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Island networks and graphs

In the course of transforming verbal propositions into images many things are made explicit that were previously implicit and hidden.

Herbert A. Simon, *Models of My Life*

Oceanists have increasingly come to recognize the limitations of the laboratory analogy that treats island societies as isolated experiments in adaptive radiation. Reconstructions of regional exchange systems (Hage and Harary 1991), archaeological evidence of sustained inter-island contacts (Kirch 1988a), firsthand accounts of traditional voyaging techniques (Lewis 1972), and the evident contradiction between neoevolutionist assumptions and the facts of Oceanic ethnography and prehistory (Friedman 1981) conduce to a network perspective that views island societies as elements of communication systems. Most islands in the Pacific are, in fact, distributed in groups, and most island societies are, or once were, connected to other island societies – as colonists, trade partners, tributaries, allies, wife-givers, and in various other ways. In acknowledging the importance of these connections many researchers, including anthropologists, archaeologists, and linguists, are now using or recommending the application of network concepts to answer a range of fundamental questions concerning

1. the settlement of island groups (Levison, Ward, and Webb 1973; Ward, Webb, and Levison 1976; Green 1979; Kirch 1988a; Irwin 1992);
2. the location of trade centers (Irwin 1974, 1978, 1983; Kirch 1988b; Hunt 1988);
3. the development of social stratification and social complexity (Reid 1977; Friedman 1981; Kirch 1984a; Lilley 1985; Graves and Hunt 1990);
4. the differentiation of cultural complexes (Green 1978);

5. the diversification of dialects and languages (Pawley and Green 1984; Marck 1986);
6. the distribution of physical and cultural traits (Terrell 1986);
7. the selection of subsistence practices (Harris 1979);
8. the evolution of kinship structures (Epling, Kirk, and Boyd 1973; Marshall 1984).

It is clear that network analysis can lead to the solution of many long-standing problems and open up entirely new lines of inquiry in Oceania studies, provided that the handful of technical terms and structural metaphors now in use are replaced by a full and consistent set of mathematical concepts that are all part of some underlying model. Without such a model, many research problems cannot be correctly formulated or even imagined.

Among all mathematical models, those from graph theory are most naturally suited for network analysis (Harary, Norman, and Cartwright 1965; Harary 1969; Hage 1979a; Hage and Harary 1983, 1991; Buckley and Harary 1990). The models, called graphs, are topological systems whose abstract properties define the significant structural features of all networks. For island networks these features include their distances, reachability, and connectedness and hence their centers, sources, cycles, orders, partitions, and spanning substructures. Graphs have the intuitive appeal of iconicity and the computational advantage of matrix methods. Graph theory contains theorems and algorithms that permit deductive approaches and precise and efficient solutions to numerous structural problems.

Our purpose is to provide a set of graph theoretic models for studying the structure and formation of island networks and certain of their kinship armatures. We show how island networks are internally connected by a variety of social, cultural, and linguistic relations, and how they are partitioned into subgroups and organized into hierarchies. We introduce elementary techniques for simulating processes of network growth, where historical data are lacking, and we clarify and extend the application of graph theoretic models to the study of culture history. We provide a large repertory of concepts for analyzing the effects of network location on the economic and political status of island communities, and we show how complex networks can be simply represented to reveal essential connections – how to see the trees obscured by the forests.

Our presentation is organized in a logical, graph theoretic manner. As in our previous books, *Structural Models in Anthropology* and *Exchange in Oceania*, the account is self-contained and readily accessible to the nonmathematical reader. We now give informal illustrations of our models together with a preview of the research presented later in this book.

Graph theoretic models

We use six basic graph theoretic models in the analysis of island networks and associated kinship structures: trees, minimum spanning trees, search trees, centrality concepts, dominating sets, and digraphs. The models, and hence our illustrations, are suggested by applications of graph theory to network analysis in several different fields: computer science, operations research, recreational mathematics, social networks, and transportation geography.

Graphs

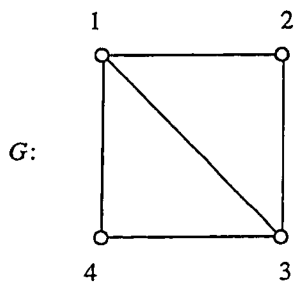
A graph G consists of a finite set V of nodes together with a set E of edges where each edge joins two nodes. This is illustrated by the diagram in Fig. 1.1a. For purposes of algebraic manipulation a graph can also be represented by an adjacency matrix, denoted $A(G)$, in which each node has a row and a column and in which the entries in the cells are either 1 or 0 to show the presence or absence of an edge joining a pair of nodes (see Fig. 1.1b). A third way to represent a graph, useful for certain algorithmic procedures and in computer implementations, is to list its edges, as shown in Fig. 1.1c.

