialgebra 536
- quasi-modular Hopf algebra 521
- quasi-Poisson–Lie group 48
- quasi-quantum Yang–Baxter equation 531
- QUE algebra 176, 187–92
 - coboundary 188
 - cocommutative 187–8
 - quasitriangular 188
 - rational form 280–1
 - triangular 188
- QUE double 190, 266–7, 270
- QUE dual 190, 267, 268–70
- quiver 307, 488–90
- rational form 280–1
 - adjoint 281
 - and R -matrices 327–9
 - simply-connected 281
- real form 117–9
 - compact 431
 - equivalent 117

650

General index

- reconstruction theorem 147–9
- reduced decomposition 565
 - adapted 488
- regular dominant weight 433
- regular isotopy 500, 512
- Reidemeister moves 496–7
- representation
 - adjoint 110
 - and symplectic leaves 180, 439, 442, 454
 - characters 315
 - completely reducible 391
 - corresponding to a Schubert cell 435, 441, 448
 - cyclic 344–8
 - dual 110–1
 - faithful 438–9
 - highest weight 314, 324, 433
 - integrable 314, 492, 556, 566
 - physical 158
 - pseudo-highest weight 396
 - regular 110
 - restricted 359
 - tame 415
 - tensor products 110
 - trivial 110
 - type 314, 394, 401
 - unitarizable 433
 - unitary 118, 329–31, 433
- *-representation 433, 437–9, 441–2
- representative functions 106, 116
- restricted enveloping algebra 304, 360
- restricted integral form 296–7
 - basis 298
 - generators and relations 299–300
 - of a representation 316
- restricted representation 359
- restricted specialization 300
 - and affine Lie algebras 360–1
 - centre 300
 - representations 351–61
- ribbon Hopf algebra 125–7, 133, 161, 276, 517
 - central elements 168–9
- ribbon tangle 146–7, 502
 - closure 164–5
 - coloured 147
 - directed 146–7
 - isotopy invariants 161–6
- right dual 111, 122
- rigidity theorems 176–7, 212
- Robinson–Schensted
 - correspondence 478
- root 563
- root space 564
- root vector 259, 282, 565
- rotation number 501
- Schouten bracket 17
- Schubert cell 41, 435, 441, 448
- segment 402
- short exact sequence 137
- simple object 137
- simple reflection 564
- simple root 563
- sink 488
- six vertex model 252–3
 - rational 252
 - trigonometric 253
- skein invariant 498
- skein relation 497
- Sklyanin bracket 60
- smooth vector 552
- soliton equations 32
- source 484
- sphere braid group 538
- state 246
- strange algebra 220, 230–2
- Steinberg’s theorem 359
- string 389–90, 482
 - length 482
- surgery 503
- Sweedler’s example 131–3
- symmetric algebra 107–8
- symmetrizable 562
- symplectic groupoid 48
- symplectic manifold 15, 18, 184
- symplectic leaves 18–21, 296, 444
 - and dressing transformations 38
 - and representations 180, 439, 442, 444, 454
 - in compact Poisson–Lie groups 39–43, 227

- in Poisson homogeneous spaces 38
- Tait number 499
- tame representation 415
- tangles 144
 - isotopy invariants of 167–8
- Tannaka–Krein duality 116, 135, 147
- Temperley–Lieb algebra 333, 335
- tensor product theorem 357–9
- tilting modules 361–72
 - and quantum dimension 365–6
- Toda system 75–7
- topologically generated 196
- topological quantum field theory 160–1
- transfer matrix 248–9
- triangular module 342
- trigonometric solutions of the CYBE 91–4
- trivial deformation 171
- truncated tensor product 160, 368–71
- twisting 129–31, 183–4, 439, 532, 536, 537
 - and group cohomology 534
 - of a quasitensor category 559–60
- type
 - of a C^* -algebra 442, 454
 - of a representation 314, 394, 401
- universal enveloping algebra 107, 112–3
- universal R-matrix 123, 129–31, 201, 263–6, 271–5, 283–4, 327, 531
 - at a root of unity 303–4
 - multiplicative properties 274
 - uniqueness 275
- vacuum representation 158
- Vassiliev invariants 526
- Verma module 314, 320–1, 356, 435, 566
- Verma’s identity 487
- vertex model 246–7, 510–4
 - and link invariants 512–4
 - integrable 250
- Virasoro algebra 31, 64–7, 556
- weight 314, 353, 383, 566
- Weyl character formula 317, 325, 355, 357, 361
- Weyl filtration 362
- Weyl group 564
- Weyl module 354–6, 362
 - quantum dimension of 365
- Whitehead’s lemma 188, 212
- writhe 499
- XXX model 252–3, 422
- XYZ model 253
- Yangians 30, 375–80
 - and infinite-dimensional groups 414–7
 - automorphisms 380–1, 384
 - centre 381
 - finite-dimensional representations 383–91
 - infinite 416–7
 - PBW basis 382
 - pseudotriangular structure 385, 391, 418–23
- Zelevinsky tensor product 407–8