

# Index\*

Barycentric subdivision of finite regular cell Abelian group, 36 complex, 121 Abramson, M. Bases of vector space, for polynomial generat-Terquem's problem (exercise), 63 ing functions, 208 Acyclotope, 290 Bayer, M. M., and Eulerian posets, 199 Adjacency matrix: Bedrosian, S. D., and counting rooted forests of digraph, 241 entries in  $A^n$  (theorem), 242 (exercise), 94 Bell, E. T., and incidence algebras, 149 Aigner, M .: and closed poset, 183 Bell number, 33 Bennett, M. K., and Eulerian posets, 197 and intervals of posets, 175 and uniform geometric lattice, 191 Betke, U., and integral convex d-polytopes, Alexander duality theorem, for simplicial com-Bijection, 11 plexes, 137 Billera, L. J., and dimension of vector sub-Alexandroff, P. S., finite posets and finite topologies, 174 space, 199 Alphabet, as finite set, 247 Binary trees: Andrews, G. E.: number of unordered (proposition), 25 counting triangles with perimeter n, 281 properties of (proposition), 24 Binomial coefficients, generating function, Euler pentagonal number formula (exercise), approach to, 14 Binomial posets: and pleasant posets, 182 and q-Dyson conjecture, 54 definition, 142 and generating functions, 140 Antichain, as subset of poset, 100 literature references, 151 Antimagic square, 272 Atiyah, M. F., and number of words in quoand subalgebra of incidence algebra (theorem), 144 tient monoid, 277 Atom, of finite lattice, 104 Birkhoff, Garrett, and posets and lattices, 149 Birkhoff-von Neumann theorem, 93 Autocorrelation polynomial, 267 Björner, A.: and Bruhat order, 199 Backelin, J., and number of words in quotient monoid, 277 and core of irreducible poset, 176 and Crapo complementation theorem, 184 Baclawski, K .: and finite supersolvable lattice, 190 and core of irreducible poset, 176 and topological properties of posets, 150 and odd composition poset, 191 Block, of set partition, 33 and topological properties of posets, 150 Baker, K. A., and dimension of poset, 176 Board, and general theory of derangement, 71 Banaschewski, Z., and MacNeille completion, Bonferroni, over and undercounts for approximation (exercise), 91 176

\*Index compiled by E. Virginia Hobbs

296



Boole, G., and partially ordered sets and lat-	Cohen-Macaulay posets:
tices, 149	and lexicographically shellable posets, 151
Boolean algebra of rank $n$ , as distributive lat-	literature references to discovery of, 150
tice, 107	shellable, 193
Bressoud, D. M.:	Coimage, 38
proof of $q$ -Dyson conjecture, 54	Combinations with repetition, and multiset, 15
and tournament (exercise), 95	Combinatorial interpretation, of polynomial
Bruggesser, H., and Ehrhart polynomial, 288	associated with Eulerian poset, 139
Bruhat order, of symmetric group, 172, 199	Combinatorial proof:
Brylawski, T., partitions of integer ordered by	definition, 11
dominance, 192	and non-combinatorial proof for same ex-
Butler, L.:	amples, 12
chains of Young diagrams, 284	Combinatorial reciprocity theorem (example),
and set of infinite words, 191	16 (see also Reciprocity theorem)
	Comparability graph, 168
Calculus (formal), for formal power series, 7	Comparable elements, of posets, 97
Calculus of finite differences, 36	Compatibility:
Canfield, E. R., and Stirling number of the	between injectivity and equivalence relation,
second kind (exercise), 57	32
Cardinality:	between surjectivity and equivalence relation
comparison between two finite sets by bijec-	32
tion, 79	Compatible:
of multiset, definition, 15	dual ρ- (lemma), 212
Carlson, F., and power series with integer	$\pi$ and P-partition (lemma), 212
coefficients, 282	Compatible permutations, and descent sets
Cartier, P.:	(lemma), 212
applications of theory of hyperplane arrange-	Completely fundamental elements:
ments, 192	of $n \times n$ magic squares (lemma), 232
and monoid, 189	of $n \times n$ symmetric magic squares (lemma)
Catalan numbers:	234
and a difference equation, 62	of rational polytope (lemma), 236
and non-crossing partitions, 169 and number of linear extensions of poset,	Composition of n:
112	definition, 14
Cayley, A., and generating function, 151	and factorization in free monoids, 250
Cellular decomposition, of Grassmann variety	Comtet, L., and convergence of generating function (exercise), 61
and partitions of vector space, 30	Connected poset, 100
Chain:	Convergent formal power series, 6
definition of poset as, 99	Convex polytope, integral (corollary), 238
in distributive lattice, 110	Convex subposet, 98
increasing and R-labeling, 133	Convolution, for locally finite poset, 113
saturated, 99	Core, of irreducible finite poset, 155
Chain condition:	Counting, subsets of a set, 13
δ, 216	Cover function, of finitary distributive lattice,
λ, 219	157
Chain-partitionable, finite graded poset, 168	Cover relations, of posets, 98
Characteristic polynomial:	Crapo, H. H.:
of adjacency matrix, 242	and closure relations, 183
of a graded poset, 128	complementation theorem, 184
Chromatic polynomial of a graph, 162	Cvetković, D. M., rooted tree (exercise), 94
Circular factorization, unique, 248	Cycles:
CL-labeling, literature references, 151	bijection for standard representation (proposi-
Closure, 159	tion), 18
Cluster, L-, 266	number of (proposition), 18
Clutter, antichain or Sperner family, 100	of permutation defined, 17
Coatom, of finite lattice, 104	standard representation of, 17
Coefficient of n, restricted to finite computa-	Th. 6 Tr.
tion, 6	Das, S. K., and meet-distributive semilattice,
Cohen-Macaulay finite poset: definition, 123	178
and Möbius function, 123	Davis, R. L., and valuation of finite dis-
and Moulus function, 125	tributive lattice, 184



