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A Primer of Algebraic *D*-modules

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For Sérgio Montenegro

CONTENTS

PREFACE

As its title says, this book is only a primer; in particular, you will learn very little 'grammar' from it. That is not surprising; to speak the language of algebraic D-modules fluently you must first learn some algebraic geometry and be familiar with derived categories. Both of these are beyond the bounds of an elementary textbook.

But you can expect to know the answers to two basic questions by the time you finish the book: *what are D-modules?* and *why D-modules?* It is particularly easy to answer the latter, because D-module theory has many interesting applications. Hardly any area of mathematics has been left untouched by this theory. Those that have been touched range from number theory to mathematical physics.

I have tried to include some real applications, but they are not by any means the ones that have caused the greatest impact from the point of view of mathematics at large. To some, they may even seem a little eccentric. That reflects two facts. First, and most important, this is an elementary book. The most interesting applications (to singularity theory and representations of algebraic groups, for example) are way beyond the bounds of such a book. Second, among the applications that were elementary enough to be presented here, I chose the ones that I like the most.

The pre-requisites have been kept to a minimum. So the book should be accessible to final year undergraduates or first year post-graduates. But I have made no effort to write a book that is 'purely algebraic'. Such a book might be possible, but it would not be true. One of the attractions of the theory of D-modules is that it sits across the traditional division of mathematics into algebra, geometry and analysis. It would be a pity to lose that. So this is a book about mathematics, and in it you will find algebras and modules, differential equations and special functions, all in easy conviviality. The introduction contains a detailed description of the pre-requisites.

While writing this book I often worried about the language. I am too fond of English not to shiver at the idea of badly abusing it. But English is not my first language, and I am also well aware of my deficiencies. I can only hope that the many revisions have spared me from sharing the fate of the master of the brigantine in Conrad's *Lord Jim* whose 'flowing English seemed to be derived from a dictionary compiled by a lunatic'.

The material in this book is not original. I have only tried to present the foundations of D-module theory for the beginner that I was, when I started working on the subject ten years ago. In a sense this book is only a compilation: while writing it I have tapped many sources. I have truly plundered the literature for results and exercises to be included in the book. Since it would be very difficult to give references for all of these, I have limited my attributions to the main results and applications.

The book grew out of notes for a basic course on algebraic D-modules taught at the Federal University of Rio de Janeiro. The constant questioning of the students greatly improved the exposition, and directed me to many interesting examples. Many people offered hints, provided references or explained some of the topics to me, particularly G. Meisters, C. Gutierrez, S. Toscano de Melo and A. Pacheco. M.B. Alves and D. Levcovitz read parts of the book and contributed many useful suggestions and corrections. D. Tranah and R. Astley, at Cambridge University Press, put up patiently with the naive questions of a novice and offered many helpful suggestions. Martin Holland read most of the book. His comments saved me from many mistakes while helping to make the exposition clearer; and his unflagging support and friendship kept me at work and made the book possible. To Andrea I owe whatever else was life.

The work in this book was partially supported by a CNPq grant, and benefited from the excellent working conditions generously provided by the Institute of Pure and Applied Mathematics (IMPA).

Rio de Janeiro, April 1995.