

## Index

- absolute value of quaternion, 60  
 action, Clifford, 257  
 action, spinor, 257  
 Adams, J.F., x, 186, 187, 285  
 adjoint, 27, 53, 96, 101  
 adjoint representation, 238  
 admissible chart, 217  
 affine form, 112  
 affine map, 6  
 affine space, 6  
 affine subspace, 6, 212  
 Ahlfors, L., 167, 176, 254, 285  
 algebra, alternative, 179  
 algebra, Cayley, ix, 180  
 algebra, Clifford, 123  
 algebra, division, 178  
 algebra, extensive, 138  
 algebra, exterior, 138  
 algebra, geometric, 123  
 algebra, Grassmann, 138  
 algebra, Jordan, 257  
 algebra, Lie, 239, 242  
 algebra, linear, 9  
 algebra, Pauli, 152  
 algebra, tensor, 124  
 algebra anti-isomorphism, 9  
 algebra isomorphism, 9  
 algebra map, 9  
 algebra-reversing map, 9  
 alternative algebra, 179  
 analytic, 209  
 angle, absolute, 39  
 angles, preservation of, 245  
 annihilator, dual, 4  
 annihilator, orthogonal, 32  
 anti-automorphism, algebra, 13  
 anti-automorphism of  $\mathbf{H}$ , 62  
 anti-involution, algebra, 13  
 anti-involution of  $\mathbf{H}$ , 64  
 anti-rotation, 27, 143  
 Arnol'd, V.I., 66, 112, 285  
 Artin, E., x, 176, 285  
 Atiyah, M.F., x, 124, 285  
 atlas, 215  
 atlas, standard, 111  
 automorphism, algebra, 13  
 automorphism of  $\mathbf{H}$ , 62  
 automorphism, linear, 3, 5  
 automorphism, orthogonal, 27  
 automorphism, triality, 267  
 axis of rotation, 35  
  
 Banach space, 202  
 basis, 2  
 basis theorem for hermitian spaces, 54  
 basis theorem for  ${}^2\mathbf{K}$ -linear spaces, 92  
 basis theorem for real quadratic spaces, 34  
 basis theorem for symmetric correlated quaternionic spaces, 77  
 Benn, I.M., 166, 285  
 bijection, 2  
 bijective, 2  
 bilinear map, 4  
 biquaternions, 174  
 bivector, 176  
 Bott, R., 124, 186, 187, 285, 287  
 Boudet, R., x, 287  
 boundary point, 192  
 bounded linear map, 194  
 Boyer, R.H., x  
 bracket, Lie, 239  
 Brackx, F., x, 285  
 Bredon, G.E., 277, 285  
 Bruck, R.H., 186, 188, 285  
 bundle, tangent, 219, 220  
  
 Cartan, É., 247, 256, 278, 285  
 Cauchy-Schwarz inequality, 39  
 Cayley, A., 180, 286  
 Cayley algebra, ix, 180

