

## CONTENTS

<b>Preface</b>	xi
<b>Introduction</b>	
1. The Weyl algebra	1
2. Algebraic $D$ -modules	3
3. The book: an overview	5
4. Pre-requisites	6
<b>Chapter 1. The Weyl algebra</b>	
1. Definition	8
2. Canonical form	9
3. Generators and relations	10
4. Exercises	12
<b>Chapter 2. Ideal structure of the Weyl algebra.</b>	
1. The degree of an operator	14
2. Ideal structure	16
3. Positive characteristic	17
4. Exercises	18
<b>Chapter 3. Rings of differential operators.</b>	
1. Definitions	20
2. The Weyl algebra	22
3. Exercises	24
<b>Chapter 4. Jacobian Conjecture.</b>	
1. Polynomial maps	26
2. Jacobian conjecture	28
3. Derivations	30
4. Automorphisms	32
5. Exercises	34
<b>Chapter 5. Modules over the Weyl algebra.</b>	
1. The polynomial ring	36
2. Twisting	38
3. Holomorphic functions	40
4. Exercises	41

<b>Chapter 6.</b> Differential equations.	
1. The $D$ -module of an equation	44
2. Direct limit of modules	46
3. Microfunctions	48
4. Exercises	50
<b>Chapter 7.</b> Graded and filtered modules.	
1. Graded rings	53
2. Filtered rings	55
3. Associated graded algebra	57
4. Filtered modules	59
5. Induced filtration	60
6. Exercises	62
<b>Chapter 8.</b> Noetherian rings and modules.	
1. Noetherian modules	65
2. Noetherian rings	67
3. Good filtrations	70
4. Exercises	72
<b>Chapter 9.</b> Dimension and multiplicity.	
1. The Hilbert polynomial	74
2. Dimension and multiplicity	77
3. Basic properties	80
4. Bernstein's inequality	82
5. Exercises	84
<b>Chapter 10.</b> Holonomic modules.	
1. Definition and examples	86
2. Basic properties	88
3. Further examples	91
4. Exercises	95
<b>Chapter 11.</b> Characteristic varieties.	
1. The characteristic variety	97
2. Symplectic geometry	100
3. Non-holonomic irreducible modules	104
4. Exercises	106
<b>Chapter 12.</b> Tensor products.	
1. Bimodules	108
2. Tensor products	109
3. The universal property	111

<i>Contents</i>	ix
4. Basic properties	113
5. Localization	117
6. Exercises	119
<b>Chapter 13.</b> External products.	
1. External products of algebras	121
2. External products of modules	122
3. Graduations and filtrations	124
4. Dimensions and multiplicities	127
5. Exercises	128
<b>Chapter 14.</b> Inverse Image.	
1. Change of rings	130
2. Inverse images	132
3. Projections	134
4. Exercises	135
<b>Chapter 15.</b> Embeddings.	
1. The standard embedding	138
2. Composition	139
3. Embeddings revisited	142
4. Exercises	144
<b>Chapter 16.</b> Direct images.	
1. Right modules	146
2. Transposition	147
3. Left modules	150
4. Exercises	153
<b>Chapter 17.</b> Kashiwara's theorem.	
1. Embeddings	154
2. Kashiwara's theorem	156
3. Exercises	160
<b>Chapter 18.</b> Preservation of holonomy.	
1. Inverse images	162
2. Direct images	165
3. Categories and functors	166
4. Exercises	170
<b>Chapter 19.</b> Stability of differential equations.	
1. Asymptotic stability	171
2. Global upper bound	173
3. Global stability on the plane	176
4. Exercises	178

x	<i>Contents</i>	
<b>Chapter 20.</b>	Automatic proof of identities.	
1.	Holonomic functions	179
2.	Hyperexponential functions	180
3.	The method	183
4.	Exercises	186
<b>Coda</b>		188
<b>Appendix 1.</b>	Defining the action of a module using generators	191
<b>Appendix 2.</b>	Local inversion theorem	194
<b>References</b>		197
<b>Index</b>		203