

Contents

Preface	xiii		
Chapter 1.			
Introduction	1		
1. Mathematical physics	1		
2. Basic concepts of continuum mechanics	3	2. Equations of continuity. Convective and diffusion flux in nonelectrolyte solutions in presence of chemical reactions. Fick's equation of diffusion in binary solutions. Diffusion of electrolytes. Nernst–Planck equation	32
3. Elements of electrostatics	12		
4. Elements of electrodynamics	14		
5. Elements of chemical kinetics	15	3. Equation of motion of continuous medium	36
6. Elements of equilibrium thermodynamics	20	4. Equation of heat conduction in continuous media. Heat conduction in moving homogeneous compressible fluid	40
7. Integral laws of conservation of extensive parameters	23		
8. Elements of thermodynamics of irreversible processes	27	5. Potential motion of inviscid incompressible liquid. Equations of vibrations of elastic body and of slightly compressible inviscid liquid	42
Problems	28		
Chapter 2. Typical equations of mathematical physics. Boundary conditions	29	6. Chain of springs oscillating in medium with friction. Wave equation	46
1. Laws of conservation and continuity. Three prototypic second-order equations of mathematical physics	29	7. Maxwell's equations of electrodynamics	47

vi	<i>Contents</i>
8. Theory of percolation of multicomponent liquids	52
9. Brownian motion. Langevin's equation and hyperbolic diffusion equation	66
10. Boundary and initial conditions	68
11. Examples of typical free boundary-value problems	77
12. Well-posedness in Hadamard's sense. Examples of ill-posed problems	83
13. Terminology. Concluding remark. Notation	88
Problems	89
Chapter 3. Cauchy problem for first-order partial differential equations	92
1. Local Cauchy problem for quasilinear equation with two independent variables	92
2. Local Cauchy problem for nonlinear first-order partial differential equation	95
3. Global Cauchy problem for quasilinear partial differential first-order equation with two independent variables. Need for broader class of generalized (discontinuous) solutions	98
4. Necessary conditions of discontinuity. Problem of decay of arbitrary discontinuity. Gelfand's heuristic theory	100
Problems	115
Chapter 4. Classification of second-order partial differential equations with linear principal part. Elements of the theory of characteristics	117
1. Classification of second-order partial differential equations	117
2. Reduction of second-order equation to canonical form	121
3. Canonical form of linear partial differential equations with constant coefficients	126
4. Cauchy problem for partial differential equations with linear principal part. Classification of equations	128
5. Cauchy problem for system of two quasilinear first-order partial differential equations with two independent variables; concept of characteristics	131
6. Characteristics as curves of weak discontinuity of second or higher order	135
7. Riemann's formula. Characteristics as curves of weak discontinuity of first order or as curves of strong discontinuity	137
Problems	142
Chapter 5. Cauchy and mixed problems for the wave equation in \mathbb{R}_1. Method of traveling waves	144
1. Small vibrations of infinite string. Method of traveling waves	144

	<i>Contents</i>	
2. Small vibrations of semi-infinite and finite strings with rigidly fixed or free ends. Method of prolongation	147	
3. Generalized solution of problem of vibration of loaded string with nonhomogeneous boundary conditions	149	
Problems	153	
Chapter 6. Cauchy and Goursat problems for a second-order linear hyperbolic equation with two independent variables. Riemann's method	155	
1. Riemann's method	155	
2. Goursat problem. Existence and uniqueness of Riemann's function	163	
3. Dynamics of sorption from solution percolating through layer of porous adsorbent. Riemann function for a linear hyperbolic equation with constant coefficients	171	
Problems	175	
Chapter 7. Cauchy problem for a 2-dimensional wave equation. The Volterra–D'Adhemar solution	176	
1. Characteristic manifold of second-order linear hyperbolic equation with n independent variables	176	
2. Cauchy problem for the 2-dimensional wave equation. Volterra–D'Adhemar solution	180	
Problems	185	
		vii
	Chapter 8. Cauchy problem for the wave equation in \mathbb{R}_3. Methods of averaging and descent. Huygens's principle	186
	1. Method of averaging	186
	2. Method of descent	191
	3. Huygens's principle	192
	Problems	194
	Chapter 9. Basic properties of harmonic functions	195
	1. Convex, linear, and concave functions in \mathbb{R}_1	195
	2. Classes of twice continuously differentiable superharmonic, harmonic, and subharmonic functions in multidimensional regions	196
	3. Hopf's lemma and strong maximum principle	201
	4. Green's formulas. Flux of harmonic function through closed surface. Uniqueness theorems	205
	5. Integral identity. Mean value theorem. Inverse mean value theorem	208
	Problems	212
	Chapter 10. Green's functions	214
	1. Definitions. Main properties	214
	2. Sommerfeld's method of electrostatic images (method of superposition of sources and sinks)	221
	3. Poisson integral	225
	Problems	227
	Chapter 11. Sequences of harmonic functions. Perron's theorem. Schwarz alternating method	229
	1. Harnack's theorems	229

