

# Index

- $(\Omega, \mathbf{P})$ , 178  
 $(n, \epsilon)$ -spanning set, 118  
 $2^\infty$  map, 40, 66, 134, 138, 218  
 $A^c$ , 1  
 $B(S, \epsilon)$ , 2  
 $B(x, \epsilon)$ , 2  
 $C^r$  smooth, 34  
 $F_J(x)$ , 205  
 $F_l$ , 107  
 $F_u$ , 107  
 $Ho$ , 101  
 $J'$ , 111  
 $J''$ , 111  
 $J(R_\rho)$ , 111  
 $K_A$ , 238  
 $K_B$ , 239  
 $K_C$ , 239  
 $P_n$ , 204  
 $R_\alpha$ , 103  
 $Rot(F)$ , 106  
 $S^1$ , 114  
 $T_\beta$ , 195  
 $[a_0, a_1, \dots, a_N]$ , 96  
 $\#A$ , 1, 184  
 $\mathbb{C}$ , 251  
 $\Delta$ , 232  
 $\mathbb{N}$ , 1  
 $\Omega$ , 179  
 $\Omega_0$ , 181  
 $\Omega_\infty$ , 69  
 $\mathbf{P}$ , 217  
 $\mathbf{P}(\omega)$ , 181  
 $\Phi_g$ , 199  
 $\Pi_\rho$ , 184  
 $\mathbb{Q}$ , 1  
 $\mathbb{R}/\mathbb{Z}$ , 101  
 $\mathbb{R}$ , 1  
 $\overline{\mathbb{C}}$ , 251  
 $\Sigma$ , 188  
 $\mathbb{Z}$ , 1  
 $\overline{\Phi}_g$ , 199  
 $\overline{\varphi}_g$ , 199  
 $\beta$ -transformation, 194  
 $\delta$ -pseudo orbit, 85  
 $\epsilon$ -shadows, 118  
 $\epsilon$ -ball, 2  
 $\epsilon$ -shadowed, 85  
 $\gamma(\omega)$ , 232  
 $\hat{x}$ , 44  
 $\langle \mathbb{N} \rangle$ , 179  
 $\langle a; b \rangle$ , 1  
 $\langle n \rangle_k$ , 179  
 $\lfloor x \rfloor$ , 98  
 $\omega$ -limit set, 25, 153, 218  
 $\omega_n(a)$ , 157  
 $\overline{Rot}(F)$ , 106  
 $\overline{\rho}_F$ , 106  
 $\|x\|$ , 184  
 $\varphi_\beta$ , 196  
 $\varphi_g$ , 198  
 $\pi$ , 101, 182, 243  
 $\psi$ , 197, 208  
 $\rho(S, T)$ , 2  
 $\succeq_L$ , 78  
 $\tau_\beta$ , 195  
 $Const(F)$ , 108  
 $\underline{Rot}(F)$ , 106  
 $\underline{\rho}_F$ , 106  
 $\{0, 1\}^{\mathbb{N}}$ , 9  
 $\{G_k\}$ -odometer, 184

- $\{S_k\}$ -expansion, 181
- $\{S_k\}$ -odometer, 181
- $\{\{x\}\} = x$ -round( $x$ ), 184
- $a(R_\rho)$ , 111
- $f_2$ , 193
- $f_\beta$ , 194
- $g_a$ , 67
- $h_i$ , 204
- $h_{top}(g)$ , 203
- $n$ -fold accessible, 257
- $n$ -path, 132
- $n$ -to-one, 54
- $\mathbb{Z}^2$ -shift, 208
- $\mathcal{C}$ , 245
- $\mathcal{P}$ , 157
- $\mathcal{R}$ , 111
- FT, 93
  
- a.e., 18
- absolutely continuous, 55
- absorbing Cantor set, 173
- abstract Hubbard tree, 270
- accumulation point, 3
- action, 79
- add and carry, 181
- adding machine, 66, 70, 178, 181
- additive, 14
- address, 27
- Adler, 194
- admissible, 78, 151
- almost everywhere, 18
- angle doubling map, 193
- attracting, 23
- attractor, 172
- Axiom of Choice, 15
  
- Baire Category Theorem, 9
- barn map, 144
- basin, 172
- basin of  $\infty$ , 252
- Bernoulli shift, 211
- binary expansion, 194
- binary graph, 135
- Birkhoff Ergodic Theorem, 62
  
- Bolzano-Weirstrass Theorem, 5
- Borel measurable, 53
- Borel set, 53
- boundary, 3
- bounded variation, 115
- bounded-to-one, 54
- branch, 75
- branch point, 267
  