- Cayley chart, 118, 174  
 Cayley heptagon, 185  
 Cayley numbers, 165  
 Cayley projective line, 187  
 Cayley projective plane, 187  
 Cayley triangle, 184  
 centre, 40  
 centre of algebra, 10  
 chain rule, 203, 207  
 chart, 176, 215  
 chart, admissible, 217  
 chart, Cayley, 118, 174  
 chart, Pfaffian, 176  
 charts on quadric Grassmannians, 114  
 chart, standard, 111, 215  
 Chevalley, C., 124, 243, 286  
 Chisholm, J.S.R., x, 286  
 choice, axiom of, 2  
 circle, unit, 40  
 circle group, 56, 57  
 classical group, 100, 103  
 classification theorem for complex quadratic spaces, 49  
 classification theorem for complex symplectic spaces, 50  
 classification theorem for hermitian spaces, 55  
 classification theorem for  ${}^2\mathbf{K}$ -correlations, 98  
 classification theorem for  ${}^2\mathbf{K}$ -semi-linear maps, 94  
 classification theorem of real quadratic spaces, first part, 34  
 classification theorem of real quadratic spaces, second part, 36  
 classification theorem for real symplectic spaces, 47  
 classification theorem for symmetric correlated quaternionic spaces, 78  
 Clifford, W.K., 123, 286  
 Clifford action, 257  
 Clifford algebra, complex, 134  
 Clifford algebra, even, 136  
 Clifford algebra, real, 123  
 Clifford embedding, 262  
 Clifford group, 141  
 Clifford homogeneous space, 262  
 Clifford matrix, 167  
 closed map, 196  
 closed set, 192  
 Cnops, J., 164, 167, 168, 172, 255, 286  
 code numbers for ten types of classical group, 103  
 code numbers for ten types of matrix algebra with anti-involution, 155  
 coimage, 1  
 Common, A.K., x, 286  
 commutative diagram, 19  
 compact space, 198  
 compactification, conformal, 247  
 compactification, projective, 246  
 companion, 267  
 companion involution, 266  
 complement, linear, 3  
 complement, orthogonal, 32  
 complex field, as real algebra, 9  
 complex group = symplectic group, 79  
 complex orthogonal group, 50  
 complex quadratic space, 49  
 complex Stiefel manifold, 265  
 complex symplectic plane, 50  
 complex symplectic space, 50  
 complexification, 86, 105  
 composite, 1  
 conformal compactification, 247  
 conformal group, 248  
 conformal map, 245  
 conformal spheres, 248  
 conformal transformation, special, 252  
 conjugate of a quaternion, 59  
 conjugation, 135, 148  
 conjugation anti-involution, 59  
 connected, 199  
 continuous map, 193  
 continuously differentiable map, 205  
 contour, 1  
 contraction lemma, 211  
 correlated  ${}^2\mathbf{K}$ -space, 95  
 correlated linear space, 24  
 correlated quaternionic space, 74  
 correlation, 52, 74  
 correlation, linear, 24  
 correlation, semi-linear, 95  
 correlation, symmetric, 24  
 ‘Correspondence from an ultramundane correspondent’, 176  
 coset space representation, 20  
 cover, double, 146  
 cover, induced, 197  
 cover, open, 197  
 cross product, 60  
 decomposition, orthogonal, 32  
 decomposition, Witt, 37, 109  
 Delanghe, R., viii, 285  
 determinant, 6, 98  
 determinant, absolute, 70  
 diagram chasing, 18  
 diffeomorphism, 211, 218  
 differentiable map, 205  
 differential, higher-order, 209  
 differential of a map, 205  
 dimension, 2  
 dimension of affine subspace, 212

- dimension of classical group, 103  
 dimension of Grassmannian, 111  
 dimension, quaternionic, 68  
 dimension of smooth manifold, 217  
 dimension of submanifold, 212  
 Dirac spinors, 165  
 direct product, 3  
 direct sum, 3  
 disconnected, 199  
 dissimilar, 14  
 distance, 39, 192  
 division algebra, 178  
 division algebra, normed, 179  
 domain, 1  
 double cover, 146  
 double field, 5, 91  
 double field  ${}^2\mathbf{R}$ , as real algebra, 9  
 dual, 52  
 dual, linear, 68  
 dual of semi-linear map, 73, 95  
 dual annihilator, 4  
 dual linear map, 4  
 dual linear space, 4  
  
 Elstrodt, J., 167, 255, 286  
 embedding, 196, 213  
 embedding, Clifford, 262  
 endomorphism, linear, 5  
 Epstein, D.B.A., 264, 276, 288  
 equation of map, 2  
 equivalent atlases, 216  
 equivalent correlations, 52  
 equivalent norms, 194  
 equivalent semi-linear correlations, 74, 95  
 equivalent smooth manifolds, 217  
 euclidean, locally, 215  
 euclidean space, 39  
 even Clifford algebra, 136  
 exact, left-coset, 20, 108  
 exact pair, 18  
 exact sequence, 18  
 exact sequence, short, 18  
 existence theorem for Clifford algebras, 131  
 extensive algebra, 138  
 exterior algebra, 138  
  
