1 Introduction

Ptolemy's reputation as one of the outstanding scientists of the ancient world rests mainly on his epoch-making treatise in astronomy, the *Mathematike Syntaxis* (usually known as the *Almagest*). Modern students of the ancient sciences know him also for his writings on optics, geography and astrology; but with a few honourable exceptions they have shown rather little interest in his *Harmonics*. Those who have examined it in detail have not, for the most part, been historians of science. Either they have been concerned less with the text in its original setting than with its afterlife in Renaissance musicology, or else they are dedicated (even fanatical) specialists in ancient musical theory; and we are rather few. But even if the subject it addresses continues to languish (as it should not) in a cobwebby corner of our gallery of the Greek sciences, the *Harmonics* itself deserves much wider attention.[1]

It is in the first place a work of real intellectual distinction, and its skilfully mustered arguments, despite their technical intricacies, are presented with a flair and panache that should commend it to any connoisseur of scientific writing. Secondly, it is quite unusually explicit and self-conscious about its own methodology and procedures. In this respect it has a good deal more to offer than the *Syntaxis*, whose overt reflections on the general features of the science are relatively brief and less directly methodological, and play a notably less prominent role in the development of the subsequent argument. The *Harmonics*, by contrast, announces and seeks to justify at the outset a sophisticated set of procedural principles which scientists in this field, so it argues, must follow if they are to produce defensible results. It also repeatedly reminds us of these principles in the course of the investigation itself, alerting readers to the ways in which each of its stages fits into the prescribed pattern of procedures, or why it is needed to ensure that the procedures are carried

[1] The problems posed by the text will be significantly eased by a detailed, scholarly study that has just been published (Solomon (2000)), but which reached me, unfortunately, too late to be taken into account while I was writing this book.

through effectively and reliably. Neither the *Syntaxis* nor even the essay *On the Criterion*, devoted though it is to issues concerning scientific understanding and the means by which it should be sought and assessed, gives such clear insights into Ptolemy's conception of the methods appropriate to a science, and of the presuppositions on which its enquiries rest.

The task I have set myself in this book is to explore the *Harmonics* from a methodological point of view. Its own pronouncements on these matters are of great interest in their own right, and demand close analysis. But it will also be necessary to ask how far the treatise is faithful to the principles it advertises, in the actual conduct of its investigations. There are grounds for some scepticism here, and special reasons why the issue should be thought important. The complex combination of rationalism and empiricism which Ptolemy professes to adopt insists, among other things, on a crucial role for experimental tests of provisional, theory-based results. Here, as we shall see, the word 'experimental' is to be construed in a strict sense that will seem surprisingly modern. I hope to show beyond reasonable doubt that Ptolemy understood very well what conditions must be met if experimental tests are to be fully rigorous, and that he had a clear and persuasive conception of the roles they should be assigned in a well conducted scientific project. I do not think that these ideas are so fully worked out and so lucidly expressed in any other surviving Greek source. What is much harder to decide is whether the experimental equipment he meticulously describes was ever actually built, whether his carefully designed and controlled experiments were ever conducted, and if they were, whether he allowed their results genuinely to modify or to put at risk the theoretically grounded conclusions which they purported to test. Greek science in general is not renowned for its adherence to experimental methods. Harmonic scientists in particular often claim that their theoretical results are confirmed 'by perception', sometimes offering geometrically conceived descriptions of instrumental devices through which (they allege) these results can be presented to the ear. But their remarks seldom inspire much confidence in the supposition that the instruments were actually built and used, still less that they were used in an experimental spirit; they seem to have been thought of, at the most, as making manifest 'rationally' excogitated truths to the senses, rather than as putting them to the test. If we are to conclude that Ptolemy not only represented the use of strict experimental techniques as an essential element in a well conducted scientific project, but also carried his programme through in practice, the case will have to be argued in detail and with the greatest caution. Certainly the author's explicit statements about his own procedures should not be taken at face value without a good deal of supporting evidence.