- Cantor set, 7, 17, 80, 114, 198, 216, 217
- cardinality, 184
- CE condition, 169
- center, 264
- central branch, 75
- characteristic function, 60
- characteristic point, 277
- characteristic polynomial, 134
- circle rotation, 115
- closed, 3
- closest precritical point, 75
- closure, 3
- co-cutting time, 165
- Collet-Eckmann condition, 169
- compact, 5
- complete measure, 18
- complex plane, 251
- complexity, 124
- complexity function, 123
- component, 263
- condensation point, 8
- Condition A, 238
- Condition B, 239
- Condition C, 239
- conformal mapping, 254
- conjugacy, 34, 54, 105, 120, 140, 169, 175, 178, 193, 196, 198, 203, 248, 251, 273
- connected, 4
- conservative, 57
- continued fraction algorithm, 98
- continued fraction expansion, 97, 111, 186

- continuous, 4
- convergent, 96
- converges, 3
- core, 38, 74, 268
- Coulet Tresser map, *see*  $2^\infty$  map
- countable, 7
- countable additive, 14
- countable subadditive, 13
- critical point, 38, 251
- critical value, 251
- critically finite, 170
- cutting time, 75
- cyclic, 274
- cylinder, 124
  
- degree one lift, 106
- Denjoy, 115
- dense, 7, 218
- depth, 33
- diffeomorphism, 115
- disconnected, 4
- distortion, 147, 158, 166
- Dominated Convergence Theorem, 62
- dyadic adding machine, 66, 71, 210
- dyadic intervals, 64
- dyadic rationals, 204
  
- ergodic, 59
- even parity, 174
- even return, 86
- eventually maximal, 187
- external angle, 250, 254, 255, 258
- external ray, 250, 251, 254, 266
  
- factor, 35, 42, 106, 120, 178, 184, 186, 198
- Farey arithmetic, 92
- Farey neighbors, 92
- Farey parents, 93
- Farey sequence, 94
- Farey sum, 92
- Farey tree, 92, 93, 98
- Fatou component, 263
  
- Fatou set, 253
- Fatou's Lemma, 62
- Feigenbaum map, *see*  $2^\infty$  map
- Feigenbaum sequence, 73
- Fibonacci combinatorics, 76
- Fibonacci map, 132
- Fibonacci sequence, 76, 179
- Fibonacci shift, 48
- filled-in Julia set, 253
- finite continued fraction, 96
- first return map, 111
- flip-half-of-the-graph trick, 195, 197
- flipping, 196
- foliation of external rays, 254
- frame, 87
- full- $n$  shift, 47
  
- golden mean shift, 48
- greatest common divisor, 94
  
- Heine-Borel Theorem, 6
- Hofbauer tower, 75, 80, 130, 147, 162, 181, 241
- homeomorphism, 6, 11, 25, 34, 47, 53, 72, 101, 114, 119, 138, 197, 202, 216, 220, 238, 260, 270
- homogeneous, 7
- homterval, 82, 148, 181
- Hubbard tree, 251, 267
- hyperbolic, 263
  
- incompressible, 57
- indicator function, 60
- infinite continued fraction, 97
- infinitely Poincaré recurrent, 57
- infinitely renormalizable, 41, 68, 173, 216
- integer part, 98, 104, 194
- interior, 3
- internal address, 250, 264
- invariant, 25
- irrational rotation, 101, 106, 111, 184

## INDEX

295

- irreducible, 50, 124
- isolated, 33
- isometry, 119
- isomorphism, 52, 54, 193, 203, 211
- itinerary, 27, 67, 68, 76, 77, 127, 148, 159, 193, 196, 201, 204, 212, 258, 266
- Julia set, 250, 253, 266
- kink, 145
- kneading invariant, 144
- kneading map, 75, 78, 79, 81, 85, 87, 184, 186, 198, 217, 221, 249
- kneading sequence, 77, 147, 149, 151, 153, 158, 168, 176, 212, 249, 250, 266
- Kolmogorov, 211
- lands, 255
- lapnumber, 129, 142, 203
- Lebesgue, 12
- Lebesgue Covering Lemma, 6
- Lebesgue integral, 61
- Lebesgue measurable, 14
- Lebesgue measure, 12
- Ledrappier's three-dot example, 208
- left child, 93
- left parent, 93
- leo, 38, 82, 140, 153, 204
- lexicographical order, 78
- lift, 101, 199
- lift-orbit, 108
- lifted  $m$ -cycle, 108
- lifted cycle, 108
- locally connected, 253
- locally constant, 108
- locally eventually onto, 35, 38
- locally expanding, 219
- locally uncountable, 85, 87, 218
- logistic family, 20, 66, 80, 145, 173, 216, 221
- longbranched, 80, 85, 155, 171, 198, 199
- Lorenz map, 196
- lower map, 107
- main cardioid, 263
- Mandelbrot set, 250, 257
- Markov chain, 140
- Markov extension, 129, 130, 137
- Markov partition, 213
- matrix norm, 125
- measurable, 14, 53
- measurable function, 53
- measure preserving, 55, 56, 63, 65, 205
- mediant, 92
- metric attractor, 173
- metric space, 1
- Middle Third Cantor set, 7, 9, 12, 16, 53
- minimal mapping, 217, 235
- minimal set, 43, 72, 80, 114, 155, 173, 186, 198, 216, 217, 219, 220, 238
- mixing, 208
- Monotone Convergence Theorem, 62
- monotone decreasing, 3
- monotone increasing, 3
- Montel's theorem, 256
- nesting, 243
- nice, 44
- nonflat, 38, 80, 168
- nonrenormalizable, 41, 80, 147, 176, 216, 238
- nonsingular, 57
- nonwandering point, 32
- nonwandering set, 37
- nonwandering set for  $f$ , 32, 149
- norm, 251
- normal number, 64
- nowhere dense, 4
- null set, 18

