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Sergio Albeverio, Jurgen Jost, Sylvie Paycha and Sergio Scarlatti
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A Mathematical Introduction to String Theory

Variational problems, geometric and probabilistic methods

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Preface

This book is intended as an introduction to certain global analytic and probabilistic aspects of string theory. Nowadays string theory is a domain where mathematics and physics meet, and proceed together concerning certain aspects. However, the theory itself is far from being complete, in fact it is suspended between purely heuristic Ansätze with little hope of mathematical justification and very advanced mathematical ideas. Our aim has been to bring together as far as presently possible the differential–geometric aspects (related to theory of harmonic maps, infinite dimensional differential geometry, Riemann surfaces) and the measure theoretical and probabilistic aspects one encounters when trying to give a sense to the heuristic "Feynman path integrals", so often used not only by physicists but also by mathematicians "to get started".

One of us (J. Jost) worked out a theory of strings with boundary as a quantization of Plateau's problem for minimal surfaces and lectured at several conferences on the geometric aspects of the theory. Two of us, S. Paycha and S. Scarlatti, have been working on relating these aspects with probabilistic ones, in connection with Ph.D. theses in Bochum/Paris and Rome respectively, under the direction of S. Albeverio [Pa1], [Sc]. The probabilistic aspects are connected with the study of a mass zero Høegh-Krohn model, and the first basic study of these aspects was undertaken by S. Albeverio, S. Paycha and S. Scarlatti in collaboration with the late R. Høegh-Krohn.

It was then natural to join efforts and to produce a book which unifies the approaches. We hope our endeavour will be appreciated by the reader. We stress once more that the book presents only a small portion of all aspects of string theory – but we have strived to present this portion as much as possible as a coherent mathematical theory.

Bochum, December 1994

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Dedication

This book is dedicated to the dear memory of Raphael Høegh-Krohn (1938–1988). He was a great mathematician and a natural philosopher who on so many occasions was able to show us the correct way, foreseeing so many of the new developments.

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