Typically, a real-world problem involving structure is modeled by a graph and then solved through the application of concepts, theorems, and algorithms from graph theory.

Trees

Trees are the simplest of all graph theoretic models. In Chapter 2 we present a theorem that gives several structurally equivalent properties of a graph, each serving to characterize a tree. For present purposes, a tree is simple because it has no cycles. Particularly useful in network analysis are a rooted tree (which has a special node called its root) and a spanning tree of a connected graph.

Rooted trees serve as models of (1) networks in which one island is distinguished from all others, (2) kinship groups defined by reference to an ego or an ancestor (Goodenough's 1970 "ego-focused" and "ancestor-focused" kinship groups), (3) hierarchical classification systems headed by a unique beginner. Fig. 1.2a shows a rooted tree, with the root indicated by an encircled node. In a collection of *nested sets*, either any two sets are disjoint or one set is a subset of the other. Fig. 1.2b shows an equivalent representation of a rooted tree as nested sets. Some anthropologists, including several whose work is discussed in Chapter



(a)

$$A(G) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(b)

List of edges:
1, 2
1, 3
1, 4
2, 3
3, 4

(c)

Figure 1.1. Alternative representations of a graph.

8, implicitly use nested sets to model kinship structures, but we will always use graphs, which have the advantages of clarity and intuitiveness.

Binary trees, as illustrated in Fig. 1.3, with the top node taken as the root, constitute an important special class of rooted trees and are often used to model classification systems. In Chapter 2 we use twin binary trees to give a single characterization of an apparently widespread and primordial type of Austronesian classification system variously referred to as "recursive dualism" (Eyde 1983), "recursive complementarity" (J. J. Fox 1989), and "perpetual dichotomy" (Hocart 1952), among other designations.

In modeling kinship structures as rooted trees we note that Evans-Pritchard (1940), who was renowned for his diagrammatic virtuosity, used several different models, all of which are equivalent to a rooted tree, to represent the segmentary lineage.[1] He used conventional kinship diagrams to describe its branching structure and nested sets to elucidate its alliance structure. He even drew pictures of trees, a method that he

1 As Geertz (1988:44) has observed, "The outstanding characteristic of E-P's [Evans-Pritchard's] approach to ethnographic exposition and the main source of his persuasive power is his enormous capacity to construct visualizable representations of cultural phenomena – anthropological transparencies."

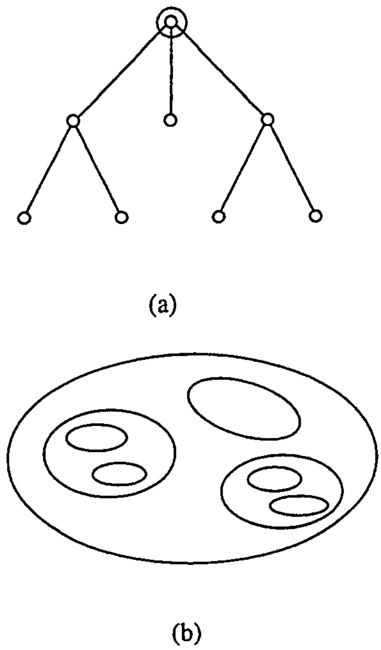


Figure 1.2. Equivalent representations of a rooted tree as (a) a graph and (b) nested sets.

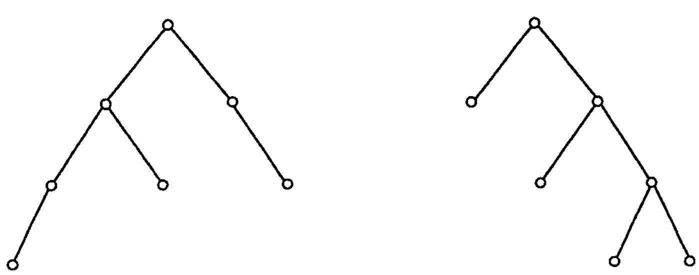


Figure 1.3. Two binary trees.

felt would "commend itself to Nuer, who sometimes speak of a lineage as *kar*, a branch." We will not use pictures here, but the native drawing in Fig. 1.4 (where the root is as usual) of moiety organization, from Pukapuka Atoll in East Polynesia, suggests that the rooted tree model would also commend itself to Oceanic thought.