- odd parity, 174
- odd return, 86
- odometer, 181, 184
- onto homeomorphism, 6
- open, 3
- orientation preserving, 26, 101, 274
- Ornstein, 211
- outer Lebesgue measure, 13
  
- parity-lexicographical order, 68, 89, 148
- Parry, 140, 194
- Pascal's triangle, 209
- perfect, 4
- periodic, 50
- periodic point, 20
- persistently recurrent, 44, 56, 80, 173
- phase portrait, 157
- piecewise monotone, 129
- planar, 274
- Poincaré, 52, 104
- Poincaré recurrent, 56
- polar angle, 251
- preperiodic, 21
- primitive, 50
- probability matrix, 213
- property drop, 231
  
- quadratic family, 20
- queer component, 264
  
- Radon-Nykodým derivative, 214
- rational rotation, 92, 106
- rectangular diagram, 208
- recurrent, 25, 35, 42, 56, 68, 154, 197, 200, 207
- renormalizable, 35, 40, 78, 153, 174
- Renyi, 194
- repelling, 23, 28, 83, 165, 256
- residual, 173
- restrictive interval, 35, 39, 143, 174
- Riemann integration, 12
- Riemann sphere, 251
  
- right child, 93
- right parent, 93
- rigid rotation, 92, 95, 101, 103, 105, 117, 119, 184, 190, 198
- rome, 135, 136
- rome matrix, 135
- rotation number, 103, 109, 114, 199
- round( $x$ ), 184
  
- S-unimodal, 168
- Schwarzian derivative, 34, 144, 145, 168
- semiconjugate, 35, 105, 120, 140, 196, 200, 254
- semimetric, 2
- sensitive dependence, 117
- separable, 7
- set of measure zero, 18
- SFT, 48
- shadowing, 74, 85
- shift map, 47, 123, 260
- shift maximal, 151, 154
- shift of finite type, 48, 123, 214
- shift space, 47, 123, 156
- shift-invariant, 47
- simple function, 60
- skeleton, 87
- slowly recurrent, 147, 169
- Smorodinsky, 194
- spectral radius, 125
- spine, 257
- star product, 174
- stereographic projection, 252
- Stern-Brocot tree, 93
- strictly unimodal, 38
- strongly invariant, 25, 43
- strongly transitive, 37
- subadditive, 13, 122
- submultiplicative, 122
- subshift, 123
- symmetric family of tent maps, 20, 34, 39, 216
- symmetric trapezoidal map, 221

## INDEX

297

- ternary expansion, 10
- three-dot pattern, 208
- Thurston's nonwandering triangle lemma, 270
- Toeplitz sequence, 73
- topological attractor, 173
- topological conjugacy, *see* conjugacy
- topological entropy, 21, 48, 119, 124, 129, 141, 144, 170, 177, 203, 211
- topologically transitive, 35
- totally disconnected, 4
- transition graph, 48
- transition matrix, 48, 213
- transitive point, 50
- trapezoidal, 220
- tree, 267
- turning point, 38
- twice renormalizable, 41
- twist lifted  $m$ -cycle, 108
- twisted lift orbit, 199
- twisted lift-orbit, 108
  
- uniformly continuous, 6, 81, 120, 121
- uniformly recurrent, 43, 68, 73, 249
- unimodal, 35, 38, 43, 44, 58, 74, 80, 86, 87, 92, 106, 129, 142, 144, 147, 168, 172, 174, 178, 184, 186, 193, 197, 204, 216, 225, 238, 250
- uniquely ergodic, 64
- upper map, 107
  
- variation, 139
  
- wandering interval, 35, 37, 80, 114, 115, 147, 202, 217, 238
- wandering set, 37, 57