To anticipate the book's conclusions on this issue, I believe that a very strong case can be made in Ptolemy's favour, and I shall do my best to provide it. If it can indeed be shown that when he wrote the *Harmonics*, Ptolemy not only had a well honed understanding of experimental methods but was also seriously committed to their use, that fact should obviously provoke the question whether this treatise is merely a freakish anticipation of later concepts of science, or whether once these methods have been drawn to our notice, we shall be able to find convincing traces of comparable procedures in other Greek works in the 'exact' or 'mathematical' sciences. Such questions have of course been asked before; but it may be that a starting-point in the *Harmonics*, where the issues are brought so insistently to our attention, will place them in a fresh perspective. My business in this book is only to provide the necessary point of departure. No doubt the wider questions are the more important, but they must be reserved for a different book and probably for a different writer. Here I intend to keep the focus as sharp as possible, restricting myself to an examination of this single text, without drawing elaborate comparisons or attempting to generate large conclusions about Greek science in general.

Ptolemy's treatment of the strictly 'rational' or 'theoretical' phases of his enquiry also raises issues relevant to the other sciences, particularly, perhaps, to astronomy. He proposes that what the ear perceives as musically admirable relations between pitched sounds are manifestations of mathematically intelligible and elegant form; and their complex and various structures can be derived, through orderly mathematical procedures, from principles of 'reason' whose credentials are accessible to the mind. This is all very fine and inspiring. But given that our initial data are simply patterns of sound which are perceived as musically satisfactory, we must plainly ask, first, how we are to represent them in ways that express their mathematical form and make them amenable to 'rational' (that is, mathematical) manipulation. We must also ask how we are to move from our initial perceptions to a grasp on the principles which govern their orderly relationships; why it is that these rational principles and no others are the appropriate ones for the task; why it is that some mathematically describable patterns are 'better' and correspond to 'finer' musical relations than others; by what procedures well formed musical systems are to be derived from the initial principles, and why (since different methods of derivation will yield different results); and so on. All these questions have their counterparts in astronomy, at least as Ptolemy conceived it, and the answers offered in the *Harmonics* may shed some light on the character of his reasoning in the *Syntaxis*, perhaps on that of other ancient astronomers too. But those issues, once again, will not be addressed in this book.

From time to time in the course of this study it will be necessary to examine rather closely some of the finer details of Ptolemy's arguments, partly for the light they shed on the nature and application of his method, and partly for their own intrinsic interest. The procedure of the *Harmonics* depends to a high degree on rigorous reasoning, and its sophisticated intricacies can on occasion provide matter for serious philosophical reflection. I shall also suggest, on the other hand, that some of his arguments fail to pass muster by the standards he purports to accept. Some of his constructive strategies seem to break down in their applications; and his criticisms of his predecessors are sometimes more rhetorically than rationally persuasive. It is not always possible to judge whether Ptolemy is merely being too hasty, or whether on some occasions he is deliberately seeking to mislead. This book is a discussion of procedures and the principles governing them, not primarily of the substance of Ptolemy's conclusions or those of other writers in this field; but it would hardly be possible to explore the issues at which I have gestured, at least in any depth, without introducing some musicological technicalities. I shall expound them, however, only to the extent that seems necessary for my main purposes, and a good deal of detail will be ignored. I shall also do my best to introduce them in ways that will be accessible to readers unversed in the conundrums of Greek harmonics, and digestible by those for whom the subject is not itself of special interest.

Because of the narrow restrictions I have placed on my project, and because the *Harmonics* has previously received little close attention from a methodological point of view, this study will seldom engage directly with the work of other modern students of Greek science. I have of course learned a great deal from them. But readers of this book will find it alarmingly free of the reassuring paraphernalia of scholarly footnotes, acknowledgements and backbiting which have become normal in academic literature. I expect to be taken to task for these omissions, but I do not apologise for them. My aim is simply to focus all attention squarely on the contents of Ptolemy's text, with as few distractions as possible.

Greek harmonic science studies a variety of topics, none of which has more than a tangential connection with 'harmony' in the modern English sense of that word. Its general field of operations is melody, the musical sequence of a single melodic line rather than an array, or a sequence of arrays, of simultaneous sounds; and the line's musical credentials and character are conceived – and were heard – as depending solely on its own structure, not on its relation to any accompanying sonorous events, real or implied. Chordal harmony, harmonic progressions and so on are notions quite foreign to the Greek experience. At the centre of the concerns of the science is the analysis of the elements from which musical attunements are constructed, and of the systematic patterns of relationships in which these elements are bound up to form an organised structure. As a rough preliminary guide, we may conceive an 'attunement' as the system of relations holding between the pitches of the strings of a lyre, for example, when they have been so adjusted that a musical melody can be played on them accurately, with all its intervals 'in tune'. It provides the pattern of pitches and intervals upon which the melody will draw.

