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The Clifford algebras of real quadratic forms and their complexifications are studied here in detail, and those parts which are immediately relevant to theoretical physics are seen in the proper broad context.

Central to the work is the classification of the conjugation and reversion anti-involutions that arise naturally in the theory. It is of interest that all the classical groups play essential roles in this classification. Other features include detailed sections on conformal groups, the eight-dimensional non-associative Cayley algebra, its automorphism group, the exceptional Lie group G_2, and the triality automorphism of Spin 8.

The book is designed to be suitable for the last year of an undergraduate course or the first year of a postgraduate course.

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Postscript: On Saturday, 6th May, 1995, the 150th anniversary of the birth of William Kingdon Clifford (1845–1879), a Celebration of this life and that of his wife Lucy (1846–1929) was held at the University of Kent at Canterbury. There we learned that Clifford was not only theoretically but also athletically expert at rotations! Dave Chisholm's delightful portrayal of Clifford performing his 'corkscrew' at Cambridge in 1869 was a foil on that occasion to an entrancing exhibition of Victorian cartoons.

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Ian R. Porteous
University of Liverpool

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Foreword

This book's parent *Topological Geometry* (Porteous (1969)), originally written in the 1960's to make propaganda for a basis-free approach to the differential calculus of functions of several variables, contained, almost by accident, a central section on Clifford algebras, a generalisation of quaternions that was at that time little known. This section was strengthened in the second edition (Porteous (1981)) by an additional chapter on the triality outer automorphism of the group $Spin(8)$, a feature which illuminates the structure of several of the other Spin groups and which is related to a property of six-dimensional projective quadrics first noticed almost a hundred years ago by Study in work on the rigid motions of three-dimensional space.

In recent years Clifford algebras have become a more popular tool in theoretical physics and it seems therefore appropriate to rework the original book, summarising the linear algebra and calculus required but expanding the Clifford algebra material. This seems the more worth while since it is clear that the central result of the old book, the classification of the conjugation anti-involution of the Clifford algebras $\mathbf{R}_{p,q}$ and their complexifications, was dealt with too briefly to be readily understood, and some of the more recent treatments of it elsewhere have been less than complete.

As in the previous version, the opportunity has been taken to give an exhaustive treatment of all the generalisations of the orthogonal and unitary groups known as the classical groups, since the full set plays a part in the Clifford algebra story. In particular, perhaps surprisingly, one learns to think of the general linear groups as unitary groups. Toward the end of the book the classical groups are presented as Lie groups and their Lie algebras are introduced. The exceptional Lie group G_2 also makes an appearance as the group of automorphisms of the Cayley algebra, a

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non-associative analogue of the quaternions that plays an essential role in the discussion of triality.

I owe a great debt not only to colleagues and students at the University of Liverpool over the years but also to new found friends at the by now regular international meetings on Clifford algebras and their applications to problems of mathematical physics, whose Proceedings have been published as Chisholm and Common (1986), Micali, Boudet and Helmstetter (1991) and Brackx, Delanghe and Serras (1993).

My interest in Clifford algebras and their use in physics was originally stimulated by discussions with my colleague at Liverpool Bob Boyer, tragically killed by a madman's bullet on the campus of the University of Austin, Texas, on August 1, 1966. Explicit classifications of both the conjugation and the reversion anti-involutions in the tables of Clifford algebras in Chapter 17 are in a Liverpool M.Sc. thesis by Tony Hampson (1969). On obtaining the answers Hampson and I wrote to my colleague Terry Wall, who was at that time on a visit to Mexico. He replied by drawing our attention to his paper (1968) which we had not read, and which presented the entire theory very succinctly! For the classical groups my main debt is to Prof. E. Artin's classic *Geometric Algebra* (1957). The observation that the Cayley algebra can be derived from one of the Clifford algebras I owe to Michael Atiyah, while the method adopted in Chapter 22 for constructing the Lie algebras of a Lie group was outlined to me by Frank Adams.

In preparing this fresh version of the material I am hugely indebted to Pertti Lounesto who has read much of the book in draft and over the years has kept me right on many points of detail. His knowledge of the history of the subject is unsurpassed. Much more recently, Chapter 23 on the conformal groups owes much to Jan Cnops, as is there acknowledged, and to the hospitality of Julius Ławrynowicz and the Banach Institute in Warsaw in 1994. An early version of some of the material of Chapter 17 has appeared as Porteous (1993).

Finally a disclaimer! I am no physicist, and therefore the reader will search in vain for particular applications to physics. On the other hand, works that are strongly biased toward applications frequently give only a fragmented and partial view of the subject. It is my belief that the subject only makes sense when the full picture is unfolded, and some of the otherwise confusing details are seen naturally to fall into place.

Ian Porteous, Liverpool, January, 1995

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