Ordinary differential equations—the building blocks of mathematical modeling—are also key elements of disciplines as diverse as engineering and economics. Although mastery of these equations is essential, adhering to any one method of solving them is not: This book stresses alternative examples and analyses by means of which the student can build an understanding of a number of approaches to finding solutions and understanding their behavior.

The text includes brief expositions of standard topics, including first-order equations, homogeneous and nonhomogeneous second-order linear equations, power series expansions about regular and regular singular points, linear systems theory, and stability concepts for both the phase plane and higher-dimensional systems. A variety of exercises and examples is included, and readers are encouraged to try alternative approaches to find solutions that integrate and build upon ideas introduced in earlier chapters. This book offers not only an applied perspective for the student learning to solve differential equations, but also the challenge to apply these analytical tools in the context of singular perturbations, which arises in many areas of application. An important resource for the advanced undergraduate, this book would be equally useful for the beginning graduate student investigating further approaches to these essential equations.

Thinking About Ordinary Differential Equations

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To Candy, Patrick, Timothy, and Daniel . . .
Great O'Malleys, blissfully unenlightened concerning differential equations

Contents

Preface

This small book is intended for use by students in the applied sciences and engineering who already have some elementary knowledge of ordinary differential equations. It aims to emphasize the variety of analytical approaches available and to teach simple techniques to use in their own technical work and in understanding the behavior of solutions to many standard problems. The exercises at the end of each chapter, in particular, are intended to be the primary learning tool, so fairly detailed solutions are provided for many of them. The important job of interpreting solutions in their underlying physical context is left to the reader.

Good calculus skills are called for. Some familiarity with numerical and/or symbolic computing and with matrix analysis would also be helpful, but is not necessary. We will not hesitate to introduce needed theory, without proof, in order to advance the reader's understanding. The fundamental perspective is that there is no best way to solve a given ordinary differential equation. Indeed, most equations that scientists encounter are solved numerically, and the traditional analytical techniques presented here remain important because they provide the basis for successful computing schemes. Readers are definitely urged to use available software to learn about the solutions of the differential equations they either need to solve or have otherwise become fascinated by.

The examples and exercises included have been collected over many years for various classes given by the author. Many were taken from others' textbooks and papers (only a few are original), so it is no longer possible to properly acknowledge the original sources. This explicit debt to earlier writers is, certainly, substantial. Likewise, little reference is made to more advanced monographs, as would be appropriate for students seeking a less-utilitarian acquintanceship with differential equations.

We hope readers will find the problems considered interesting and challenging. Moreover, we hope they will learn enough about how to solve differential

equations to be able to apply the tools introduced in substantial applications. Reading this book is a do-it-yourself project; some scribbling between steps is expected, and fooling around with toy problems is encouraged.

Special thanks are extended to all those who have helped me learn this material and prepare this manuscript. They include Shepley Ross and Bob Owens (years ago), Frances Chen (most recently), and many colleagues and students (over the years).

Robert E. O'Malley, Jr.
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