

The Physics of Fluids and Plasmas

An Introduction for Astrophysicists

A good working knowledge of fluid mechanics and plasma physics is essential for the modern astrophysicist. This graduate textbook provides a clear, pedagogical introduction to these core subjects. Assuming an undergraduate background in physics, this book develops fluid mechanics and plasma physics from first principles.

This book is unique because it presents neutral fluids and plasmas in a unified scheme, clearly indicating both their similarities and their differences. Also, both the macroscopic (continuum) and microscopic (particles) theories are developed, establishing the connections between them. Throughout, key examples from astrophysics are used, though no previous knowledge of astronomy is assumed. Exercises are included at the end of chapters to test the reader's understanding.

This textbook is aimed primarily at astrophysics graduate students. It will also be of interest to advanced students in physics and applied mathematics seeking a unified view of fluid mechanics and plasma physics, encompassing both the microscopic and macroscopic theories.

Arnab Rai Choudhuri is an Associate Professor of Physics at the Indian Institute of Science in Bangalore. After obtaining his Ph.D. at the University of Chicago in 1985, he spent two years at the High Altitude Observatory in Boulder and then joined the faculty of Indian Institute of Science in Bangalore. During his tenure in Bangalore, he has held visiting positions at the University of Chicago, the University of St Andrews and the Kiepenheuer Institut in Freiburg (as an Alexander von Humboldt Fellow). In the field of solar magnetohydrodynamics, he is one of the world's best-known scientists of his generation.

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To **Gene Parker**
from whom I learnt
much of what is
in this book

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Preface

Hydrodynamics, magnetohydrodynamics, kinetic theory and plasma physics are becoming increasingly important tools for astrophysics research. Many graduate schools in astrophysics around the world nowadays offer courses to train graduate students in these areas. This was not the case even a few years ago—say around 1980—when it was rare for an astrophysics graduate school to teach these subjects, and the students who needed the knowledge of these subjects for their research were supposed to pick up the tricks of the trade on their own. With increasing applications of these subjects to astrophysics—especially to understand many phenomena discovered in the radio, X-ray or infrared wavelengths—the need is felt to impart a systematic training in these areas to all graduate students in astrophysics.

When I joined the faculty of the Astronomy Programme in Bangalore in 1987, I argued that a course covering these subjects should be introduced. My colleague and friend, Rajaram Nityananda, shared my enthusiasm for it, and we together managed to convince the syllabus committee of the need for it. From then onwards, this course has been taught regularly in our graduate programme, the responsibility of teaching it falling on my shoulders on several occasions. When I taught this course for the first time in 1988, I had to work very hard preparing lectures from different sources. I was lucky to have taken such a course myself as a graduate student in Chicago in 1981—taught by E. N. Parker—although it was somewhat unusual at that time for astronomy departments in the U.S.A. to offer such courses. The lecture notes which I had taken in that course and preserved carefully were a great help in preparing my course. I had to tell the students to use at least half-a-dozen basic books (such as Landau and Lifshitz's *Fluid Mechanics*, Chandrasekhar's *Hydrodynamic and Hydromagnetic Stability*, Cowling's *Magnetohydrodynamics*, Huang's *Statistical Mechanics*,

Chen's *Plasma Physics*) for different parts of the course. Ever since that time, I have been thinking of writing a comprehensive textbook based on my course. While I was trying to get some time off to write my book, the first textbook on the subject by Shu (1992) appeared. When I first saw the book, my heart sank. I thought that the book I had been contemplating had been written, and there would now be no point in my writing a book. However, on looking at Shu's book more closely, I realized that my point of view has been sufficiently different and there should be scope for more textbooks with different perspectives on such an important subject.

While writing such a textbook for astrophysics graduate students, two alternative approaches are possible. One is to take some astrophysical topics as central themes, and then develop fluid mechanics and plasma physics primarily as tools to be used. The other approach is to present fluid mechanics and plasma physics as logically coherent subjects, with some astrophysical examples to illustrate the applications of basic principles. I have followed the second approach, whereas the approach followed in Shu's book is closer to the first option. Fluid mechanics and plasma physics are venerable subjects with beautiful structures, and I personally believe that it is important for astrophysics students to appreciate the beauty of these structures rather than regarding these subjects only as tools for solving problems. Astrophysicists often have to deal with situations where it may not be obvious whether macroscopic continuum models work. Hence I have developed both the microscopic (particles) and the macroscopic (continuum) theories, establishing the connection between them.