deBruijn, N. G., decomposition of permutations of sets (exercise), 47, 57	Duality theorem, master for Eulerian posets, 138
Dedekind, R., and posets and lattices, 149	Duffus, D., and core of irreducible poset, 176
Degree of formal power series, 6	Dumont, D., finite differences (exercise), 61
Dehn-Sommerville equations:	Durfee square, definition for partition, 58
literature references, 151	Dwork, B., and Weil conjectures, 279
and Whitney numbers of second kind (propo-	Dyson, F. J., and q-Dyson conjecture, 54
sition), 136	
Deligne, P., and Weil conjectures, 279	Edelman, P. H.:
Delta-chain condition:	and Bruhat order, 199
definition, 216	counting certain posets, 178
theorem, 218	and maximum Möbius function, 187
Denominator, of a rational vector, 236	and meet-distributive meet-semilattice, 179
Deodhar, V. V., and Bruhat order, 199	and non-crossing partitions, 197
Derangement, definition, 67	and odd composition poset, 191
Derangement problem, and use of Principle of	simple reduction of permutation set, 199
Inclusion–Exclusion, 67	weak Bruhat order, 199
Derivative (formal), definition for formal power	Edwards, R. D., and finite regular cell complex
series, 7	(proposition), 150
Descent set:	Ehrhart polynomial, 238
and Inclusion-Exclusion, 69	Ehrhart quasi-polynomial, of rational polytope,
of multiset permutation, 25	235
number of permutations (proposition), 22	Eigenvalue:
of permutation, 21	of adjacency matrix, 242
Determinants, 82	of circulant matrix, 273
Determinations, definition by example, 1	Enting, I. G., and chromatic polynomial, 292
Dickson, L. E., binomial coefficients and	Equivalent sets, 31
primes (exercise), 53	Euler characteristic, definition of reduced, 120
Difference of $f$ , formula for $k$ -th, 37	Eulerian lattice, a fundamental example of, 136
Difference operator, 36	Eulerian number:
Difference table:	f, 209, 22
and bases for polynomials, 209	as count of excedances, 23
for finite differences of a function, 37	Eulerian polynomial:
Digraph, 241, 293	f, 209, 22
Dilworth, R. P.:	for multiset, 26
and modular lattice, 186	Eulerian posets:
and order ideals, 182	definition, 122, 135
Dilworth, theorem of, 110	figure, 138
Dimension, of finite poset, 176	literature references, 151
Dimers, 273	and zeta polynomial (proposition), 135
Diophantine equations, linear homogeneous,	Euler number, count of alternating per-
221	mutations, 149
Direct product:	Euler pair, 48
twisted (figure), 175	Euler pentagonal number formula, 89
of two posets (figure), 101	Euler's totient function, 62
Dissects, literature references, 151	exercise, 48, 161
Distinguishable, elements of set, 31	Excedance, of permutation, 23
Dominance, definition of order by, 166	Exceptional set, and rational functions (proposi
Doob, M., rooted tree (exercise), 94	tion), 205
Double partition, 77	Exponential generating function:
Doubly stochastic matrices, 240	and binomial posets, 141
Dowling lattice, 191	definition, 3
Dowling, T.:	Extensions:
and uniform poset, 191	number of, and chains, 110
and Whitney number inequalities, 185	number of, and permutations, 112
Down-set, subset of poset, 100	number of, and recurrence, 112
Dreesen, B., and comparability graph, 194	number of linear, for posets (figure), 111
Dual:	
of Möbius inversion formula, 116	Factor, of word, 266
of poset, 101	Factorial function, of P as number of maximal
for Principle of Inclusion-Exclusion, 66	chains, 142