 fibre, 1, 212  
 field, 2  
 field, complex, 2  
 field, double, 5, 91  
 field, real, 2  
 Fillmore, J., 167, 255, 286  
 finite-dimensional, 2  
 five, 154  
 form induced by correlation, 74  
 form, hermitian, 52  
 form, quadratic, 22  
  
 form, sesqui-linear, 95  
 free, linearly, 92  
 Freudenthal, H., 278, 286  
 Freudenthal's principle of triality, 278  
 Frobenius, G., 186  
 Fuchs, L., 13, 286  
  
 general linear group, 7, 98  
 general linear group, thought of as a  
     unitary group, 91  
 generators, set of, 12  
 geometric algebra, 123  
 Gilmore, R., 243, 277, 286  
 grade involution, 135  
 gradient norm, 194  
 Grassmann, H., 138, 286  
 Grassmann algebra, 138  
 Grassmannian, 110  
 Grassmannians as coset spaces, 113  
 Grassmannian, quadric, 112  
 Graves, J., 180, 286  
 group, circle, 56, 57  
 group, classical, 100, 103  
 group, Clifford, 141  
 group, complex orthogonal, 50  
 group, conformal, 248  
 group  $G_2$ , ix, 270  
 group, general linear, 7, 98  
 group, Lie, viii, 100, 103, 231  
 group, linear, 7  
 group, Lipschitz, 141  
 group, Lorentz, 238  
 group, Möbius, 248, 255  
 group, orthogonal, 27  
 group, Pin, 146  
 group, quaternionic, 78  
 group, real symplectic, 47  
 group, special linear, 7, 99  
 group, special unitary, 55  
 group, Spin, 146  
 group, topological, 225  
 group, unitary, 55, 104  
 group action, 21  
 group action, Lie, 233  
 group map = group homomorphism, 20  
 group map, Lie, 231  
 group map, topological, 225  
 Grunewald, F., 167, 255, 286  
  
 Haantjes, J., 246, 286  
 half-spinor space, 134  
 Hamilton, Sir W.R., 58, 286  
 Hamilton triangle, 183  
 Hampson, A., ix, 286  
 Hausdorff space, 192  
 Heine-Borel theorem, 197  
 Helgason, S., 243, 277, 286

Cambridge University Press

0521551773 - Clifford Algebras and the Classical Groups

Ian R. Porteous

Index

[More information](#)

292

Helmstetter, J., x, 287  
 heptagon, Cayley, 185  
 Hermann, R., 247, 286  
 hermitian form, 52  
 hermitian product, 52  
 hermitian space, 52  
 higher-order differential, 209  
 homeomorphism, 193  
 homogeneous space, 226  
 homogeneous space, Clifford, 262  
 Hopf, H., 122, 286  
 Hopf map, 111, 258, 259  
 Hurwitz, A., 164, 165, 186, 286  
 Hurwitz pair, 164  
 hyperbolic, 14  
 hyperbolic plane, 26  
 hyperbolic plane, standard, 23  
 hyperplane at infinity, 246

ideal, left, 11  
 ideal, right, 11  
 idempotent, 14  
 idempotent, primitive, 14  
 image, 1  
 immersion, 213, 222  
 immersive, 213  
 increment, 209  
 increment formula, 211  
 index, 37  
 index, Witt, 37, 109  
 induced cover, 197  
 induced topology, 193, 196  
 infinity, hyperplane at, 246  
 infinity, point at, 111  
 inflation, 254  
 injection, 2  
 injective, 2  
 injective criterion for smoothness, 212  
 inverse, 2  
 inverse of a vector, 24  
 inverse function theorem, 211  
 inversion, 254  
 invertible vector, 24  
 involution, algebra, 13  
 involution, companion, 266  
 involution, grade, 135  
 involution of  $\mathbf{H}$ , 64  
 involution, linear, 5  
 involution, main, 135  
 irreducible, 15  
 irreducible semi-linear map, 95  
 isomorphic quadratic spaces, 26  
 isomorphism, semi-linear, 73  
 isotropic quadratic space, 23  
 isotropy subgroup, 21

Jacobi identity, 242

*Index*

Jacobian matrix, 205  
 Jacobson, N., 277, 286  
 James, I.M., 264, 287  
 Jordan algebra, 257  
 juxtaposition, 266