A spanning tree of a graph G contains all the nodes of G. This concept was discovered by the pioneering physicist Gustav Kirchhoff in the

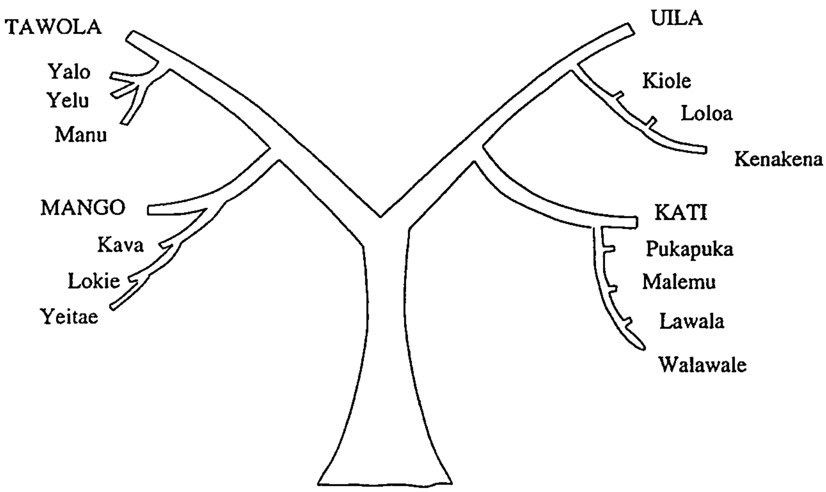


Figure 1.4. A native model of social organization in Polynesia: "Branching of maternal sub-lineages (*keinanga*), from informant's drawing. Main trunk of tree represents total population; main branches four main groupings of maternal lineages (*wua*); remaining small branches individual sub-lineages" (from E. and P. Beaglehole 1938).

course of solving a network problem in electrical engineering. The circumstances of his discovery provide an example of graph theoretic modeling that can be interpreted generically. Kirchhoff (1847) developed the theory of trees in 1847 in order to solve the system of simultaneous linear equations that give the current in each branch (edge) and around each circuit (cycle) of an electrical network. The resulting graph is often called the "topology of the network." As a physicist he abstracted and deliberately oversimplified an electrical network, with its resistances, condensers, inductances, and so forth, and replaced it with its corresponding combinatorial structure consisting only of nodes and edges without any indication of the type of element represented by individual edges. Thus, in effect, Kirchhoff replaced each network with its underlying graph and showed that it is not necessary to consider every cycle in the graph of a network separately in order to solve the system of equations. Instead, he pointed out, by a simple but powerful construction which has since become standard procedure, that the independent cycles of a graph, determined by any one of its spanning trees, will suffice. A contrived electrical network *N*, its underlying graph *G*, and a spanning tree *T* are shown in Fig. 1.5.

In graph theory, the independent cycles of a graph determine its cycle

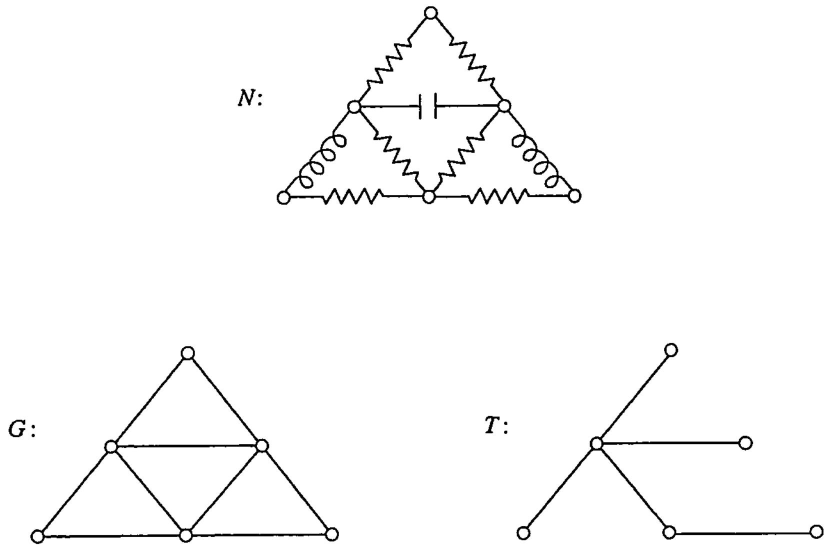


Figure 1.5. An electrical network (N), its underlying graph (G), and a spanning tree (T).

rank. In geography, cycle rank provides a standard measure of the connectedness of a transportation network known as the alpha index of a graph (Garrison and Marble 1962). In Chapter 2 we give this index a firm mathematical foundation by supplying the necessary theorem on cycle rank in graphs and propose it as a tool for studying the relation between network connectedness and linguistic and cultural differentiation. In Chapter 4 we give a political interpretation of the alpha index as the potential for elite control of an exchange network.