Fall, as element of permutation, 24	Ganter, B., and modular lattice, 186
Farmer, F. D., and poset of normal words, 199	Garisa, A. M.:
Fatou, P., and Fatou's lemma, 275	and chain-partitionable posets, 193
Fatou's lemma, and primitive series, 275	and vector partitions, 282
Fences, or zig-zag posets, 109	Gaussian polynomial, or q-binomial coefficient,
Ferrers boards, 74	26
with equal rook polynomials, 75	Geissinger, L.:
Ferrers diagram,	and a convex polytope associated with a
of conjugate partition, 39	poset, 199
of a partition of n, 29	and valuation of finite distributive lattice,
Fibonacci number, Eulerian posets and dimen-	184
sion of vector subspace, 172	Generating function:
Filter, order ideal subset of poset, 100	and adjacency matrix (theorem), 242
Finitary distributive lattice:	advantages of using, 10
definition of, 107	and binomial posets, 140
and Pascal's triangle, 112	chromatic, 142
Finite difference, to define excluded objects	and closed walks (corollary), 243
from set (proposition), 68	cluster, 267
Finite graded poset, Eulerian, 122	and completely fundamental elements
Finite graph:	(theorem), 228
definition, 293	doubly-exponential, 141
edges, 293	Eulerian, 141
loop, 293	for evaluating counting function, 3
simple, 293	exponential, 141
vertices of, 293	extracting information from, 10
Finite regular cell complex:	fundamental, and Jordan-Hölder set
definition, 121	(theorem), 214
figures, 122	fundamental for P-partition, 212
Fishburn, P. C., and dimension of poset, 176	for monoid, 222
Fixed points, for permutations, 68	for monoid, specialization (theorem), 231
Fixed point set, counted in terms of weight	for number of closed walks of length n, 244
function, 80	for number of partitions of $n$ , 38
Foata, D.:	and number of P-partitions, 216
and monoid, 189	ordinary, 141
multiset permutations, 57	and quasi-generators (corollary), 227
Forest, and Ehrhart polynomial, 272	rational, and adjacency matrices, 245
Forests, 93	reasons for using, 3
rooted, 89	for restricted partitions, 39
spanning, 93, 273	r-exponential, 141
Formal power series:	sets associated with, 143
definition, 3	for simplicial monoids (corollary), 227
as integral domain, 4	and weighted edges of digraph, 258
primitive, 275	Geometric lattice:
treated as functions, 8	definition of, 105
validity of identities (example), 5, 6	and partition lattice, 128
Frankl, P., conjecture on cardinality of dual	Geometric representation, of permutations, 23
order ideals, 186	Gessel, I. M.:
Freely generated submonoid, 247	and duality theorem for Eulerian posets,
very pure, 248	198
Free monoids, factorization in, 247	and formal power series for rational function
Free R-module, 167	275
Freese, R., and complements in a finite lattice,	and a recurrence relation, 282
184	and strong fixed point of permutation (ex-
Fundamental property of rational functions:	ercise), 61
simplification (corollary), 204	tournament (exercise), 94
theorem, 202	and vector partitions, 282
	Gleason, A., and direct products of posets,
Gale's evenness condition, and faces of convex	175
polytope, 199	Good, I. J., proof of Dyson conjecture, 54
Γ, triangulation of pointed convex polyhedral	Gordon, B., and pleasant posets, 182
cone, 223	Goresky M intersection (co)homology 100