Kaplansky, I., 243, 287  
 kernel, 3  
 kernel of quadratic space, 25  
 kernel rank, 3, 51, 73  
 Kervaire, M., 186, 187, 287  
 Kleinfeld, E., 186, 187, 287  
 Kuiper, N.H., 247, 287

Lagrangian, 112  
 Lawrynowicz, J., x, 164, 287  
 left linear space, 67  
 left-coset exact, 20, 108, 227  
 Leibniz notation, 205  
 length, 39  
 level set, 1, 212  
 Lie algebra, 215, 239, 242  
 Lie bracket, 239  
 Lie group, ix, 100, 103, 215, 231  
 Lie group, exceptional, 277  
 Lie group action, 233  
 Lie group map, 231  
 line complex, 79, 113  
 linear algebra, 9  
 linear map, 2  
 linear map, left, 68  
 linear map, right, 68  
 linear space, 1, 2, 5  
 linear space, left, 67  
 linear space, right, 67  
 linear subspace, 3  
 linear subspace of general linear group, 164  
 linked spheres, 42  
 Liouville, J., 245, 287  
 Liouville's theorem, 245  
 Lipschitz, R., 176, 287  
 Lipschitz chart, 176  
 Lipschitz group, 141  
 Liverpool, the University of, x  
 locally euclidean, 215  
 Loos, O., 270, 279, 287  
 Lorentz group, 238  
 Lounesto, P., ix, 176

Möbius group, 248, 255  
 Möbius map, 246  
 Möbius transformation, 250  
 main involution, 135  
 main involution of  $\mathbf{H}$ , 64  
 Majorana spinors, 166  
 Majs, J., 167, 172, 255, 287  
 manifold, smooth, 215, 217

- manifold, Stiefel, 244, 262  
 manifold, topological, 215  
 map, 1  
 map, affine, 6  
 map, bilinear, 4  
 map, conformal, 245  
 map, Hopf, 258, 259  
 map, identity, 2  
 map, linear, 2  
 map, Möbius, 246  
 map, orthogonal, 25  
 map, semi-linear, 72  
 matrix, 3  
 matrix, Jacobian, 205  
 matrix, quaternionic, 69  
 matrix algebra, full, 9  
 mean value theorem, 211  
 Mennicke, J., 167, 255, 286  
 Meusnier, J.-B.-M.-C., 246, 287  
 Micali, A., viii, 287  
 Milnor, J., 186, 187, 287  
 minimal left ideal, 12, 45, 133  
 module, 5  
 module,  ${}^2\mathbf{K}$ , 5  
 module map, 5  
 modulus of quaternion, 60  
 monoid, 168  
 Montgomery, D., 283, 287  
 Morton, H.R., 277  
 motion, rigid, 173, 281  
 Moufang, R., 190, 287  
 Moufang identities, 190  
  
 negative-definite, 23  
 neighbourhood, 192  
 neutral quadratic space, 23  
 neutral space, 38  
 non-degenerate correlation, 25, 52, 74, 98  
 non-degenerate orthonormal subset, 126  
 non-degenerate quadratic form, 25  
 norm, 191  
 norm, gradient, 194  
 norm, product, 194  
 norm, quadratic, 145  
 norm of quaternion, 60  
 normed division algebra, 179  
 North pole, 40  
 null quadratic space, 23  
 nullity, 3  
  
 octaves, 180  
 octonions, 180  
 open cover, 197  
 open map, 196  
 open set, 191, 192  
 orbit, 21  
 orientation-preserving, 7  
 orientation-reversing, 7  
 orthogonal complement, 32  
 orthogonal decomposition, 32  
 orthogonal group, 27  
 orthogonal group, special, 27  
 orthogonal isomorphism, 26  
 orthogonal map, 25  
 orthogonal quaternionic group, 78  
 orthogonal vectors, 23  
 orthonormal subset, 126  
  