A well formed attunement differs from a random collection of pitches in two principal ways. There are limitations, most obviously, on the ways in which adjacent pitches can be related to one another, that is, on the sizes of the intervals that can lie between them, and which a musician can use as basic melodic 'steps'. Though the Greeks admitted in their music a much greater variety of elementary, 'scalar' intervals than are found in later Western practice, not every physically constructible interval was recognised as capable of taking an aesthetically intelligible role in a melodic sequence. There is a distinction to be drawn, then, between musical and unmusical intervals. Secondly, not every possible arrangement of melodically admissible intervals constitutes a well formed attunement. A series of pitches becomes a melody only when they are located within a structure of relations which is recognised as musically coherent; and this structure must be exhibited in the attunement on which the melody is based. Different patterns of relations between pitches and intervals constitute different forms of attunement, each of which is the matrix for a different class of melodies. Several types, distinguished according to criteria of more than one sort, were regularly employed by Greek musicians. But their diversity, like that of the elementary intervals, was not unlimited. Only some arrangements of intervals could be accepted as exhibiting the structure of a musical attunement.

From this perspective, harmonic science had two fundamental tasks. The first is to identify with the greatest possible precision those intervals and structures from which attunements are constituted, marking out the boundaries between them and the realm of the unmusical. The second is to look for the principles on which this distinction between the musical and the unmusical is founded, principles to which a musically acceptable arrangement of intervals must conform. In calling these enquiries 'first' and 'second' I do not mean to imply anything about the order in which they are to be undertaken, or about the logical or epistemological priority of the one over the other. These were issues on which opinions differed; and for Ptolemy, as we shall see, the postulation of principles is in the most important respects prior to the accurate identification of musical intervals.

The part of the programme which I have called 'the first' could be extended in a number of directions, most importantly into enquiries about the ways in which systems of intervals could be grouped and understood in relation to one another – the ways in which small intervallic structures, for instance, could be linked in a series, and the ways in which they could not; or again, the criteria according to which certain structures should be conceived as fundamental and others as derived from them by acceptable processes of transformation. As to the enquiry into principles, Greek theorists of all persuasions were confident that such principles exist, that the differences between the musical and the unmusical are objective and accessible to scientific enquiry, and that they will be found to be orderly and intelligible. They are not dependent on temporary social conventions, or on whim or personal taste, though we, as imperfect human individuals and ones whose perceptions are partly conditioned by our education and social experience, may disagree about where the boundaries lie. If we do, that shows only that we are unreliable judges in the matter, not that the boundaries themselves are flexible or vague. But there was much dispute about the identity of these governing principles, and more radical dispute about the general character of principles proper to the field of harmonics, and of the domain to which they belong.

Though many nuances of opinion on this issue existed, they fell broadly into two groups. Some theorists, notably Aristoxenus and his followers, held that musical principles are autonomous, specific to music itself, not applications to a particular subject matter of laws holding in a wider domain. Others, in a tradition associated with Plato and with a school of thought loosely labelled 'Pythagorean', took the opposite view. Conceiving the relations between pitches in an attunement as essentially quantitative, and the structures formed by them as constituted by relations between quantities, they looked to the science of quantity, mathematics, for the principles by which 'harmoniously' coordinated systems of relations are distinguished from incoherent jumbles. These principles were located specifically in number-theory and in the theory of ratio and proportion. On this latter approach, musical principles are not autonomous. They are to be understood as applications or special cases of principles governing quantities in general, and referring in the first instance to numbers. The perceptible harmoniousness of a musical system is thus a reflection of the intelligible, mathematical coherence of the pattern of quantitative relations between its elements.