Gould, H. W., and formal power series (ex-	Incidence algebra (continued)
ercise), 62	of a locally finite poset, 113
Goulden, I. P.:	subalgebra of and binomial poset (theorem),
and binomial poset, 201	144
and cluster generating function, 280	Incident, edges and vertices, 293
Graded posets:	Inclusion-Exclusion:
definition, 99	dual form, 66
ranks of (table), 99	Principle of, 64
Graph:	Principle of (theorem), 64
definition for permutation, 71	Indecomposable permutation, 49
directed, 241	Index, greater (major), of permutation, 216
Greene, C.:	Indik, R., and a recurrence relation, 282
and rank inequality for finite geometric lat-	Indistinguishable, elements of set, 31
tice, 185	Inequivalent injective functions, number of,
and geometric sublattice, 178	definition, 32
and maximum size antichains, 182	Inequivalent sets, 31
and meet-distributive meet-semilattice, 179	Integral convex d-polytope, and relative
and Möbius algebras, 150	volume, 239
and Möbius functions, 149	Interior, of simplicial submonoid, 226
partitions of integer ordered by dominance,	Inverse:
192	of elements of formal power series, 4
and subsets of finite lattice, 186	of matrix for Stirling number of the second
and weak Bruhat order, 199	kind, 35 Inversion:
Grimson, E., and Möbius function, 188	
Gross, O. A., chains in boolean algebra, litera-	definition of, for multiset permutation, 26 definition of, for permutation, 21
ture references, 151 Guibas, L. J., and autocorrelation polynomial,	number of, for permutations (corollary), 21
280	and permutation statistics, 20
Gunson, J., and Dyson conjecture, 54	Inversion table, of permutation, 21
Guison, 7., and Dyson conjecture, 5.	Involution principle, for converting proofs to
Hadamard product:	combinatorial proofs, 80
and generating function with weighted edge,	Involutions, 79
258	construction of for intersecting lattice paths,
for rational power series (proposition), 207	84
and sums over compositions, 258	Irreducible finite poset, 155
Haiman, M., and order ideals, 182	Isomorphic posets, 98
Hajos, G., combinatorial proof (exercise), 52	
Hall, P., and Möbius inversion formula, 149	Jackson, D. M.:
Hanlon, P., and Gaussian poset, 287	and cluster generating function, 280
Harbater, David, and integral power series, 276	and descents of permutations, 201
Hasse diagram:	Jambu, M., and intersections of hyperplanes,
for $B_4$ (figure), 108	192
for direct product of two posets (figure), 101	Jeu de taquin, 179
edges of and R-labeling, 133	Join, for elements of poset, 102
of finite poset, 98	Join-irreducible:
method for drawing, finite order ideals of	element of lattice, 106
poset, 108	of finite distributive lattice (proposition), 106
method for drawing (figures), 109	Jordan, Charles, number of objects of finite set
Hasse diagrams, of lattices (figures), 102	(exercise), 91
Hawkins, T., and history of group representa-	Jordan, J. H., counting triples, 281
tions, 290	Jordan-Hölder set:
Hilton, P. J., and duality for abelion groups,	definition of, 131
190	and P-partitions (lemma), 213
Hock, J. L., and dimers, 291	and supersolvable lattices, 164
Horizontally convex polyominos, 256	k-composition:
Hurwitz, A., and Fatou's lemma, 275	definition, 14
Hyperplane arrangement, 166	weak, 15
Igusa, JI., and congruences mod $p^n$ , 279	k-composition of $n$ , schematic representation,
Incidence algebra:	15
and algebra of upper triangular matrices, 114	k-discordant permutations, problem of, 74