viii	<i>Contents</i>
2. Complete classes of (continuous) superharmonic and subharmonic functions	232
3. Basic Perron theorem	237
4. Existence theorem for Dirichlet problem. Barriers.	241
5. Schwarz alternating method	246
Problems	257
Chapter 12. Outer boundary-value problems. Elements of potential theory	258
1. Isolated singular points of harmonic functions	258
2. Regularity of harmonic functions at infinity	259
3. Extension of the fundamental identity to unbounded regions. Liouville's theorem	263
4. Electrostatic potentials	265
5. Integrals with polar singularities	269
6. Properties of electrostatic volume potential	274
7. Properties of electrostatic potentials of double and single layers	277
8. Dirichlet and Neumann boundary-value problems. Reduction to integral equations. Existence theorems	286
Problems	295
Chapter 13. Cauchy problem for heat-conduction equation	296
1. Fundamental solution of Fourier equation. Heaviside unit function and Dirac δ function	296
2. Solution of Cauchy problem for 1-dimensional Fourier equation. Poisson integral	302
3. Moments of solution of Cauchy problem. Asymptotic behavior of the Poisson integral as $t \uparrow \infty$	309
4. Prigogine principle, Glansdorf–Prigogine criterion, and solution of Cauchy problem for heat-conduction equation	312
5. Fundamental solution of multidimensional heat-conduction equation	318
Problems	324
Chapter 14. Maximum principle for parabolic equations	325
1. Notation	325
2. Weak maximum principle	328
3. Nirenberg's strong maximum principle	335
4. Vyborny–Friedman analog of Hopf's lemma	341
5. Uniqueness theorems. Tichonov's comparison theorem	344
6. Remarks on time irreversibility in parabolic equations	350
Problems	353
Chapter 15. Application of Green's formulas. Fundamental identity. Green's functions for Fourier equation	354
1. Fundamental identity	354

	<i>Contents</i>		ix
2. Application of first Green's formula and uniqueness theorems	358	according to Stefan–Boltzmann law	428
3. Green's functions	359	Problems	431
4. Relationship between Green's functions of Dirichlet problem in \mathbb{R}_3 , corresponding to Laplace and Fourier operators (Tichonov's theorem)	365	Chapter 18. Sequences of parabolic functions	435
5. Examples of Green's functions	367	1. Parabolic analogs of Harnack's theorems	435
Problems	379	2. Space of continuous super- and subparabolic functions	441
Chapter 16. Heat potentials	380	3. Perron–Petrovsky's theorem. Parabolic barriers	448
1. Volume heat potential	380	4. Case of cylindrical region. Tichonov's theorem. Duhamel test	456
2. Heat potentials of double and single layers	387	5. Application of Schwarz alternating method to solution of Dirichlet problem for heat-conduction equation in noncylindrical region	460
Problems	398	Problems	467
Chapter 17. Volterra integral equations and their application to solution of boundary-value problems in heat-conduction theory	399	Chapter 19. Fourier method for bounded regions	468
1. Reduction of first, second, and third boundary-value problems for Fourier equation to Volterra integral equations. Existence theorems	399	1. Vibration of a bounded string. D'Alembert's solution and superposition of standing waves. Formal scheme of the method of separation of variables	468
2. Asymptotic behavior of solution of first boundary-value problem and respective integral equations	406	2. Heat transfer through a homogeneous slab	473
3. Solution of quasilinear Cauchy problem	413	3. Two-dimensional Dirichlet problem for Poisson equation in a rectangle	476
4. One-dimensional one-phase Stefan problem with ablation	421	4. Vibration of circular membrane with rigidly fixed boundary under action of instant point impulse initially applied at an interior point of membrane	480
5. Determination of temperature of half-space $z > 0$ radiating heat		5. Heat transfer through two-layer circular disk with Newtonian irradiation from	