This distinction must be explored a little further and linked with certain others. For Aristoxenus and his successors, and perhaps also for those earlier theorists whom he called *harmonikoi*, melodic relations are characterised by their conformity to principles specific to the musical domain. This means that the forms of coherence exhibited by acceptable attunements, the rules governing musical successions of intervals, and so on, are not ones that hold in a wider field and apply to musical relations because they fall within it, or because they constitute one of various modes of being in which its forms are instantiated. Specifically, relations between pitches are not to be conceived, for the purposes of musical analysis, as relations between quantities, subject to principles of organisation drawn from mathematics. The reason is essentially a simple one. Melody exists in a dimension accessible only to the hearing, and it exists in forms of relation that the ear is capable of recognising. Even if it is true from the perspective of physics that sounds are disturbances in the air, and that they differ in pitch through quantifiable variations in some aspect of these disturbances, it cannot be the relations between these quantities, as such, that give certain sound-patterns their character as melodic, since in hearing them as constituting melody we do not perceive their pitches in the guise of things differing in quantity. If the relations between the notes of music were essentially relations between quantities, then since we do not hear their differences in pitch *as* differences in quantity, we would be unable to hear anything at all as constituting a melody.

Aristoxenus studied with Aristotle, and seems to have been much influenced by his work in metaphysics and the philosophy of science. Here a rudimentary sub-Aristotelian analogy may help. Melody is made up of pitched sounds; they are the 'matter' out of which it is put together. Animals, similarly, are made of certain material stuffs, ones which are compounds, at the most basic level, of the four material elements. Some of the properties of animals are due to their material constitution. But to be a tiger or a kangaroo is not merely to be an assemblage of these elements in some special combination. A kangaroo has properties that are not derived from those of its material constituents, but depend on its possession of a 'nature' additional to theirs, one that actively determines the organism's structure, the course of its development to a determinate mode of completion, and the forms of activity characteristic of its kind. Just so, according to Aristoxenus, though not, as it happens, according to Aristotle himself, 'the melodic' or 'the well attuned' is a *phusis*, a nature, a form of reality independent of all others and obedient to principles peculiar to itself; it organises its materials, sounds, according to rules and patterns of its own. Studies in the physics of sound, that is, of the properties of the materials out of which melodies are made, will no more inform us of what it is to be a melody than studies of the properties of the four elements can reveal what it is to be a kangaroo, and what principles govern such a creature's organisation and behaviour.

Important consequences flow from this position. One is that in a broad sense the science of harmonics must be empirical in its procedures, drawing on the phenomena accessible to the sense of hearing for its data, seeking the principles that govern them through some form of induction or abstraction from these data, and assessing general hypotheses by means of empirical tests. It cannot proceed by seeking to derive them, by reasoning, from principles of an allegedly higher sort. There are no higher principles from which it follows that melody must have the form and nature which it does, any more than there are principles allowing us to derive the special properties of kangaroos from theorems holding of some more inclusive class of beings. Aristoxenus is not an 'empiricist' in the crude sense in which some of his predecessors apparently were. Nevertheless, he insists on the authority of perception as the criterion of what is and what is not harmonically correct. In the end, we can have no reason to believe that this or that principle must necessarily hold, except that attentive listening and reflection on what we hear convince us that it actually does.

A second consequence has to do with the language in which harmonic science is to be articulated and the concepts within which it is framed. If melody is an autonomous phenomenon existing only in the realm of what is heard, the way in which the phenomena are described must reflect the manner in which they strike the ear when they are perceived *as* melody. Musical relations hold between sounds in their guise as audible items, and melodic relations hold only in respect of those aspects of their audible character which affect our perception of them as melodic or unmelodic. For the latter reason, some features of sounds, such as their loudness or brightness, are irrelevant to harmonics even though they are audible; the same melodic relations are involved irrespective of volume or timbre. For the former reason, and more importantly, there is nothing to be learned in harmonics from aspects of sounds that are not perceived as such by the ear, in particular from their character as movements of air which differ in some quantitative manner. Hence sounds conceived under that description have no place in harmonics; the language of the physicist is inappropriate and misleading in the context of this enterprise. In practice, the language of Aristoxenian harmonics is a fusion of a theoretical terminology drawn from Aristotelian natural science with a sophisticated extension of that used by musicians themselves. It represents the elements and relations proper to music in terms that are meaningful and familiar to those most sensitive to their musically significant nuances.

Platonist and 'Pythagorean' approaches to harmonics contrast with that of Aristoxenus at every step. As I said earlier, the principles governing musical relations are on their view not autonomous, but belong to mathematics. Relations between pitches are to be conceived quantitatively; in essence they are ratios between numbers. The language in which these relations are to be expressed, for the purposes of scientific harmonics, is not that of musicians, which is incurably imprecise, and is designed to reflect only the impressions of the senses; it is the exact terminology of number-theory. Finally, the proper criterion of correctness in musical relations is not auditory perception but pure mathematical reason. A correct attunement is one formed according to principles which are intelligible to the mind, and whose privileged status in this domain can be explained and justified by reason. The question whether human perception will recognise it as correct or musically acceptable is of little importance, or none.