Kahn, J., and order polynomial, 285 Kasteleyn, P. W., and dimers, 291 Katz, N. M., and Weil conjectures, 279 Kelly, D., and dimension of poset, 176 Kelmans, A. K., and counting rooted forests	Lattice path, 82 diagram, 82 enumeration of, 44, 111 intersecting, 82 Laurent series, and rational generating function,
(exercise), 94	206
Kendall, M. G., and tournament (exercise), 94 Kim, D., and chromatic polynomial, 292 Kirdar, M. S., proof for partition counts (ex-	Lexicographically shellable posets and CL- labeling, 151 Lie algebras, semi-simple, 288
ercise), 59	Lieb, E. H., and dimers, 291
Klarner, D. A.: decomposition of permutations of sets (ex-	Lindström, B., and meet-semilattice determi- nant, 184
ercise), 47, 57	Linear extension, of poset, 110
and dimers, 291	Linear ordering, of elements of multiset, 16
and rational generating functions, 276	Link, in a finite simplicial complex, 121
Kleitman, D. J.:	Linked sets, literature references, 151
combinatorial proofs (exercise), 52 and free distributive lattice, 181	Loop, of digraph, 241
and maximum-size antichains, 182	Lucas, E., binomial coefficients and primes
and meet-distributive meet-semilattice, 179	(exercise), 53 Lucas number, and permutation enumeration,
and meet-irreducible elements of finite lat-	246
tice, 177 and non-isomorphic <i>n</i> -element posets, 174	Lyndon, R. C., and multiset permutations (exercise), 57
and order ideals, 182	M 1 11 7 6
and outdegree sequences, 290 Knuth, D. E.:	Macdonald, I. G.:
inversion polynomial (exercise), 61	constant term of product expansion (exercise), 54
and monoid, 189	cycle types of permutations (exercise), 61
multiset permutations, 57	and number of words in quotient monoid,
and number of words in quotient monoid, 277	and pleasant posets, 182
Koblitz, N., and Weil conjectures, 279	and volume of convex polytopes, 290
Koh, K. M., and finite distributive lattice,	MacMahon, P. A.:
182	and recurrence formula (exercise), 58
Koszul complex, 92	and Young's lattice, 195
Koszul relation, 92 Kratzer, C., Möbius function of lattice of sub-	MacNeille completion, of irreducibles of finite
groups, 191	lattice, 176
Kreweras, G.:	MacPherson, R., intersection (co)homology, 199
and non-crossing partitions, 197	Magic squares, number of (proposition), 232
and Young's lattice, 195	Mani, P., and Ehrhart polynomial, 288
Kummer, E., binomial coefficients and primes	Markowsky, G., and free distributive lattice,
(exercise), 53	181
Kung, J., and subsets of finite lattice, 186  L-labeling, literature references, 150	Maximal chain, and supersolvable finite lattice,
Lascoux, A., and weak Bruhat order, 199	Maximal chains:
Lattice, 102	in distributive lattice, 110
atomic, 104	number of and R-labeling (theorem), 133 McMullen, P.:
complemented, 104	and cyclic polytopes, 199
complete, 103	and Ehrhart quasi-polynomial, 290
distributive, 105	McQuistan, R. B., and dimers, 291
finitary distributive, definition, 107	Meet-distributive, meet-semilattice, 156
finitary distributive and FTFDL (proposition), 107	Meet-semilattice, conditionally a lattice (proposition), 103
finite distributive, fundamental theorem for,	Meuser, D., and congruences mod $p^n$ , 279
106 finite geometric, 105	Milne, S., Stirling number of the second kind
finite semimodular, 104, 105	(exercise), 60
free distributive, 158	Möbius algebras: definition for lattice, 124
and Möbius algebras. 124	theorem 124



Möbius function:	Order ideal:
classical, 9, 119	definition of, 100
computing (proposition), 119	dual, 100
and count of permutations (lemmas), 147	principal, 100
determination of, for a boolean algebra, 145	Order polynomial:
and Eulerian posets (lemmas), 136	and chain length (corollary), 220
and Eulerian posets (proposition), 137	definition, 130
and finite regular cell complex (proposition),	fundamental property (theorem), 219
122	and λ-chain condition (corollary), 221
function, 9	reciprocity theorem, 194, 219
for lattice faces of triangulation, 224	strict, 218
and meet of coatoms (corollary), 126	Order-reversing, strict, 211
of poset, 116	Order-reversing map, 211
of semimodular lattice (proposition), 126	Ordinary generating function, 3
techniques for computing, 117	Orientation:
Möbius inversion, and Euler characteristic, 119	acyclic, 273
Möbius inversion formula, and zeta function,	of edges of finite graph, 273
116	Orlik-Solomon-Terao conjecture, 192
Modular lattice, 104	Outdegree sequence, of orientation, 273
as poset of subspaces of a vector space, 127	
Monjardet, B., and meet-distributive meet-	P-compatible permutation, 171
semilattice, 179	P-partition:
Monoid:	definition, 211
definition, 163	and linear homogeneous equations, equiva-
free, factorization in, 247	lence of, 222
and homogeneous linear equations, 221	and strict P-partition (lemma), 220
positive, 222	strict P-partitions and $\delta$ -chain condition, 218
Moon, J. W., and counting rooted forests (ex-	Partial G-partition, of a set, 165
ercise), 94	Partial partition, of a set, 165
Moser, W. O. J., Terquem's problem (ex-	Partition:
ercise), 63	of an integer, 28
Multichains:	definition of self-conjugate, 58
of poset, 100	of finite set, definition, 33
and zeta function, 115	and $q$ -binomial coefficient (propositions), 29
and zeta polynomial, 130	with restricted parts, 38
Multinomial coefficients:	Partitions of <i>n</i> -set:
definition, 16	definition, 33
q, 26	lattice of, 97, 127
Multiset:	ordered, 34
definition, 15	Pascal's triangle, generalized, 112
generating function approach, 15	Peak, as element of permutation, 24
permutations of, 16	Peirce, C. S., and partially ordered sets and
Multiset cycle, 47	lattices, 149
Multiset permutation, 47	Permutation enumeration and binomial posets,
Multisets, permutations of, 25	147
1 162	Permutations: alternating and number of complete binary
n-coloring, of a graph, 162	trees, 24
n-stack, 76 Nelson, R. B., and volume of a convex poly-	equivalent, 89
	and factorization method, 252
tope, 290 Non-crossing partitions, 197	k-discordant, and factorization method, 253
Non-crossing partitions, 197	mapped to inversion table (proposition), 21
Odlyzko, A. M.:	of a multiset, 16
and autocorrelation polynomial, 280	of a multiset and inversions (proposition), 26
count of subsets mod $n$ (exercise), 60	number, with specified number of maxima
Open subset of preposet, 153	(corollary), 20
Order by inclusion, poset of subsets, 97	standard, 93
Order by refinement:	Permutation statistics, 17
partitions of a set, 97, 127	cycle structure, 17
partitions of n, 166	descents, 21
Order complex, of poset, 120	inversions, 20