x	<i>Contents</i>
medium of prescribed temperature	482
6. Application of Fourier method to solution of mixed problems. Reduction to denumerable system of algebraic equations. Perfect systems	487
Problems	495
Chapter 20. Integral transform method in unbounded regions	497
1. Integral transforms in solution of boundary-value problems in unbounded regions	497
2. Fourier transform, sine and cosine Fourier transform. Double Fourier integral and Fourier–Lebesgue theorem. Fourier transform of derivatives	501
3. Use of Fourier transforms to solve Cauchy problem of heat conduction	503
4. Fourier–Bessel (Hankel) transform and solution of boundary-value problems with cylindrical symmetry. Fundamental solution of heat-conduction equation with forced convection, generated by continuously acting source of incompressible liquid	505
5. Laplace–Carson transform and its simplest properties	511
6. Relationship between Laplace and Fourier transforms. Bromwich integral and Jordan lemma	516
7. Relationship between limits of functions and their transforms. Asymptotic expansion	522
Problems	525
Chapter 21. Asymptotic expansions. Asymptotic solution of boundary-value problems	527
1. Solution of Cauchy problem for 1-dimensional Fourier equation. Short relaxation time asymptotics for solution of hyperbolic heat-conduction equation	527
2. Asymptotic sequences. Expansions in asymptotic series. Definitions and preliminary statements	532
3. Regular and singular perturbations. Differential equations depending on parameters. Scaling. Outer and inner expansions. Matching	535
4. Electrodiffusion and the nonequilibrium space charge in the 1-dimensional liquid junction	543
Problems	549
Appendix 1. Elements of vector analysis	551
1. Definitions	551
2. Gauss divergence theorem and Stokes's theorem	553
3. Orthogonal curvilinear coordinate systems. Lamé coefficients. Basic operators of vector analysis	554
Appendix 2. Elements of theory of Bessel functions	560
1. Introduction. Euler's gamma function	560

	<i>Contents</i>		xi
2. Generating functions and Bessel functions of first kind. Neumann functions	561	eigenfunctions of regular Sturm–Liouville operator	600
3. Bessel and Lipschitz integrals	566	4. Remarks on case of singular operator	610
4. Neumann’s addition theorem	567	5. Expansions into Fourier–Bessel and Dini series	612
5. Potential of double layer of dipoles distributed with unit density along surface of infinitely long circular cylinder. Discontinuous Weber–Schafheitlin integral. Fourier–Bessel double integral	569	Problems	616
6. Bessel functions of imaginary argument. Spherical Bessel functions	572	Appendix 4. Fourier integral	617
7. Asymptotic behavior of Bessel functions	575	1. Riemann–Lebesgue lemma	617
8. Method of averaging. Weber’s integrals	577	2. Fundamental Fourier theorem	618
9. Representation of Bessel functions by contour and singular integrals	582	3. Fourier transform of function of exponential growth at infinity. Relationship between double Fourier integral and Fourier series	620
10. Asymptotic representation of Bessel functions in complex plane	583	4. Convolution theorem and evaluation of definite integrals	622
11. Hint for solution of cylindrical Stefan problem	586	5. Abel-summable integrals and solution of problems with concentrated capacity	624
Problems	588	Problems	627
Appendix 3. Fourier’s method and Sturm–Liouville equations	589	Appendix 5. Examples of solution of nontrivial engineering and physical problems	628
1. Separation of variables and eigenvalue problem	589	1. Heat loss in injection of heat into oil stratum	628
2. Elementary theory of regular Sturm–Liouville equations	592	2. Nonlinear effects in electrodiffusion equilibrium. Saturation of force of repulsion between two symmetrically charged spheres in electrolyte solution	635
3. Expansion of functions in \mathcal{M}^* in series of		3. Linear stability of Neumann’s solution of	

Cambridge University Press

978-0-521-55846-4 - Partial Differential Equations in Classical Mathematical Physics

Isaak Rubinstein and Lev Rubinstein

Table of Contents

[More information](#)

xii	<i>Contents</i>		
two-phase Cauchy–Stefan problem	650	Index	672
References	666		