 Paige, L.J., 257, 287  
 paravector, 140  
 partition, 1, 195  
 partner, 10  
 Pauli algebra, 152  
 pentagon, rebracketing, 2  
 periodicity theorem for Clifford algebras, 133  
 Pfaffian, 175  
 Pfaffian, complete, 174  
 Pfaffian chart, 176  
 physics, x  
 physics, theoretical, 165  
 Pin group, 146  
 point at infinity, 111  
 Pontrjagin, L.S., 238, 243, 287  
 Porteous, I.R., ix, x, 14, 191, 202, 287, 288  
 positive-definite, 23, 55  
 positive-definite correlated space, 78  
 power-associative, 179  
 preservation of angles, 245  
 primitive idempotent, 14  
 product, 5  
 product induced by correlation, 74  
 product, algebra, 9  
 product, cross, 60  
 product, direct, 3  
 product, hermitian, 52  
 product, scalar, 22  
 product, tensor, 81  
 product norm, 194  
 product topology, 196  
 projection, 3, 196  
 projection, stereographic, 41, 249  
 projection, tangent, 220  
 projective compactification, 246  
 projective line, 110  
 projective line, Cayley, 187  
 projective plane, 110  
 projective plane, Cayley, 187  
 projective point, 110  
 projective quadric, 112  
 projective space, 110  
 pseudo-determinant, 168

- pure part of quaternion, 58  
 pure quaternion, 58
- quadratic form, 22  
 quadratic norm, 22, 145  
 quadratic space, complex, 49  
 quadratic space, null, 23  
 quadratic space, real, 23  
 quadric, projective, 112  
 quadric Grassmannian, 112  
 quadric Grassmannian, affine, 112  
 quadric Grassmannians as coset spaces, 116  
 quasi-sphere, 106, 229, 248  
 quaternion, pure, 58  
 quaternion product, 58  
 quaternionic group, 78  
 quaternionic group, orthogonal, 78  
 quaternionic group, symplectic, 78  
 quaternions, algebra of, 58  
 quotient, 195  
 quotient topology, 196
- radius, 40  
 Radon, J., 165, 288  
 Radon-Hurwitz sequence, 165  
 Randriamihamon, L.-S., 164, 288  
 rank, 3, 51, 73  
 rank, kernel, 3  
 rank of quadratic space, 25  
 real part of quaternion, 58  
 real quadratic space, 23  
 reciprocity law, 282  
 recognition of subalgebras, 88  
 reducible anti-automorphism, 15  
 reducible automorphism, 15  
 reducible permutation, 15  
 reducible semi-linear map, 95  
 reflection, 34  
 reflection, hyperplane, 34  
 reflexive correlation, 75, 96  
 reflexive scalar product, 43  
 Rembieliński, J., 164, 287  
 representation, adjoint, 238  
 representation, Vahlen, 254  
 reversion, 135, 162  
 reversion of  $H$ , 64  
 Reziprozitätsgesetz, 282  
 right linear space, 67  
 rigid motion, 173, 281  
 ring, 5  
 rotation, 27, 143  
 rotation of  $\mathbb{R}^3$ , 62  
 rotation of  $\mathbb{R}^4$ , 65
- Samelson, H., 283, 287  
 scalar, 2
- scalar product, reflexive, 43  
 scalar product, skew, 43  
 scalar product, symmetric, 22  
 scalar triple product, 61, 182  
 section, 2  
 Segre, B., 13, 288  
 self-adjoint, 53, 75  
 semi-linear map, 51, 72  
 semi-linear maps over  ${}^2K$ , 94  
 semi-neutral, 112  
 semi-neutral quadric Grassmannian, 11  
 semi-orientation-preserving, 36  
 semi-simple, 277  
 Serras, H., x, 285  
 sesqui-linear, 52, 74  
 Shapiro, A., 124, 285  
 signature, 37  
 signature of correlated space, 78  
 signature of hermitian space, 55  
 signature theorem, 36  
 similar, 14  
 simple, 277  
 skew, 16, 52  
 skew correlation, 74, 95  
 skew scalar product, 43  
 skew-adjoint, 53, 75  
 Skornyakov, L.A., 186  
 smooth manifold, 215, 217  
 smooth map, 209, 218  
 smooth structure, 216  
 smooth submanifold, 212  
 smooth subset, 212  
 smooth, infinitely, 209  
 source, 1  
 source of a map, 194  
 South pole, 40  
 space of half-spinors, 134  
 space, linear, 1, 2, 5  
 space of spinors, 133  
 special complex orthogonal group, 50  
 special linear group, 7, 99  
 special orthogonal automorphism, 27  
 special orthogonal group, 27  
 special unitary automorphism, 55  
 special unitary group, 55  
 sphere, 40  
 spheres, conformal, 248  
 sphere, unit, 40  
 Spin group, 146  
 spinor action, 257  
 spinor space, 133, 154  
 Springer, A., 167, 286  
 Spurgeon, C., 277, 288  
 square, twisted, 99  
 square of vector, 22  
 stabiliser, 21  
 standard atlas, 111