Permutations with descent set S, determinant expression for, 69 Permutations with restricted position, 71 Permutation with k cycles, and Stirling number, 18	Product (continued) of words, 247 Product theorem, and Möbius function, 118 Provan, J. S., and chain-partitionable posets,
Philip Hall's theorem (Proposition 3.8.5), 119 π-compatible, 212 Plane trees, and face-lattice of an n-cube, 170 Pleasant poset, 158	193 Pseudo-polynomial, or quasi-polynomial, 210
Poguntke, W., and comparability graph, 194 Point lattice, 104	q-analogue, for number of permutations with descent set $S$ , $70$
Pollack, J., and dimers, 291 Pólya, G.:	q-binomial coefficient, 26 and partitions (proposition), 29
and power series with integer coefficients, 282	q-Dyson conjecture, 54, 95 q-multinomial coefficient, and multisets, 26
and value of a determinant, 184 Pólya's enumeration theorem, 48	Quasi-generators, 226 completely fundamental elements, 228
Polynomials, 208	Quasi-ordered set, 153
bases for, 208	Quasi-period, of quasi-polynomial, 210
as coefficients of rational generating func- tions, 208	Quasi-polynomial:
Polynomials of degree $n$ , coefficients as finite	and convex polytope (theorem), 237 definition, 210
differences (proposition), 38	and number of P-partitions (theorem), 216
Polyominos:	properties of (proposition), 210
definition, 256	Quotient, of poset relative to closure, 159
enumerating horizontally convex, 257 Polytope:	R-labelings:
cyclic, 199	definition, 133
integral, 238	literature references, 150
rational, 235	r-stemmed V-partition, 89
Poset:	Rank, of finite poset, 99
basic concepts, 96	Rank-generating function, 99
defining axioms, 97	definition of, 99
definition of isomorphic, 98 extension of to total order, 110	and simplicial posets, 135
Gaussian, 270	Rank-selected Möbius invariant: direct combinatorial interpretation, 133
and λ-chain condition, 220	and lattice of subspaces of a vector space,
locally finite, 98	132
table of examples, 105	and permutations in Jordan-Hölder set
Power series:	(theorem), 132
composition defined, 6	of a poset, 131
rational of one variable, 202	Rank-selected subposets:
Prefabs, literature references, 151 Preferential arrangement, definition, 146	definition, 131
Preposet, 153	literature references, 150 Rational generating function:
Principle of Inclusion-Exclusion:	coefficients at negative integers, 206
and binomial posets, 145	definition, 202
and maximal chains in rank-selected sub-	exceptional set, 204
poset, 131	Rational power series in one variable, 202
and Möbius inversion on boolean algebra,	Read, R. C., and dimers, 291
Prins, G., and direct products of posets, 175	Reciprocity theorem: for Ehrhart quasi-polynomials, 238
Problème des ménages:	for linear homogeneous diophantine equa-
lemma, 73	tions, 230
and non-attacking rooks, 73	for order polynomials, 194, 219
Proctor, R.:	for P-partitions (theorem), 215
and Gaussian posets, 288 and zeta polynomial, 179	Recurrence:
Product: of convergent formal power series (proposi-	for computing coefficients, negative $n$ , 207 for computing coefficients of rational func-
tion), 6	tion, 205 defining for Möbius function, 116