Minimum spanning trees

When we assign numerical values, or "weights" or "costs," to the edges of a graph G, we obtain a network N, also called a weighted graph, W. A minimum spanning tree, denoted MST, is a most economical spanning tree of a network. An intuitively appealing illustration of an MST, and one which immediately suggests anthropological applications, is that of Boruvka (1926a, b). Boruvka discovered MSTs when he was asked to give a mathematical solution to the problem of finding the most economical electrical network for a region. He described his method using the example in Fig. 1.6. The points (nodes) are in the plane, and the distances between them are euclidean. Boruvka writes,

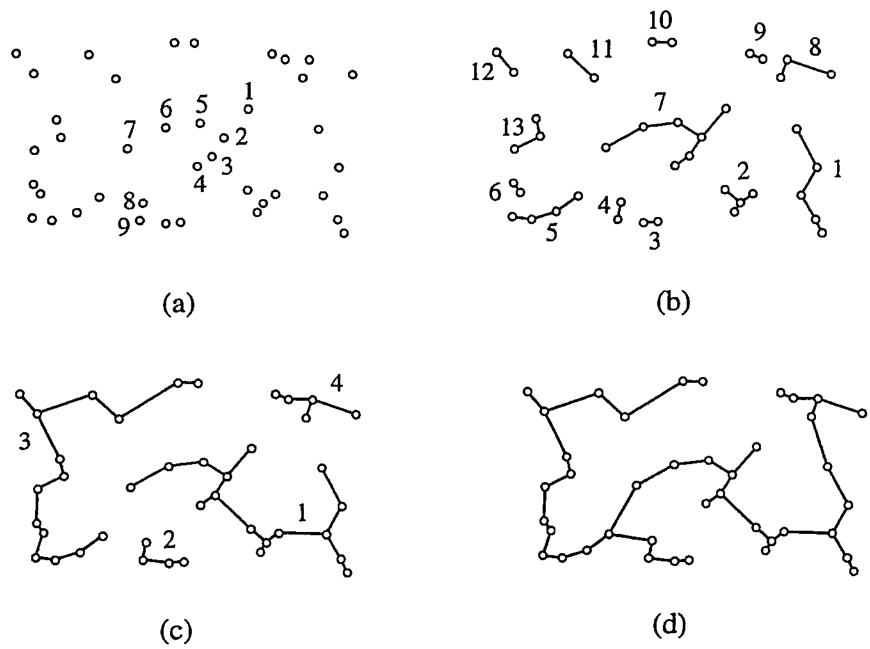


Figure 1.6. Boruvka's illustration of a minimum spanning tree algorithm (from Graham and Hell 1985).

I will join each of the given points with the point nearest to it. Thus, for example, point 1 with point 2, point 2 with point 3, point 3 with point 4 (point 4 with point 3), point 5 with point 2, point 6 with point 5, point 7 with point 6, point 8 with point 9 (point 9 with point 8), etc. I will obtain a sequence of polygonal strokes [i.e., fragments] 1, 2, . . . , 13 [Fig. 1.6b].

I will join each of these in the shortest possible way with the stroke nearest to it. Thus for example, stroke 1 with stroke 2 (stroke 2 with stroke 1), stroke 3 with stroke 4 (stroke 4 with stroke 3), etc. I will obtain a sequence of polygonal strokes 1, 2, . . . , 4 [Fig. 1.6c].

I will join each of these in the shortest possible way with the stroke nearest to it. Thus stroke 1 with stroke 3, stroke 2 with stroke 3 (stroke 3 with stroke 1), stroke 4 with stroke 1. I will finally obtain a single polygonal stroke [Fig. 1.6d], which solves the given problem (Boruvka 1926a, translated by and quoted in Graham and Hell 1985:51–2).

Boruvka's method is based on an *algorithm*, that is, a finite, systematic (step-by-step) procedure for solving a problem.[2] Boruvka's procedure is one of three standard algorithms for solving the minimum spanning tree problem (MSTP), as it is called in applied combinatorics. As informally stated by Graham and Hell (1985:44), they are:

ALGORITHM 1 *(two nearest fragments)*. Add a shortest edge which joins different fragments.

