Precision Cosmology

The First Half Million Years

Cosmology seeks to characterise our Universe in terms of models based on well-understood and tested physics. Today we know our Universe with a precision that once would have been unthinkable. This book develops the entire mathematical, physical and statistical framework within which this has been achieved.

It tells the story of how we arrive at our profound conclusions, starting from the early 20th century and following developments up to the latest data analysis of big astronomical datasets. It provides an enlightening description of the mathematical, physical and statistical basis for understanding and interpreting the results of key space-based and ground-based data. Subjects covered include general relativity, cosmological models, the inhomogeneous Universe, physics of the cosmic background radiation, and methods and results of data analysis.

Extensive on-line supplementary notes, exercises, teaching materials, and exercises in Python make this the perfect companion for researchers, teachers and students in physics, mathematics, and astrophysics.

Bernard J. T. Jones is Emeritus Professor at the Kapteyn Astronomical Institute of the University of Groningen. His research has covered many areas of astrophysics, both theoretical and observational, with a strong emphasis on cosmology, where he is widely published and cited.

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To the memory of my teenage daughter Suzie,
in fulfilment of my promise to her.

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Preface

The subject matter in this book covers the theory and observations relating to the earliest epochs of our evolving Universe, from its singular origin to just beyond the epoch of recombination: the first half million years. This is a fundamental period of the Universe's lifetime, and the period from which the key parameters describing it can be determined. As our understanding of the Universe grew, and new observations were found to rule out the simplest models, new physical entities and concepts had to be introduced to ensure consistency between theory and observation. The subsequent period, from recombination to the present epoch, is the era during which stellar and galactic physics takes place, and is not covered in this volume.

Precision Cosmology is a teaching book and a technical reference book which aims to provide a clear and understandable description of the applied mathematics and physics required to establish the high level of precision which can now be achieved for the study of the Universe – Precision Cosmology. It details the background and mathematical tools needed to enable a student embarking on advanced studies and research to understand what others have done and published.

This book is also an experiment in presentation. The printed text is accompanied by extensive online supplements, which provide a high level of didactic mathematical material too extensive to be included in the main text. These supplements are freely available for download and are intended as a source of teaching material for both teachers and students across a range of topics in physics, mathematics and statistics. They are suitable for all levels of university study, from undergraduate to postdoctoral research. The online supplements and other support material are available from

`www.BernardJones.net/PrecisionCosmology.html`

Figures in the text marked 🅭 are available on this website and are free for use under the terms of the CC-BY-SA Creative Commons License. Exercises and slides for teaching purposes are also to be found on this site.

In this way, much of the book should be accessible to physics and mathematics undergraduates who have had courses in mechanics, electromagnetism and special relativity. The aim is to take the student beyond this basic knowledge, right up to the level of being able to follow and understand research papers, and thus to understand today's cosmology in its proper context as a branch of physics. In this way the text should be equally useful to those postgraduates wishing to embark on research into cosmology.

Our understanding of the Universe, the science of cosmology, has evolved remarkably over the past century and, in particular, over the years since the discovery in 1965 of the Cosmic Microwave Background Radiation (CMB). That discovery established the basic paradigm of modern cosmology: the Hot Big Bang theory. So, by the turn of the 21st century another revolution was already under way: a revolution based on big science and big data, with data gathering projects consuming considerable resources in terms of telescopes, computers, detectors, etc., and involving big teams of engineers and scientists, numbering in their hundreds or even thousands. Cosmology today is largely about the acquisition, modelling and analysis of data with which we may confront our theories.

This book describes the science that forms the basis for our understanding of the Universe and drives the design of experiments to test that understanding. The main focus is on the methods and tools that are are available to model and analyse the data from these experiments. This is the basis for our profound conclusions about the nature of our Universe.

Good science needs corroboration from alternative directions. However, unlike almost every other branch of physics, we have only one example of the object of our study: our Universe is unique and we cannot study any other for the purposes of corroboration. We cannot just take another sample. Corroboration therefore requires considerable ingenuity and that, to some extent, has come via simulations of the Universe, from analysis of the distribution of galaxies and from gravitational lensing studies. These very different approaches to determine the cosmological parameters yield very similar results. Yet none of these, taken alone, would provide convincing evidence that our interpretation is correct. However, taken collectively, and combining also with the CMB, Baryonic Acoustic Oscillations (BAOs) and supernova data, our conviction that everything is self-consistent is strengthened.

Fifty years before the publication of this book, cosmology was a small but important branch of Einstein's General Theory of Relativity (GR). Observational cosmology at that time was once described as merely being 'a search for two numbers' that would define which of the relativistic models would best describe the Universe. Fifty years before that our Galaxy was believed to be the entire Universe: all the nebulae, clusters and stars were thought to be part of the one super-system, enshrined first as the 'Kapteyn Universe' and a little later as Harlow Shapley's 'meta-Galaxy'. At that time, Einstein's theory had only just achieved its final form.