Recurrence (continuea)	Secant number, or Euler number, 149
and master duality theorem for Eulerian	Self-conjugate partition, 58
posets, 138	Semi-Eulerian poset, 122
for Möbius function (corollary), 125	Semilattice, meet- and join-, 103
for number of linear extensions of poset, 112	Semimodular lattice:
•	
for partitions into k parts, 28	Möbius function of, 126
and Stirling number of the second kind, 33	and R-labeling (figure), 135
Reduced Euler characteristic:	as R-poset, 134
definition, 120	seven-element (figure), 104
and topological significance of Möbius func-	Sequences and paths, literature reference, 151
tion, 121	Series-parallel posets, 100
Reduction:	Serre, JP., and rational generating functions,
	276
of a permutation, 172	
simple, 172	Sets and multisets, 13
Refinement, of poset, 98	Shapiro, L.:
Regev, A., constant term of product expansion	and duality theorem for face lattice of an n-
(exercise), 54	cube, 198
Regular local ring, one-dimensional complete, 4	finite differences (exercise), 62
Relative volume, of integral polytope, 239	Shearer, J. B.:
Remmel, J. B., derangement proof (exercise),	and Möbius function, 188
92	and number of words in quotient monoid,
Restricted direct product, definition and Möbius	277
function of a poset, 119	Shephard, G. C.:
Restriction, of maximal chain, 168	and cyclic polytopes, 199
Rise, as element of permutation, 24	and zonotopes with integer vertices, 290
Rival, I.:	Shift operator, $E$ , 36
and core of irreducible poset, 176	Shuffle:
finite atomic and coatomic lattice, 177	definition, 70
and modular lattice, 186	inverse of permutation, 70
Robbins, D., and pleasant posets, 182	Sieve equivalence, as variation of involution
Roberts, F. S., and dimension of poset, 176	principle, 80
Rook numbers, for Ferrers board (theorem), 74	Sieve method, for determining cardinality, 64
Rook polynomial:	Simplicial complex, and Alexander duality
	theorem, 137
equality for Ferrers boards, 75	
and factorization method, 255	Simplicial finite poset, 135
and k-discordant permutations, 74	Simplicial submonoid, 226
and transfer-matrix method, 253	Skyrme, T. H. R., proof for partition counts
Rooks, non-attacking, 71	(exercise), 59
Roos, JE., and number of words in quotient	Smith, B. Babington, and tournament (ex-
monoid, 277	ercise), 94
Rota, GC.:	Solomon, L.:
and chromatic polynomial, 187	and Möbius algebra of poset, 150
and closure relations, 183	and subgroups of $\mathbb{Z}^k$ of finite index, 190
and homology theory for posets, 150	Spanning subsets of a vector space, number of,
and valuation of finite distributive lattice,	127
184	Species, as alternative to binomial posets, refer-
Rothschild, B. L., and non-isomorphic n-	ences, 151
element posets, 174	Sperner family, or antichain, 100
Row-reduced echelon form, and partitions of	Standard permutation, 93
integers, 29	Standard representation, of permutation, 17
	Stanley, R.:
Sachs, H., and counting rooted forests (ex-	and chain-partitionable Cohen-Macaulay
ercise), 94	posets, 194
Saks, M., and order polynomial, 285	and chain-partitionable posets, 193
Saturated chain, 99	and comparability graph, 194
Scheid, H., and bounds on Möbius function,	convex polytope and Eulerian lattice, 199
186	and counting certain permutations, 93
Schröder, E., and posets and lattices, 149	count of subsets mod n (exercise), 60
Schützenberger, MP.:	cycle types of permutations (exercise), 61
and jeu de taquin, 179	and Ehrhart (quasi-)polynomial, 290
and weak Bruhat order, 199	and an extremal finite distributive lattice, 178