ALGORITHM 2 *(nearest neighbor)*. (A vertex [node] v is arbitrarily chosen.) Add a shortest edge which joins the fragment containing v to another fragment.

ALGORITHM 3 *(all nearest fragments)*. For every fragment add the shortest edge which joins it to another fragment.

MST algorithms provide models for simulating processes of network growth and for analyzing clustering in geographically and culturally defined networks. In Chapter 3 we use Algorithm 3 to simulate the evolution of overseas chiefdoms in the Lau Islands, Fiji. Then Algorithm 1 and the concept of clustering in a minimum spanning tree describe the partitioning of the Tuamotu archipelago into dialect groups and marriage isolates. We regard this clustering as one way of interpreting the Pawley and Green (1984) "network-breaking model" of diversification in the Austronesian languages. In an archaeological application, we show that the complicated method of "close-proximity analysis" developed by Renfrew and Sterud (1969) in Mediterranean archaeology and applied by Green (1978) to the analysis of pottery-design motifs in the Lapita network in Oceania, is an independent discovery of an MST algorithm. As such it can be much more simply stated and efficiently computed using Algorithm 2.

All three algorithms are "greedy" in the sense that they add shortest edges of a network first. At the conclusion of Chapter 3 we note two additional algorithms that proceed dually by removing longest edges first. These could serve as models of network devolution.

Search trees

Search trees are rooted trees in which all the nodes are ordered in a specified way. They are basic tools of computer science, used in sorting col-

2 The term derives from the name of a ninth-century Arabian mathematician, Abu Ja' far Mohammed Ibn Mûsâ al-Khowarizm, who wrote an important text entitled *Kitab al jabr w' al-muqabala*. For a fascinating discussion of algorithmic methods and artificial intelligence, see Penrose (1989).

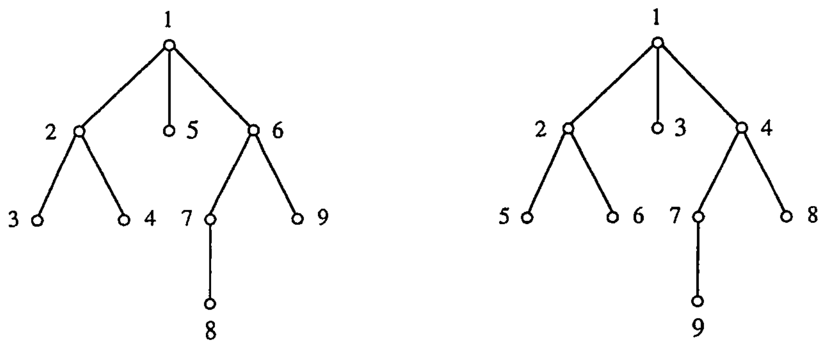


Figure 1.7. Depth-first and breadth-first search trees.

lections of items into their natural orders (Roberts 1984).[3] They should seem congenial to anthropologists, since they are conventionally described in the language of kinship, with nodes referred to as "fathers and sons" and "ancestors and descendants." Thus a computer file search resembles a genealogical search. In a breadth-first search tree (BFST) the nodes are ordered horizontally, whereas in a depth-first search tree (DFST) they are ordered vertically, as illustrated in Fig. 1.7. Each of these search trees has a directional dual, obtained by reversing the ordering of labeling. It is also possible to combine breadth-first and depth-first searches of a rooted tree.

Search trees provide ideal models of hierarchically structured kinship groups. In Chapters 4 and 5 we use DFSTs to model rank in the conical clan, a stratified type of descent group that is to Oceanists what the segmentary lineage is, or once was, to Africanists. The conical clan constitutes the basic structural design of many Polynesian societies (Sahlins 1958; Goodenough 1959), of Ancestral Polynesian Society (Kirch 1984a; Kirch and Green 1987),[4] and, we conjecture, of Proto-Nuclear Micronesian and Proto-Oceanic society as well. In spite of its theoretical importance to neoevolutionists, social anthropologists, and Oceanists, the conical clan has never been accurately defined, and as a result it has been independently discovered, but only partially characterized, on numerous occasions. Our analysis is structural, historical, and comparative. In depicting the conical clan as a DFST, we provide a precise, clear, intuitively appealing model capable of including all of its variants as de-

3 Search trees are also basic models in artificial intelligence, where they are used to represent decision-making processes.
4 Kirch and Green use the terms "protolanguage," "ancestral culture," and "parental population" to distinguish between linguistic, cultural, and biological reconstructions.