I have attempted to present a unified discussion of neutral fluids and plasmas. One could think of writing a textbook in which theories of neutral fluids and plasmas are developed simultaneously. In such an approach, similar topics in fluid mechanics and plasma physics would be discussed together. I have followed the other approach of developing the theory of neutral fluids in Part 1 and the theory of plasmas in Part 2. Within each part, first I begin from microscopic theories and then develop the macroscopic continuum models from thereon. The main reason for presenting neutral fluids and plasmas separately is that the mathematical theory of neutral fluids is relatively simpler and I have tried to give a reasonably full account of it. On the other hand, the mathematical theory of plasmas is much more involved and it has often been necessary to leave some gaps in the arguments. Especially, the connection between microscopic and macroscopic theories of plasmas is an immensely complex subject which is still not on a rigorous and firm footing. I felt that readers

would be in a better position to appreciate the complexities and subtleties of plasma physics if they already have a knowledge of the corresponding topics in the theory of neutral fluids. Throughout the book, however, I have emphasized both the similarities and the differences between the theories of neutral fluids and plasmas. The reader will find occasional discussions of stellar dynamics as well, pointing out how the techniques of stellar dynamics compare with the techniques used in the studies of neutral fluids and plasmas. A full treatment of stellar dynamics, however, is beyond the scope of this book.

Nowadays, it is often regarded as a great virtue of a textbook if its chapters are completely independent of each other. A reader should be able to read any chapter without reading any other chapter! I admit to committing the grievous sin of writing a book like an interwoven tapestry with connecting threads running from almost everywhere to everywhere. I know that only a foolish author of a technical book in this busy age would expect a reader to read his book from the first page to the last page. Once a reader develops some familiarity with this book, I do believe that it will be possible for him/her to find his/her way through the book to topics of personal interest without reading everything preceding it. I should, however, point out that I have taken particular pains to show as to how the various topics discussed in the book are connected to each other. These connections will be best appreciated by a reader who reads through major blocks of the text in the sequence in which they are presented. This book contains about 30% more material than what I can comfortably cover in a semester of fourteen weeks, lecturing three hours a week. So the instructor of a one-semester course should have ample opportunity of planning a suitably tailored course based on this textbook.

In a book of this size, it is not possible to start from the basics and cover the research frontiers properly. So I have tried mainly to emphasize the basics, although I hope to have avoided giving the impression that this is a closed classical subject in which everything has already been established for posterity. The reader should at least be able to form an imperfect impression of the research frontiers. In keeping with this philosophy of emphasizing the basics, instead of giving references to the most recent publications surveying the research frontiers, I have mainly given references to important classic papers in which new concepts emerged for the first time or new discoveries were reported for the first time. While urging the readers to go through Faraday's original papers, Maxwell (1891, Vol. I, p. xi) wrote: "It is of great advantage to the student of any subject to

read the original memoirs on that subject, for science is always most completely assimilated when it is in the nascent state." I hope that my book would encourage the student to read the important landmark works in this field. While giving references in my publications, I, as a principle, try to look up the original materials. In the case of a few references of historical value, however, I had to rely on other authors who quoted these references, as they were not available in the libraries to which I had access.

This book is primarily meant for astrophysics graduate students. I do, however, hope that it will also be of interest to students of fluid mechanics and plasma physics as one of the rare textbooks treating neutral fluids and plasmas from a unified point of view, developing both the microscopic and macroscopic theories. Several years ago, I was attending a workshop on turbulence in Boulder. The workshop featured speakers on both fluid turbulence and plasma turbulence. The communication gap between fluid dynamicists and plasma physicists was very apparent, and it became particularly embarrassing when many fluid dynamicists were found absconding during the plasma turbulence sessions. A distinguished plasma physicist made some scathing remarks about the undesirability of the situation and suggested that somebody ought to write an elementary textbook stressing the unity of neutral fluids and plasmas, to help students develop a good attitude from the beginning. I leave it for others to judge if my book fits the bill. To make sure that the book is accessible to non-astrophysicists, astronomical jargon has been kept to a minimum and even the astrophysics examples discussed in the book should be understandable without any previous formal training in astrophysics.

Any comments or suggestions on this book may be sent to my e-mail address: arnab@physics.iisc.ernet.in

Arnab Rai Choudhuri
Bangalore

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This book is dedicated to E. N. Parker, whose course in Chicago served as the model for my course in Bangalore. Several portions of this book are, to a large extent, based on his lectures in that course.

I do not think that I have been able to capture the magic of his teaching, but I hope that this book is at least not unworthy of being dedicated to him.

Arnab Rai Choudhuri