- standard chart, 111, 215  
 standard complex symplectic plane, 50  
 standard real symplectic plane, 47  
 Steenrod, N.E., 264, 276, 288  
 stereographic projection, 41, 249  
 Stiefel manifold, 244, 262  
 Stiefel manifold, complex, 265  
 Study, E., ix, 256, 281, 282, 288  
 Study's principle of triality, 282  
 subalgebra, 9  
 subgroup, topological, 225  
 submanifold, 212  
 submanifold, smooth, 212  
 submersion, 214, 222  
 submersive, 214  
 submodule, 5  
 subspace, affine, 6  
 subspace, linear, 3  
 subspace topology, 193  
 sum, direct, 3  
 sum of squares, 36  
 superalgebra, 86  
 superfield, 134  
 surjection, 2  
 surjective, 2  
 surjective criterion, 213  
 swap, 14, 154  
 symplectic, etymology of, 79  
 symmetric correlation, 24, 52, 74, 95  
 symplectic group, complex, 50  
 symplectic group, real, 47  
 symplectic map, 47, 50  
 symplectic plane, complex, 50  
 symplectic plane, real, 47  
 symplectic plane, standard complex, 50  
 symplectic plane, standard real, 47  
 symplectic quaternionic group, 78  
 symmetric second differential, 210  
 symplectic space, complex, 50  
 symplectic space, real, 46  
  
 tangency, 202  
 tangent bundle, 219, 220  
 tangent bundle map, 221  
 tangent bundle space, 219  
 tangent map, 219  
 tangent projection, 220  
 tangent space, 212, 219, 221  
 tangent vector, 221  
 target, 1  
 target of a map, 194  
 Taylor series, 209  
  
 ten, 100, 102, 155  
 tensor algebra, 124  
 tensor product, 81  
 theoretical physics, 165  
 theta triad, 267  
 Tits, J., 282, 288  
 topological group, 225  
 topological group map, 225  
 topological manifold, 215  
 topological space, 191  
 topological subgroup, 225  
 topology, 191  
 topology, induced, 193  
 topology, product, 196  
 topology, quotient, 196  
 topology, subspace, 193  
 transformation, Möbius, 250  
 transitive actions on spheres, 256  
 transitive group action, 21, 226  
 translation, 254  
 transpose, 4  
 transposition, 4, 24  
 Trautman, A., 166, 288  
 triad, theta, 267  
 triality, 256  
 triality, Freudenthal's principle of, 278  
 triality, Study's principle of, 282  
 triality automorphism, 267  
 triangle, Cayley, 184  
 triangle, Hamilton, 183  
 triangle inequality, 192  
 triple product, scalar, 61, 182  
 Tucker, R.W., 166, 285  
 twisted square, 99  
 type of orthonormal subset, 126  
  
 unit quaternion, 60  
 unitary group, 55, 104  
 unitary linear map, 55  
 universal Clifford algebra, 130  
  
 Vahlen, K.Th., 167, 254, 288  
 Vahlen representation, 254  
 van der Waerden, B.L., 176, 288  
 vector, 2, 140, 167  
  
 Wall, C.T.C., x, 288  
 Waterman, P.L., 167, 288  
 Weyl, H., 79, 243, 288  
 Weyl spinors, 166  
 Witt decomposition, 37, 109  
 Witt index, 37, 109