I have stated these contentions in their most radical form. There was, however, no single, monolithic 'school' of mathematical theorists. Their views varied in detail more, perhaps, than did those of the rival tradition, at least after about 200 BC. By that time (if not earlier) musicologists of an empirical persuasion had apparently come to treat the writings of Aristoxenus as carrying overwhelming authority, whereas no single figure had comparable status on the mathematical side. Doctrines attributed to the semi-legendary Pythagoras were indeed accorded unfailing respect, but too little was known of him for this to determine more than the most general outline of their approach. Plato, too, carried considerable weight, but his authority was not felt as binding by all mathematical theorists; and the relevant passages of his dialogues are in any case enigmatic enough to legitimise a variety of interpretations and extensions of Platonic views.

Nevertheless, these theorists shared a good deal of common ground. Their starting point was the observation that differences in pitch are correlated directly with quantitative differences in certain physical variables, and that the pitch-relations most fundamental to musical structures correspond to strikingly simple ratios between values of the variables involved. The relevant facts are familiar. The notes produced by plucking two lengths of a uniform string, one twice as long as the other, will sound exactly an octave apart, the greater length giving the lower pitch. The ratio exemplified here is thus 2:1. The two other structurally crucial intervals in Greek harmonics are the concords of the perfect fifth and the perfect fourth; and these are found to correspond respectively to the ratios 3:2 and 4:3. And just as a fifth and a fourth together make up an octave, so the product of the ratios 3:2 and 4:3 is 2:1. (That is, if a length of string is increased in the ratio 3:2, and the resulting length increased in the ratio 4:3, the final length will be twice as long as the original.)

It could scarcely be a coincidence that the three simplest and most basic musical relations correspond to this orderly set of the simplest ratios of whole numbers. From here it was a short step to the hypothesis that musically acceptable relations are so precisely *because* they correspond to numerical ratios of some privileged sort. In that case, of course, it could not be the ratios between lengths of string, as such, that are responsible for this harmoniousness. The ratios must in some sense belong to the pairs of sounds themselves, and sounds are not lengths of string; and melody can be produced by other means, for instance through pipes or by the agency of the human voice. Sounds are produced by any of these agents, however (so it was argued from the time of Plato onwards), through impacts that create movement in the air. A sound's pitch came to be treated, then, as a quantitative feature of this movement. But a further step was necessary. If the ratios accessible to observation in lengths of string and the like are to have any musical significance, there must also be a direct correlation between relative lengths of a string (or of a pipe, or appropriate dimensions of other sound-producing agents) and relative values of the physical variable that constitutes a sound's pitch. Several hypotheses were offered about the nature of this variable property of movement; but most theorists related it in one way or another to degrees of rapidity, greater speeds going with higher pitches and slower ones with lower. On some views, this 'speed' is simply the rate of a sound's transmission through the air, so that a sound of a given pitch, on this hypothesis, travels exactly twice as fast as the sound pitched an octave below it. According to others, the relevant variable is the rapidity with which impacts on the air are repeated in the production of a sound, for instance by the oscillating string of a lyre, so that pitch is associated with the frequency with which the resulting impulses follow one another through the medium. In either case, there was no great difficulty in arguing that changes in the value of the relevant variable must be directly correlated with changes in the dimensions of the sound-producing agent. Longer strings, for example, generate slower and less frequent impacts on the air.

The details of these different opinions need not concern us. If relations between pitches are ratios between numbers that measure the values of some such variable, be it speed of transmission, frequency of impact or anything else, and if musical relations are distinguished from others by the form of the ratios that characterise them, the most obvious challenge to the mathematical theorist is that he identify the ratios corresponding to every acceptable musical interval. But this identification, by itself, would leave crucial questions unanswered. If the musicality of the intervals is due to the mathematical form of their ratios, we need to enquire what 'form' it is that is shared by all these ratios and by them alone, what special mathematical features these ratios possess and others lack. Musical ratios must be 'musical' in virtue of their conformity to