Today, at the start of the 21st century, we have explored, mapped and modelled our Universe using vast quantities of data obtained and analysed by teams of thousands of engineers and scientists. We have determined the six basic parameters that describe our model with remarkable accuracy, often to two or three significant digits. While this effort has successfully yielded a convincing paradigm, it has revealed a number of important issues such as the mysteries of dark matter and dark energy. There is now strong evidence that 72% of the Universe comprises what is referred to as 'dark energy', and we have no idea what this is. A lot of ingenuity will go into future research to discover its nature.

The Structure of the Book

The text of the book is divided into five fairly independent parts, and each part is then divided into chapters. The five parts are:

I. 100 years of Cosmology
A recap on the highlights of the past century. Much of this could be regarded as introductory material for the parts that follow, especially as regards setting the material in a holistic context.
II. Newtonian Cosmology
Simple Newtonian cosmological models and what they can explain.
III. Relativistic Cosmology
Special and general relativity and the greater precision it gives.
IV. The Physics of Matter and Radiation
The physics of the cosmic background radiation.
V. Precision Tools for Precision Cosmology
Statistical methods, precise observations and data processing techniques.

Bibliographic Material

The bibliography is hardly complete, despite there being some 1000 references – there are today tens of thousands of papers that are relevant to cosmology. The cited papers are mainly the ones on which I have drawn in order to write this book. Generally, such papers are either unique, or they have a nice way of explaining their goals and methods that I will have borrowed for my text. The citation of those latter authors is one way I can acknowledge my gratitude for their clarity of exposition.

I also prefer to cite papers that are generally available over the internet either through an openly accessible archive, or through the generosity of the publisher. I have avoided citing papers that are only available to those having subscriptions to journals or who are willing to pay for downloading copies of the articles. I hold to the belief that the tax payer has already paid generously for the science we do and, as a consequence, should have full and free access to its results.

Use of Internet Resources

Much of the information that is on the periphery of my own area of expertise has been culled from the numerous excellent web pages of astronomers, physicists and statisticians around the world. This material is mostly in the form of conference presentations, lecture notes, diagrams and e-prints. This has obviously played a part in my own education both as an astrophysicist and as a teacher. It would be impossible to even recall where most of the acquired knowledge came from, let alone cite it. I can only thank all those people without specific acknowledgement.

This book would not have been possible without access to that vast and freely available resource: the Internet. Access to the SAO/NASA Astrophysics Data System Digital Library (ADS) has been invaluable, facilitating my trawling through thousands of articles. The help of Edwin Henneken of ADS has been much appreciated. Of course, there are also

Wikipedia, NASA, and the e-print arXiv who make and keep information freely accessible. These are an essential part of scientific research that can only be fully appreciated by those of us who are old enough to have been without them.

Personal Acknowledgements

Much of the material here has been culled from my own research, review articles, scientific papers (published and unpublished), text books, resources on the world wide web, and even from the lecture courses I attended as an undergraduate and graduate student many years ago. Inevitably the clearest methods and explanations I have learned from these sources have found their way into my own lecture courses and hence into this text. In particular I am indebted to my own teachers of long ago whose clarity enthused me about relativity and cosmology through their lectures and their books: C.W. Kilmister, F.A.E. Pirani, H. Bondi, D.W. Sciama, M.J. Rees, G.F.R. Ellis. Their superlative lectures are still burned into my brain. Throughout my career I have been fortunate to have mentored and supervised a large number of talented graduate students. That process has been a key part of my own education both as a teacher and a scientist.

I am indebted to Jan Tauber, Planck Project Scientist, for allowing me to use Planck pictures and edit them for publication in grey tones. Indeed, I would like to thank all those who granted permission to use their figures and those who offered advice.

On a more personal level it is important that I acknowledge the help from my wife, Janet, in editing this book and making it a reality. Without her stringent hands-on help I would still have over 1500 pages of text and no book. She was able, despite protestation, to throw out material until it got down to some 700 pages, just short of the Cambridge University Press 750 page limit. Her corrections to my grammar, compiling lists of errors, and for general encouragement when things got difficult or even seemed impossible, are in retrospect highly appreciated. She made it happen. Indeed, Cambridge University Press (CUP) have been incredibly tolerant of my ever-shifting deadlines, starting in the mid-1970s when this project was first conceived. Cosmology changed rapidly since that time, and evidently faster than I could write. The efforts of my Copy-Editor, Dr Richard Smith, in bringing this volume into shape are particularly appreciated.

Rien van de Weygaert has played a singular role in this long term project, for which I am eternally grateful. He also provided me with a ‘home’ in Groningen. Rien has acted as my mentor throughout that period, and his help, advice and support have been more appreciated than I can put into words. Most of the book in its present form was written while at the Kapteyn Institute. I am proud that I should be able to put the Kapteyn Institute as my affiliation on the title page of this book.