Stanley, R. (continued) and finite supersolvable lattice, 190	Temperley, H. N., and counting rooted forests (exercise), 94
free R-module and supersolvable poset, 192 and incidence algebras of locally finite	Terao, H.: and arrangements of hyperplanes, 192
posets, 183	Removal Theorem, 192
and integral convex d-polytopes, 288	Terquem's problem (exercise), 63
intersection (co)homology, 199 and order ideals of $\mathbb{N}^m$ , 277	Thevenaz, J., Möbius function of lattice of subgroups, 191
and order polynomial, 181	Tournament, 90
and pleasant posets, 182	Transfer-matrix method, 241
and toric varieties and hard Lefschetz theorem, 198	Transition matrix, for polynomial bases, 35, 209
and trapezoidal chains and antichains, 179	Tree:
and weak Bruhat order, 199	definition, 293
Stembridge, J.:	definition of binary, 23
and trapezoidal chains and antichains, 179	Triangular board, and rook number (corollary),
and a zeta polynomial, 179	75
Stirling number of the first kind, 18	Triangular poset, 173
and calculus of finite differences, 36 and exponential generating function (proposi-	Triangulation of pointed convex polyhedral cone, 223
tion), 19 as number of integer sequences (proposition),	Trotter, W. T., Jr., and dimension of poset, 176
20	Twelvefold Way, 31-40
and polynomial generating function, 208, 209	table, 33
recurrence (lemma), 18	Two tree representations, and permutation sta-
and Twelvefold Way, 35	tistics, 23
Stirling number of the second kind:	
and characteristic polynomial of $\Pi_n$ , 128 exercise, 57, 47	Underlying space, of finite regular cell com-
and Möbius function of poset, 128	plex, 121
and partitions of a set, 33	Unfixed points, for permutations and depen-
and polynomial generating function, 208, 209	dence on $n$ , 68
Stirling numbers, examples of matrices, 36	Uniform poset, 165
Stonesifer, J. R., and uniform geometric lattice,	Unimodal sequence: expression for and V-partition (proposition),
Stong, R. E., and core of irreducible poset, 176	78
Strauss, E. G., and basis for group of polyno-	number of analogous to partitions of $n$ , 76 of weight $n$ , 76
mials (exercise), 62 Strong fixed point, definition of, for permuta-	Unique factorization domain, 4
tion, 49	
Submonoid:	V-partitions:
of monoid, 247	definition of r-stemmed, 89
simplicial (lemmas), 226	of <i>n</i> , definition, 77
Subposet:	number of and sieve-theoretic formula, 78
convex, 98	V-partitions and unimodal sequences, 76
induced, 98	Valley, as element of permutation, 24
of P, S-rank selected, 131	Valuation, 160
weak, 98	van der Poorten, A. J., and rational generating
Subspaces of vector space, number of (proposi-	functions, 276
tion), 28	Veress, P., combinatorial proof (exercise), 52
Sum, of convergent formal power series (proposition), 6	Verma, DN., and Bruhat order, 199 Very pure monoids, characterization (proposi-
Supersolvable finite lattice:	tions), 249
additional properties, 163-164	
definition, 134	Wachs, M., and Bruhat order, 199
non-semimodular (figure), 134	Walch, R., counting triangles with perimeter $n$ ,
Szegő, G., and value of a determinant, 184	281
T	Walk:
Tangent number, or Euler number, 149	closed, 241, 293
Tanny, S. M., and counting certain per-	definition, 293
mutations (exercise), 93	in digraph, 241



### 306 Index

Word: Walker, J., and topological properties of posets, 150 as finite sequence of letters, 247 Ward, M., and Möbius inversion formula, 149 to represent a permutation, 16 Weak excedance, of permutation, 23 Worley, Dale, and a recurrence for partitions Weak subposet, 98 (exercise), 59 Weight, of multiset cycle, definition, 47 Weighted cardinalities, 66 Yoder, M. F., count of subsets mod n (ex-Weight function: ercise), 60 for set, 65 Young diagram, and partitions of n, 29, on set of edges of digraph, 241 30 Weil, A., and Weil conjectures, 279 Young's lattice, 168 Weil conjectures, 279 Weisner, L., and Möbius inversion formula, Zaslavsky, T.: and the acyclotope, 290 149 Weisner's theorem (corollary 3.9.3), 125 and Dowling lattice, 191 and hyperplane arrangements, 192 Well-defined operations (example), 5 Wensley, C. D., Möbius function of subgroup Zeilberger, D.: and cluster generating function, 280 lattice of symmetric groups, 191 West, D., and chains in posets, 175 proof of q-Dyson conjecture, 54 Whitney number of the first kind, coefficient of and tournament (exercise), 95 Zeta function: characteristic polynomial, 129 Whitney number of the second kind: definition, 114 and Dehn-Sommerville equations (proposiand zeta polynomials, 130 Zeta polynomials, 129 tion), 136 and binomial posets, 145 and number of elements of rank k, 129 Whitworth, W. A., and counting certain perdefinition, 129 mutations (exercise), 93 literature references, 150 Wilf, H., and meet-semilattice, 184 and number of chains, 129 and order polynomial, 130 Wilson, K., and q-Dyson conjecture, 54 Wilson, R.: Ziegler, G.: and Möbius function, 187 and permutation of finite lattice, 185 and Whitney number inequalities, 185 partitions of integer ordered by refinement, 192 Winkler, P., and comparability graph, 194 and self-dual poset, 175 Winston, K., and outdegree sequences, 290 Wisner, R. J., counting triangles with perimeter and zigzag posets, 181 n, 281Zonotope, 290