Last but not least, my entire family is owed a tremendous vote of thanks for their encouragement and support, despite their wondering whether it would ever be possible for me get this book finished. Well it’s now finished, and a testament to the value of their support.

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Notation and Conventions

Throughout this book I have used ‘astronomer’s units’ on the grounds that these are the units in which most of the astrophysics literature and textbooks are written. So brightness is given in ‘magnitudes’ and distances of astronomical object in parsecs, and so on. However, there are some instances that are a matter of choice of particular conventions.

Special Relativity

In the chapters on relativity and the conventions I have adopted there are best described as ‘MTW’ ((Misner *et al.*, 1973)) since that is a commonly used textbook for general relativity. Those conventions also accord with Hawking and Ellis (1975), Wald (1984) and the wonderful *Varenna Lectures* of Ellis (1971) (reprinted as Ellis (2009b)).

Throughout the section on General Relativity we use geometrised units in which $G = 1$ and $c = 1$ unless explicitly shown otherwise. Where particularly relevant, as in discussing observed data, these units will be converted to the dimensional units as are typically used in astronomy (i.e. astronomer’s units).

The *signature of the metric* will be taken as $(-+++)$ and space-time coordinates in this system will be described as 4-dimensional vectors as in (x^0, x^1, x^2, x^3), the x^0 coordinate representing the time-like coordinate. This is the space-like $(1,3)$ format. In that format the Minkowski metric for the line element ds when the coordinates are labelled (t, x, y, z) is

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2 = -c^2dt^2 + d\mathbf{r}^2,$$

with which the *proper time*, $d\tau$, is defined as

$$d\tau^2 = -ds^2.$$

With this, *time-like* intervals have $c^2dt^2 > d\mathbf{r}^2$, i.e. $d\tau^2 > 0$ and, conversely, *space-like* intervals have $ds^2 > 0$, i.e. $d\tau^2 < 0$. Accordingly the *proper time* corresponding to a time-like interval separating two events is

$$d\tau = \sqrt{dt^2 - dr^2/c^2},$$

while the space-like separation between two events separated by a space-like interval is

$$d\ell = \sqrt{dr^2 - c^2dt^2}.$$

General Relativity

Lower case Latin indexes $a, b, c, \ldots$ on vector and tensor quantities, as in u^a, and T^{ab} will take values $0, 1, 2, 3$, while lower case Greek indices $\alpha, \beta, \gamma, \ldots$ will take on the values $1, 2, 3$. The summation convention, implying summation over repeated indices, is used on lower case Latin and lower case Greek indices, but is suppressed when using upper case Latin indices.

We need to define some important symbols and tensors. Firstly, given a metric tensor g_{ab} we define the Christoffel symbols Γ^a_{bc} as

$$\Gamma^a_{bc} = \tfrac{1}{2} g^{ae}[g_{eb,c} + g_{ec,b} - g_{cb,e}].$$

With this we can define the Riemann Tensor as

$$R^a{}_{bcd} = \Gamma^a_{bd,c} - \Gamma^a_{bc,d} + \Gamma^a_{mc}\Gamma^m_{bd} - \Gamma^a_{md}\Gamma^m_{bc}.$$

The Ricci tensor R_{ab} and Ricci scalar R are the following contractions of the Riemann tensor:

$$R_{ab} = R^m{}_{amb} \qquad R = R^m{}_m.$$

The Einstein tensor is

$$G_{ab} = R_{ab} - \tfrac{1}{2} g_{ab} R.$$

Covariant derivatives with respect to the metric g_{ab} are denoted by the symbol ∂_a, or, using subscripts:

$$u^a_{;b} \equiv \partial_b u^a = \frac{\partial u^a}{\partial x^b} + \Gamma^a_{mb} u^m = \partial_b u^a + \Gamma^a_{mb} u^m = u^a_{,b} + \Gamma^a_{mb} u^m.$$

Covariant derivatives do not commute:

$$u^a_{;bc} - u^a_{;cb} = -R^a{}_{mbc} u^m.$$

The signature used here is $(-+++)$, as in MTW. To transform from the equally widely used $(+---)$ signature, change the signs of the following:

$$g_{ab}, \quad \Box = g^{ab}\partial_a\partial_b, \quad R^a{}_{bcd}, \quad R_{ab}, \quad T^a{}_b,$$

but leave these unchanged:

$$R_{abcd}, \quad R_a{}^b, \quad R, \quad T_{ab}.$$

Electromagnetism

The electromagnetic units used are also ‘astronomer’s units’, as opposed to the widely taught SI units. The varieties of electromagnetic units are discussed at some length in Appendix A. Since the goal here is to discuss Maxwell’s equations and not to discuss the experimental side of electromagnetism, the units used here follow the system that is the most convenient and concise for this purpose.